2.80

Solución

Ejercicios Propuestos

Capitulo Uno

1.- Las proposiciones no pueden ser preguntas, exclamaciones, ni expresiones poeticas

a) Si
b) No
c) Si
d) No
e) Si
f) Si
g) Si
h) Si
i) Si
j) Si
K) No
l) Si

2.-
a) Si
b) Si
c) Si
d) Si
e) No

3.-
a) No
b) No
c) No
d) No
e) Si

4.-
a) Si
b) Si
c) No
d) Si
e) Si

5. a) Sí
b) Sí
c) Sí
d) No
e) Sí

6.
I: No
II: Sí
III: Sí
IV: No
d) Correcto

7.
a) Si Elizabeth cumple sus obligaciones, entonces no es verdad que si aprueba el examen, no se va de vacaciones o trabaja.

b) Elizabeth aprueba el examen y no es verdad que decir que Elizabeth trabaja es igual a decir que no cumple con sus obligaciones ; ó ; si Elizabeth se va de vacaciones o trabaja, entonces trabaja y no come.

c) Si Elizabeth se va de vacaciones, entonces, Elizabeth cumple sus obligaciones es igual a decir que es trabajadora y.

d) Elizabeth cumple sus obligaciones y aprueba el examen es igual a decir que se va de vacaciones o si trabaja, come.

8.-

a.-) Si como espinaca y la lógica es fácil, es igual a decir me divierto con esta lección.

b.-) Si la lógica es fácil y me divierto con esta lección, entonces como espinaca

c.-) Si no como espinaca, entonces, la lógica es difícil o no me divierto con esta lección

9.-

a	v	b
0	0	0

a) Verdadero

10.- $\neg(\underbrace{\underbrace{a \vee b}_{0}}_{1})$ $a: 0$ $b: 0$

b) Falso

11.- p: estudio consentemente

q: apruebo el curso nivel cero

Traducción: $p \Rightarrow q$

contrarrecíproca $\neg q \Rightarrow \neg p$ si no apruebo el curso, no estudio consient.

$\therefore q \vee \neg p$ No estudio concientemente o apruebo el curso

b) Falso

12.- a) p: la decisión depende del juicio
q: la decisión depende de la intuición
r: la decisión depende del dinero

Traducción: $(p \vee q) \wedge \neg r$

b) p: iré al estadio
q: iré al cine
r: consigo dinero

Traducción: $r \Rightarrow (p \wedge q)$

c) p: el sol brilla
q: es el día del amor

Traducción: $q \Rightarrow p$

d) p: Juan le agrada el ejercicio
q: Juan no puede resolver el ejercicio

Traducción: $q \Rightarrow \neg p$

13.- a: la información es correcta
b: existe un incremento en los costos de producción
c: el analista tiene un error de apreciación.

Traducción ①: $\neg a \Rightarrow (b \vee c)$

Traducción ②: $\neg(b \vee c) \Rightarrow a$

14.-

a) p: Quito es capital de Argentina.

q: Buenos Aires es capital de Ecuador.

traducción: $$\underbrace{\underset{0}{p} \vee \underset{0}{q}}_{0} \equiv 0$$

b) p: 5 es menor que 10

q: 8 no es un número primo.

traducción: $$\underbrace{\underset{1}{p} \wedge \underset{1}{q}}_{1} \equiv 1$$

c) p: $9 - 16 = (3-4)(3+4)$

q: $(-5)(-2) > 0$

Traducción: $$\underbrace{\underset{1}{p} \vee \underset{1}{q}}_{1} \equiv 1$$

15.-

a) p: $2(3+5) = 16$

q: $5(6+1) = 35$

Traducción: $$\underbrace{\underset{1}{p} \Rightarrow \underset{1}{q}}_{1} \equiv 1$$

b) $p: (4+5) = 20$
$q: (6+7) = 12$

Traducción : $p \Rightarrow q \equiv 1$
0 0
1

c) $p: (9+5) = 14$
$q: (6+5) = 11$

Traducción : $p \Rightarrow q \equiv 1$
1 1
1

d) $p: 9(4+2) = 54$
$q: 9(9+1) = 14$

Traducción : $p \Rightarrow q \equiv 0$
1 0
0

e) $p: 3(4+5) = 28$
$q: 7(6+5) = 37$

Traducción : $p \Rightarrow q \equiv 1$
0 0
1

16.- p: Carlos llega impuntual
q: Carlos se levanta tarde

Traducción : $q \Rightarrow p$
recíproca : $p \Rightarrow q$

Si Carlos llega impuntual, entonces se levanta tarde

b) Correcto

17.– m: tu eres inteligente.
n: tu actuas con prudencia.
p: tú eres. un ignorante en la materia.

traducción: $(m \wedge \neg n) \Rightarrow p$.

$\therefore \neg(m \wedge \neg n) \vee p$.

$\therefore \neg m \vee n \vee p$.

$\therefore \neg m \vee (n \vee p)$.

$\therefore m \Rightarrow (n \vee p)$ a) Correcto.

18.– p: tengo hambre
q: no tengo tiempo para comer
r: no me siento bien
s: no puedo estudiar

traducción: $(p \wedge q) \Rightarrow (r \wedge s)$

contrarecíproca: $\neg(r \wedge s) \Rightarrow \neg(p \wedge q)$

$\therefore (\neg r \vee \neg s) \Rightarrow (\neg p \vee \neg q)$

Si me siento bien o puedo estudiar, entonces no tengo hambre o tengo tiempo para estudiar

e) correcto

19.– p: El pais está bien economicamente
q: yo tengo empleo

traducción: $\neg p \vee q$

$\therefore p \Rightarrow q$ → condición necesaria

b) correcto

20.

p: es función diferenciable.

q: es función continua

Traducción: $p \Rightarrow q$.

a) $p \Rightarrow q$.

b) $q \Rightarrow p$

c) $q \Rightarrow p$

d) $p \Rightarrow q$.

e) NO

21.

I) p: Compro un traje gris

q: Uso el traje gris.

r: me pagan el traje gris.

traducción: $r \Rightarrow (p \wedge q)$.

recíproca: $(p \wedge q) \Rightarrow r$ a). Correcto

inversa: $\neg r \Rightarrow \neg(p \wedge q)$.

$\neg r \Rightarrow (\neg p \vee \neg q)$ d) Correcto

Contrarecíproca: $\neg(p \wedge q) \Rightarrow \neg r$.

$(\neg p \vee \neg q) \Rightarrow \neg r$. b) Correcto

22.

a: Hoy es lunes.

b: Obtengo un buen resultado.

Traducción: $a \Rightarrow b$.

b) Falso.

23.- m: la municipalidad realiza obras.

n: los ciudadanos colaboren en el aseo de las calles

p: Guayaquil mejora su imagen.

Traducción: $(m \vee n) \Rightarrow p$.

b) Falso

24.- a: Utilizo mis habilidades Matemáticas.

b: resuelvo bien los ejercicios

c: hago un buen examen.

Traducción: $(b \wedge c) \Rightarrow a$

b) Falso.

25.- m: Mis padres me compran un carro

n: me porto bien.

p: apruebo este curso.

traducción: $m \Rightarrow (n \wedge p)$.

b) Correcto.

26.- $\neg(p \wedge \neg q \wedge \neg r) \equiv 0$

$(\underbrace{p}_{1} \wedge \underbrace{\neg q}_{1} \wedge \underbrace{\neg r}_{1}) \equiv 1$

p: 1
q: 0
r: 0

$\underbrace{\underbrace{p}_{1} \Rightarrow \underbrace{(\underbrace{q}_{0} \wedge \underbrace{r}_{0})}_{0}}_{0} \equiv 0$

b) Correcto.

27. m: Viajo al exterior.

n: apruebo el curso de nivel cero

p: obtengo una beca

Traducción: $m \Rightarrow (n \wedge p)$

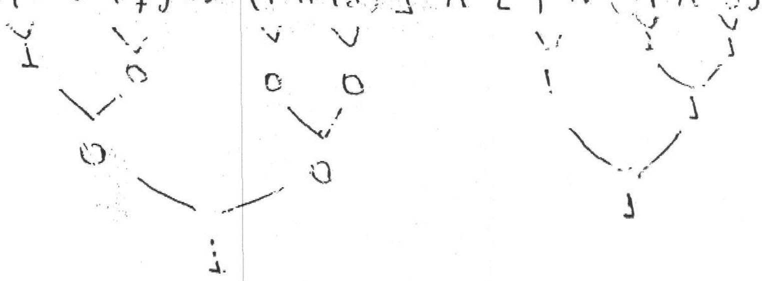
$\therefore \neg(n \wedge p) \Rightarrow \neg m$

$\therefore (n \wedge p) \vee \neg m$

d) Correcto

28.

$$[(p \Rightarrow \neg q) \Rightarrow (r \wedge \neg s)] \wedge [p \wedge (\neg r \wedge s)] \equiv 1$$



$p: 1$
$r: 0$
$s: 1$
$q: 1$

a) Falso
b) Verdadero
c) Falso
d) Verdadero
e) Falso

29.

a: te gustan las matematicas.

b: te gusta este deber.

a) No gustas de las matematicas ó gustas del deber

b) Gustas de las matematicas y no gustas del deber.

c) Si gustas de las matematicas, gustas del deber.

d). Si no gustas del deber, no gustas de las matemáticas

e) Si gustas o no de las matematicas, gustas del deber.

30.-

p: Necesito un doctor.
q: Necesito un abogado.
r: Tengo un accidente
s: estoy enfermo.

Traducción: $(s \Rightarrow p) \wedge (r \Rightarrow q)$.

b) Correcto.

31.-

p: la guerra se detiene.
q: sigo estudiando
r: sigo trabajando

Traducción: $p \Rightarrow (q \vee r)$.

Negación: $\neg[p \vee \neg(q \vee r)]$
: $\neg(\neg p \vee (q \vee r)]$
: $\neg(\neg p \vee q \vee r)$
: $p \wedge \neg q \wedge \neg r$.

e) Correcto.

32.-

p: Pedro realizó un paseo en grupo
q: Pedro preparó el mejor informe de la clase
r: Yo encontré a Pedro visitando el CC. San Marino.

Traducción: $r \Rightarrow \neg(p \wedge q)$

$r \Rightarrow (\neg p \vee \neg q)$

b) Correcto.

33.- p : Hoy es domingo.

q : Tengo que estudiar teorías de aprendizaje.

r : Aprobaré el curso.

Traducción : $(p \wedge q) \vee \neg r$.

$\neg r \vee (p \wedge q)$.

$r \Rightarrow (p \wedge q)$.

d) Correcto.

34.- a : Luis llega a tiempo.

b : Luis se levanta temprano.

c : Luis desayuna

Traducción : $(a \wedge c) \Rightarrow b$.

$\neg(a \wedge c) \vee b$.

$\neg a \vee \neg c \vee b$.

d) Correcto

35.-

m : Se realiza una gran fiesta.

n : Salgo bien en esta lección

p : Mis amigos estén de acuerdo.

Traducción : $m \Rightarrow (n \wedge p)$

$\therefore \neg m \vee (n \wedge p)$.

$\therefore (n \wedge p) \vee \neg m$.

d) Correcto

36.- p: Estudio historia.

q: Estudio geografía.

r: estudio matematicas.

Traducción: $(p \vee q) \Rightarrow r$

$\therefore\ \neg(p \vee q) \vee r$

$\therefore\ (\neg p \wedge q) \vee r$

$\therefore\ (\neg p \vee r) \wedge (\neg q \vee r)$

$\therefore\ (p \Rightarrow r) \wedge (q \Rightarrow r)$

a) Correcto.

37.- $\overbrace{[\overbrace{(\underset{1}{a} \wedge \underset{1}{\neg b})}^{1} \Rightarrow \overset{0}{\underset{0}{d}}]}^{0} \vee \overbrace{\neg(\underset{0}{d} \vee \underset{1}{e})}^{0} \equiv 0$

a) $\underset{0}{b} \vee \underset{1}{g} \equiv 1$

b) $\underset{0}{\neg e} \vee \underset{1}{\neg d} \equiv 1$

c) $\underset{0}{d} \vee \underset{1}{g} \equiv 1$

d) $\underset{1}{a} \Rightarrow \underset{0}{d} \equiv 0$

d) Correcto.

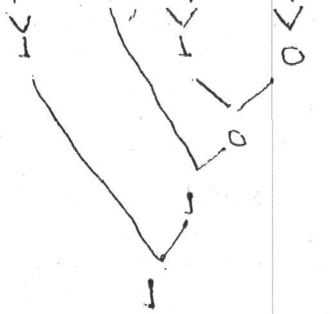
38.-

$\underset{1}{p} \Rightarrow \underset{0}{q} \equiv 0$

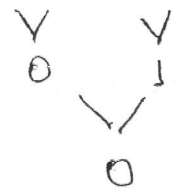
$p: 1$

$q: 0$

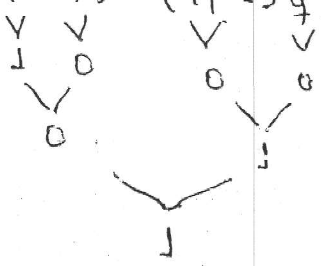
a) $p \vee \neg(\neg q \wedge \neg p)$



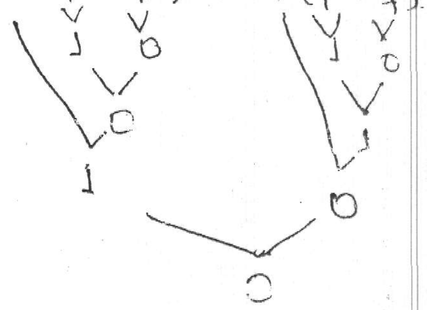
b) $\neg p \wedge \neg q$



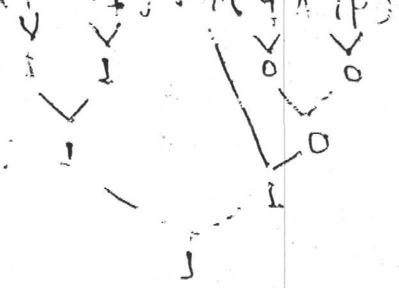
c) $(p \wedge q) \vee (\neg p \Rightarrow q)$



d) $\neg(p \wedge q) \Rightarrow \neg(p \vee q)$



e) $(p \wedge \neg q) \vee \neg(q \wedge \neg p)$

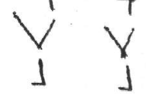


39. a) p: es un numero divisible para 2
q: es un numero primo.
Traducción: $p \Rightarrow \neg q$

b) p: estudias.
q: aprendes
r: te arrepientes
traducción: $(p \Rightarrow q) \wedge (\neg p \Rightarrow r)$

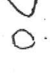
c) p: x satisface la ecuación
q: el Δ es rectangulo.
r: la longitud de la hipotenusa es 4.
s: No hay manera de calcular el área
traducción:
$(p \Rightarrow (q \wedge r)) \wedge (\neg p \Rightarrow s)$

40. $\neg p \wedge q \equiv 1$



p: 0
q: 1

a) $p \Rightarrow (\neg q \wedge r) \equiv 1$



b) $q \vee (\neg p \Leftrightarrow r) \equiv 1$

$1 \quad 1$

1

e) $p \vee (q \vee r) \equiv 1$

$0 \quad 1$

1

c) $q \Rightarrow (p \wedge q) \equiv 0$

$1 \quad 0 \quad 1$

0

0

d) $\neg p \veebar q \equiv 0$

$1 \quad 1$

0

41.– $p \Rightarrow q \equiv 0$ $\quad$ $p: 1$

$1 \quad 0$ $\quad$ $q: 0$

a) $p \vee \neg(\neg q \wedge \neg p) \equiv 1$

$1 \quad 1 \quad 0$

0

1

1

b) $\neg q \wedge \neg p \equiv 0$

$1 \quad 0$

0

c) $(p \wedge q) \vee (\neg p \Rightarrow q) \equiv 1$

$1 \quad 0 \quad 0 \quad 0$

$0 \quad 1$

1

d) $\neg(p \wedge q) \Rightarrow \neg(p \vee q) \equiv 0$

$1 \quad 0 \quad 1 \quad 0$

$0 \quad 1$

$1 \quad 0$

0

e) $(p \wedge \neg q) \vee \neg(q \wedge \neg p) \equiv 1$

$1 \quad 1 \quad 0 \quad 0$

$1 \quad 0$

1

1

42.- Si $f(p,q,r)$ es tautología, cualquier valor de una de las proposiciones deberá ser 1

b) correcto

43.- Si $f(p,q,r)$ es contradicción, cualquier valor de una de las proposiciones deberá ser 0

b) correcto

44.- $\underbrace{f(1,0,1,1)}_{0} \Rightarrow \underbrace{f(0,1,0,0)}_{0}$

1

b) correcto

45.- $\neg p \Rightarrow (q \vee \neg r)$ No se puede asegurar si es contradicción

0, 0, 0; 0; 1

b) correcto

46.- $[\overset{1}{(\neg p \vee q)} \wedge \overset{1}{(\neg r \Rightarrow q)}] \Rightarrow \overset{0}{(p \Rightarrow r)}$

p: 1
q: 1
r: 0

No es tautología

b) correcto

47.— a) $(\neg q \Rightarrow \neg p) \Rightarrow (\neg p \vee q)$

p: 1
q: 0 . Tautología.

b) $(p \vee q) \Rightarrow (\neg p \Rightarrow q)$

p: 0
q: 0 Tautología

c) $[(p \Rightarrow q) \wedge p] \Rightarrow q$

p: 1
q: 0 Tautología

d) $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$

q: 1 Falacia
p: 0

e) $[(p \wedge q) \wedge r] \Rightarrow [(p \vee r) \wedge (q \vee r)]$ Tautología

48.— a) $\neg(\neg p \wedge \neg q)$

$\therefore p \vee q$ ~~Fa~~lacia

b) $\neg(\neg p \wedge q)$

$\therefore p \vee \neg q$ Falacia

c) $p \vee (p \wedge q)$

Falacia

d) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ Tautología

49.- $[p \wedge (p \Rightarrow q)] \Rightarrow M$

a) correcto $p \wedge q$

50.- a) $(p \vee q) \Rightarrow (\neg p \Rightarrow q)$

p: 0 tautología
q: 0

b) $[(p \Rightarrow r) \wedge (q \Rightarrow r)] \Rightarrow [(p \vee q) \Rightarrow r]$

p: 1 Tautología
q: 0
r: 0

c) $[(p \vee q) \wedge \neg p] \Rightarrow q$ tautología

q: 0
p: 0

d) $[(\neg q \Rightarrow \neg p)] \Rightarrow \neg q$

Falacia

e) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

p: 1
q: 0 Tautología
r: 0

1.— a) Falso

b) Falso

c) Falso

d) Verdadero

e) falso

2.—

a) $[(p \Rightarrow q) \wedge (r \Rightarrow s)] \Rightarrow [(p \wedge r) \Rightarrow (q \wedge s)]$ tautología

b) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ tautología

c) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \wedge r)$ No es tautología

3.— p: estoy satisfecho

q: me dieron el aumento de sueldo

Traducción: $\neg q \Rightarrow \neg p$

$\therefore p \Rightarrow q$

$\therefore \neg p \vee q$

a) $q \Rightarrow p$ Falso

b) $\neg q \Rightarrow \neg p$

c) $p \Rightarrow q$

d) $q \vee \neg p$

e) $\neg p \vee q$

54.— a) $\neg(p \vee q) \Rightarrow (\neg p \vee \neg q)$ Tautología

b) $(\neg p \wedge \neg q) \Rightarrow \neg(p \vee q)$

p: 0
q: 0

Tautología

c) $\neg(\neg p \wedge \neg q) \vee \neg q$ Tautología

d) $(\neg p \wedge [\neg p \vee q]) \vee p$ tautología

e) $[p \wedge (p \vee q)] \Rightarrow q$ Falacia

55.— $(p \Rightarrow q) \wedge \neg(q \Rightarrow r)$

p	⇒	q	∧	¬	q	⇒	r
0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	1
0	1	1	1	1	1	0	0
0	1	1	0	0	1	1	1
1	0	0	0	0	0	1	0
1	0	0	0	0	0	1	1
1	1	1	1	1	1	0	0
1	1	1	0	0	1	1	1

Contingencia

1 — $[(p \wedge q) \Rightarrow r] \Rightarrow [p \Rightarrow (q \Rightarrow r)]$

$p: 1$
$q: 1$
$r: 0$

Tautologia

2 — a) $(r \wedge s) \vee \neg s$

r	$\wedge$	s	$\vee$	$\neg s$
0	0	0	1	1
0	0	1	0	0
1	0	0	1	1
1	1	1	1	0

c) $\neg p \Rightarrow (p \wedge q)$

$\neg p$	$\Rightarrow$	p	$\wedge$	q
1	0	0	0	0
1	0	0	0	1
0	1	1	0	0
0	1	1	1	1

b) $(p \Rightarrow q) \Rightarrow (q \Rightarrow \neg p)$

p	$\Rightarrow$	q	$\Rightarrow$	q	$\Rightarrow$	$\neg p$
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	1	1	0	1	0	0

d) $(p \wedge q) \wedge (p \Rightarrow \neg q)$

p	$\wedge$	q	$\wedge$	p	$\Rightarrow$	$\neg q$
0	0	0	0	0	1	1
0	0	1	0	0	1	0
1	0	0	0	1	1	1
1	1	1	0	1	0	0

e) $(p \vee q) \Rightarrow [p \vee (\neg p \wedge q)]$

p	$\vee$	q	$\Rightarrow$	p	$\vee$	$\neg p$	$\wedge$	q
0	0	0	1	0	0	1	0	0
0	1	1	1	0	1	1	1	1
1	1	0	1	1	1	0	0	0
1	1	1	1	1	1	0	0	1

58. — i) $\neg a \wedge (a \Leftrightarrow b)$

$\neg a$	$\wedge$	a	$\Leftrightarrow$	b
1	1	0	1	0
1	0	0	0	1
0	0	1	0	0
0	0	1	1	1

Contingencia

II) $(p \wedge q) \Rightarrow (p \Rightarrow \neg q)$

p	$\wedge$	q	$\Rightarrow$	p	$\Rightarrow$	$\neg q$
0	0	0	1	0	1	1
0	0	1	1	0	1	0
1	0	0	1	1	1	1
1	1	1	0	1	0	0

Contingencia

III) $(a \wedge b) \wedge \neg c$

a	$\wedge$	b	$\wedge$	$\neg c$
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
1	0	0	0	1
1	0	0	0	0
1	1	1	1	1
1	1	1	0	0

Contingencia

IV. $[(p \Rightarrow q) \wedge \neg b] \Rightarrow \neg b$

p	$\Rightarrow$	q	$\wedge$	$\neg b$	$\Rightarrow$	$\neg b$
0	1	0	0	0	1	0
0	1	0	1	1	1	1
0	1	1	0	0	1	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	0	0	1	1	1
1	1	1	0	0	1	0
1	1	1	1	1	1	1

Tautología

59.— p: me comporto bien en mi hogar

q: soy un buen hijo

traducción: $p \Rightarrow q$

negación: $\neg(p \Rightarrow q)$

$\therefore \neg(\neg p \vee q)$

$\therefore p \wedge \neg q$ a) verdadero

60.— a) correcto

61.— a) correcto

62.— a) verdadero

63.—

a) $a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$

a	$\wedge$	$(b$	$\vee$	$c)$
0	0	0	0	0
0	0	0	1	1
0	0	1	1	0
0	0	1	1	1
1	0	0	0	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

$(a$	$\wedge$	$b)$	$\vee$	$(a$	$\wedge$	$c)$
0	0	0	0	0	0	0
0	0	0	0	0	0	1
0	0	1	0	0	0	0
0	0	1	0	0	0	1
1	0	0	0	1	0	0
1	0	0	1	1	1	1
1	1	1	1	1	0	0
1	1	1	1	1	1	1

Son iguales

b) $\neg(a \wedge b) \equiv \neg a \vee \neg b$

$\neg$	$(a$	$\wedge$	$b)$
1	0	0	0
1	0	0	1
1	1	0	0
0	1	1	1

$\neg a$	$\vee$	$\neg b$
1	1	1
1	1	0
0	1	1
0	0	0

Son iguales

1.— p: se es inteligente

q: se es estudioso

r: se es aplicado

Traducción: $(p \vee q) \Rightarrow r$

b) $(p \Rightarrow r) \wedge (q \Rightarrow r)$

$\therefore (\neg p \vee r) \wedge (\neg q \vee r)$

$\therefore r \vee (\neg p \wedge \neg q)$

$\therefore r \vee \neg(p \vee q)$

$\therefore (p \vee q) \Rightarrow r$

65.– a) correcto

66.– p: te gustan las matemáticas

q: eres hábil para la geometría

traducción: $\overbrace{(p \Rightarrow q)}^{1} \Rightarrow \overset{0}{\neg p}$

b) Falso

67.– p: Trabajo arduamente

q: gano un buen sueldo

traducción: $[\overbrace{(p \Rightarrow q)}^{1} \wedge \overset{1}{\neg q}] \Rightarrow \overset{0}{\neg p}$

p: 1

q: 0

a) correcto

68.– p: lo intento con ahínco

q: tengo talento

r: me convierto en músico

s: seré feliz

a) $\{\overbrace{[(p \wedge q) \Rightarrow r]}^{1} \wedge \overbrace{[r \Rightarrow s]}^{1}\} \Rightarrow \overset{0}{s}$ FALSO

b) $\{\overbrace{[(p \wedge q) \Rightarrow r]}^{1} \wedge \overbrace{[r \Rightarrow s]}^{1}\} \Rightarrow \overbrace{(r \Rightarrow p)}^{0}$ FALSO

$\{[(p\wedge q)\Rightarrow r]\wedge[r\Rightarrow s]\}\Rightarrow \neg r$

$r:1$ Falso

$s:1$

$\{[(p\wedge q)\Rightarrow r]\wedge[r\Rightarrow s]\}\Rightarrow \neg q$

Falso

$q:1$

$\{[(p\wedge q)\Rightarrow r]\wedge[r\Rightarrow s]\}\Rightarrow (\neg s\Rightarrow(\neg p\vee \neg q))$

CORRECTO

$s:0$

$p:1$

$q:1$

p: apruebo todas las materias

q: me voy de vacaciones por un mes

r: compraré muchos recuerdos

$\{[p\Rightarrow q]\wedge[q\wedge r]\}\Rightarrow \neg q$ Falso

$q:1$

$\{[p\Rightarrow q]\wedge[q\wedge r]\}\Rightarrow (p\wedge r)$

Falso

$\{(p\Rightarrow q)\wedge(q\wedge r)\}\Rightarrow (\neg p\wedge \neg q)$

Falso

$\{(p\Rightarrow q)\wedge(q\wedge r)\}\Rightarrow q$

correcto

70.— p: el gobierno realiza las gestiones apropiadas
q: el concurso de miss universo se realiza en nuestro país
r: el turismo se reactiva en el país.

a) $[(\underset{0}{\neg p} \Rightarrow \underset{0}{\neg q}) \wedge \underset{1}{r} \wedge \underset{1}{q}] \Rightarrow \overset{0}{\neg q}$

q:1 Falso
r:1

b) $[(\underset{0}{\neg p} \Rightarrow \underset{0}{\neg q})_{1} \wedge \underset{1}{r} \wedge \underset{1}{q}] \Rightarrow \underset{0}{\neg r}$ Falso

c) $[(\underset{0}{\neg p} \Rightarrow \neg q)_{1} \wedge \overset{1}{r} \wedge \overset{1}{q}] \Rightarrow \overset{0}{(\underset{1}{r} \wedge \underset{0}{\neg p})}$ p:1 Falso

d) $[(\underset{0}{\neg p} \Rightarrow \neg q)_{1} \wedge \underset{1}{r} \wedge \underset{1}{q}] \Rightarrow \underset{0}{\neg p}$ p:1 Falso

e) $[\overset{1}{(\neg p \Rightarrow \neg q)} \wedge \overset{1}{\underset{0}{r}} \wedge \overset{1}{q}] \Rightarrow \overset{0}{(\underset{1}{p} \Rightarrow \underset{0}{r})}$ Correcto

71.— a) $[\overset{1}{\underset{1}{p}} \wedge \overset{1}{(\underset{1}{p} \Rightarrow \underset{1}{q})_{1}}] \Rightarrow \overset{0}{\neg q}$ q:1 Falso

b) $[\underset{1}{p} \wedge (\underset{1}{p} \Rightarrow \underset{1}{q})_{1}] \Rightarrow \overset{0}{(\underset{0}{\neg p} \wedge \underset{1}{q})_{0}}$ Falso

c) $[\underset{1}{p} \wedge (\underset{1}{p} \Rightarrow \underset{1}{q})_{1}] \Rightarrow \overset{0}{(\underset{0}{\neg p} \wedge \underset{0}{\neg q})}$ Falso

d) $[\underset{1}{p} \wedge (\underset{1}{p} \Rightarrow \underset{1}{q})_{1}] \Rightarrow \overset{0}{(\underset{1}{p} \wedge \underset{1}{q})_{1}}$ Correcto

7.— p: estudio

q: apruebo nivel cero

r: viajo a Galapago

a) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow \neg q$ $q:1$ Falso

b) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow (\neg p \wedge \neg q]$ Falso

c) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow (p \wedge r)$ Falso

d) $\{[p \Rightarrow q] \wedge [q \wedge r]\} \Rightarrow q$ Correcto

— a) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (p \wedge q)$

$p:1$
$r:0$ Falso

b) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (p \Rightarrow q)$

$p:1$
$q:0$ Falso
$r:0$

c) $[(\neg p \Rightarrow q) \wedge (p \wedge \neg r) \wedge (\neg p \Rightarrow r)] \Rightarrow (\neg p \Rightarrow q)$

$p:0$
$r:0$

Correcto

74.— p: se concluye con éxito la construcción del nuevo parque en el barrio del centenario

q: se coopera para el embellecimiento de la urbe.

r: se incrementa la captación de turistas

a) $\{(p \Rightarrow r) \wedge (q \wedge r)\} \Rightarrow \neg q$ q:1 Falso

b) $\{(p \Rightarrow r) \wedge (q \wedge r)\} \Rightarrow q$ correcto

75.— $p: \alpha = \beta$ $\quad u: \beta \neq 90$

$q: \beta = 45$

$r: \alpha = 90$

$s: \theta = 90$

$t: \beta = 90$

a) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow p$ Falso

b) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow \neg p$ p:1 Falso.

c) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow s$ Falso

d) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \vee u) \wedge \neg s] \Rightarrow q$ Falso

e) $[(p \Rightarrow q) \wedge (q \Rightarrow r) \wedge (t \veebar u) \wedge \neg s] \Rightarrow \cancel{x}$

Correcto

— p: el reloj está andando

q: juan llegó antes de las 10

r: juan vió partir el coche de Andrés

s: Andrés dice la verdad

t: Andrés estaba en el edificio en el momento del crimen.

$[(p \Rightarrow (q \wedge r)) \wedge (\neg s \Rightarrow \neg r) \wedge (s \vee t) \wedge p] \Rightarrow \neg q$ Falso

q: 1

p: 1

r: 1

s: 1

$[(p \Rightarrow (q \wedge r)) \wedge (\neg s \Rightarrow \neg r) \wedge (s \vee t) \wedge p] \Rightarrow t$ Falso

$[(p \Rightarrow (q \wedge r)) \wedge (\neg s \Rightarrow \neg r) \wedge (s \vee t) \wedge p] \Rightarrow (\neg r \vee t)$ Correcto

— p: estudio

q: soy un genio

r: apruebo nivel 0

s: puedo Tomar nivel 100

$\{[(p \vee q) \Rightarrow r] \wedge (r \Rightarrow s)\} \Rightarrow (\neg s \Rightarrow \neg q)$

Válido

78.— p: el congreso asigna fondos

q: el proyecto será ejecutado

r: hay consenso ~~entre~~ los diputados

$$\{[p\Rightarrow q]\wedge[p\Rightarrow r]\wedge\neg r\}\Rightarrow\neg q$$

q: 1

r: 0

p: 0

Falso

79.— p: esta ley será aprobada en esta sesión del congreso

q: la ley es apoyada por la mayoría legislativa

r: el presidente se opone a ella

s: la ley será pospuesta en deliberaciones del congreso

$$\{(p\Rightarrow q)\wedge(q\vee r)\wedge(r\Rightarrow s)\}\Rightarrow(p\vee s)$$

Falso

80.— p: la dolarización es difícil

q: la dolarización no les gusta a muchas personas

r: las medidas económicas son viables

$$\{(p\wedge q)\wedge(r\Rightarrow\neg p)\}\Rightarrow(\neg r\Rightarrow q)$$

Válido

(81) $p \Rightarrow q \equiv 1 \quad \therefore \quad p \neq 1 \wedge q \neq 0$

esta condición al mismo tiempo

$\neg p \equiv 1 \qquad q \equiv 0$

* Tenemos: $0 \Rightarrow 0 \quad \equiv 1$

~~a)~~ Verdadero $\qquad$ b) Falso

(82) a) $(p \Leftrightarrow q) \equiv (p \vee q) \Rightarrow (p \wedge q)$

* $(p \vee q) \Rightarrow (p \wedge q)$ · Tautología Útil

$\neg(p \vee q) \vee (p \wedge q)$ · De Morgan

$(\neg p \wedge \neg q) \vee (p \wedge q)$ - Distributiva

$[(\neg p \wedge \neg q) \vee p] \wedge [(\neg p \wedge \neg q) \vee q]$

$[\underbrace{(\neg p \vee p)}_{**} \wedge (\neg q \vee p)] \wedge [(\neg p \vee q) \wedge \overbrace{(\neg q \vee q)}^{**}]$

En ** Ley del Tercero o Medio Excluido

$[1 \wedge (\neg q \vee p)] \wedge [(\neg p \vee q) \wedge 1]$ - Identidad

$(\neg q \vee p) \wedge (\neg p \vee q)$ - Tautología Útil

$(q \Rightarrow p) \wedge (p \Rightarrow q)$ - Conmutativa

$(p \Rightarrow q) \wedge (q \Rightarrow p)$ - Equivalencia

$p \Leftrightarrow q$ - L. Q. Q. D.

b) $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$

$\neg(p \vee q) \vee (\neg p \wedge q)$ - De Morgan

$(\neg p \wedge \neg q) \vee (\neg p \wedge q)$ · Distributiva

$\neg p \wedge (\neg q \vee q)$ - Tercero o Medio excluido

$\neg p \wedge 1$ - Identidad

$\neg p$ - L. Q. Q. D.

83 Utilizar el método de reducción al absurdo para demostrar las siguientes proposiciones:

a) $\underbrace{[\underbrace{(\underset{1}{p} \Rightarrow \underset{0}{q})}_{1} \wedge \underbrace{p}_{1}]}_{1} \Rightarrow \underbrace{q}_{0}$ $\qquad p \equiv 1$

$q \equiv 0$

$[(1 \Rightarrow 0) \wedge 1] \Rightarrow 0$

$(0 \wedge 1) \Rightarrow 0 \qquad 0 \Rightarrow 0 \qquad \equiv 1$

b) $\underbrace{[\underbrace{(\underset{0}{p} \vee \underset{0}{q})}_{1} \wedge \underbrace{\neg q}_{1}]}_{1} \Rightarrow \underbrace{p}_{0}$ $\qquad \neg q \equiv 1 \quad q \equiv 0$

$p \equiv 0$

$[(0 \vee 0) \wedge 1] \Rightarrow 0$

$(0 \wedge 1) \Rightarrow 0$ $\qquad$ ▲ ▲ Contradicción $\equiv$ Falacia

$0 \Rightarrow 0 \qquad \equiv 1$

c) $\underbrace{\underset{0}{p}}_{1} \Rightarrow \underbrace{(\underset{0}{p} \vee \underset{0}{q})}_{0}$ $\qquad p \equiv 0$

$q \equiv 0$

$0 \Rightarrow (0 \vee 0)$

$0 \Rightarrow 0 \qquad \equiv 1$

* En los literales a, b y c todas han resultado ser tautología.

** Este método indica si una forma proposicional es tautológica o no, por medio de determinados valores para las proposiciones presentes.

*** En el caso de no ser una tautología, se deberá resolver por medio de las tablas de verdad para determinar si la forma proposicional es falacia o contingencia

CONJUNTOS

84.-

e) $x / \; x \neq x$ → no existen elementos que cumplan esta condición, por lo tanto la solución es vacío

85.-

a) Falso

b) Verdadero

c) Falso

d) Verdadero

e) Verdadero

86.-

a) $5 = \{5\}$ → subconjunto FALSO

b) $\{\ \} \in \phi$ FALSO

c) $1 \in \{\{1,4\},\{2,4\}\} \Rightarrow 1 \in \{1,4\}$ FALSO

d) $\{4, 8, 2^3, 3\} = \{(-2)^2, 8, 3\}$ VERDADERO

e) $\underbrace{\{2, 4\}}_{\text{elementos}} = \{\underbrace{\{2\}, \{4\}}_{\text{subconjuntos}}\}$ FALSO

87.- $A = \{a, \{b\}, c, \{d, e\}\}$,

$B = \{b, c\}$

a) Verdadero

b) Falso; $b \notin A$.

c) Falso

d) Verdadero

e) Falso; $b \in B$.

88.-

a) $A = \{a, i, o, u\}$

b) $B = \{a, e, i, o\}$

c) $C = \{a, e, o\}$

d) $D = \{a, e, i, o, u\}$

I) Verdadero

II) Falso

III) Falso

89.- a) Finito; tiene inicio y fin

b) infinito; tiene inicio, pero no fin

c) finito

d) unitario; es el 2

CUANTIFICADORES

10.- $Re = \{1, 2, 3, 4, 5\}$

a) FALSO; no hay elementos

b) FALSO; 2, 3, 4, 5 no cumplen la condición

c) FALSO; 1 no cumple la condición

d) VERDADERO

e) FALSO; 2, 4, 5 no cumplen la condición.

?.- a) Todo vegetariano come zanahorias

b) Algunos vegetarianos comen zanahorias

c) No todo vegetariano come zanahorias

d) $\forall x [p(x) \vee \neg q(x)]$

$\forall x [\neg q(x) \vee p(x)]$

$\forall x [q(x) \Rightarrow p(x)]$.: todo vegetariano come zanahorias

e) $\forall x [\neg p(x) \vee \neg q(x)]$

.: $\forall x [p(x) \Rightarrow \neg q(x)]$.: todos los que comen zanahorias no son vegetarianos

92) a) $A = \{1,2,3,4\}$

$P(A) = \{\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}$
$\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\}, A, \phi\}$

$N\,P(A) = 2^4 = 2\times2\times2\times2 = 16$

b) $B = \{\square, \circ, \Delta\}$

$P(B) = \{\{\square\},\{\circ\},\{\Delta\},\{\square,\circ\},\{\square,\Delta\},\{\circ,\Delta\}, B, \phi\}$

$N\,P(B) = 2^3 = 2\times2\times2 = 8$

c) $C = \{\phi, \{\phi\}\}$

$P(C) = \{\{\phi\},\{\{\phi\}\}, C, \phi\}$

$N\,P(C) = 2^2 = 2\times2 = 4$

93) $A = \{a, \{b\}\}$

a) $\{\phi\} \subseteq A$; falso

b) $a \in A$; falso

c) $\{b\} \in A$; falso

d) $N(P(P(A))) = 16$; Falso

e) $\{\{b\}\} \in P(A)$; Verdadero.

94.-

a) $A \subseteq B$; todo elemento de A está en B

VERDADERO

b) $A \subseteq B \therefore A = B \therefore \underbrace{\underbrace{(A \subseteq B)}_{1} \wedge \neg \underbrace{(B \subseteq A)}_{0}}_{0}$ FALSO

c) $\underbrace{\underbrace{(A \subset B)}_{1} \Rightarrow [\underbrace{\underbrace{(A \subseteq B)}_{0} \wedge \underbrace{(B \subseteq A)}_{0}}_{0}}_{0}$ FALSO

d) $\underbrace{\underbrace{(x \in \emptyset)}_{1} \Rightarrow \underbrace{(x \notin A)}_{1}}_{1}$; VERDADERO

e) $\underbrace{\underbrace{(x \in \emptyset)}_{1} \Rightarrow \underbrace{(x \in A)}_{1}}_{1}$; VERDADERO

f) $(A \subseteq B) \Leftrightarrow \underbrace{[\forall x [\underbrace{\underbrace{(x \in A)}_{1} \Rightarrow \underbrace{(x \in B)}_{1}}_{1}] \wedge \exists x [\underbrace{\underbrace{(x \in B)}_{1} \wedge \underbrace{(x \in A)}_{0}}_{0}]]}_{0}$

FALSO

g) $\underbrace{\underbrace{(A = B)}_{1} \Rightarrow \underbrace{[(A \subset B) \wedge (B \subset A)}_{0}}_{0}$

FALSO

95.- a) A subconjunto propio de B, es equivalente a: si x elemento de A entonces x elemento de B.

b) Si A subconjunto propio de B, entonces A subconjunto propio de B y no es verdad que B sea subconjunto propio de

c) Si A subconjunto de B, entonces, A subconjunto propio de B y B subconjunto propio de A.

d) Si x elemento de vacío, entonces x no elemento de A

e) Si x elemento de vacío, entonces x elemento de A

f) A subconjunto propio de B es equivalente a para todo elemento de A, entonces x elemento de B y para algunos x elemento de B, entonces x elemto de A.

g) Si A=B, entonces: A subconjunto de B ∧ B subconjunto A

96.- $B = \{*, \alpha\}$

a) $N(P(B)) = 2^2$
$\Rightarrow 4$

$N(P(P(B))) = 2^4$ FALSO
$\Rightarrow 16$

b) $* \in P(B)$ Falso, $\{*\} \in P(B)$

c) Verdadero

d) $\{\{*, \alpha\}\} \in P(B)$

e) Falso

87.- I) $p(x)$: x es número
$q(x)$: x es impar

$\forall x [\, p(x) \Rightarrow q(x)]$ FALSO

II)

$\exists x\ (x \in \mathbb{Z} / 3x^2 - 5 = 0)$

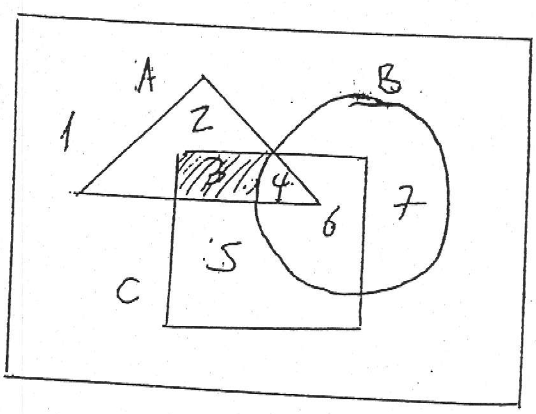
III) $\exists x\ (x \in \mathbb{Z} / x+1 < 0) \Rightarrow \forall x\ (x \in \mathbb{N} \Rightarrow x \in \mathbb{Z})$

98.- $A = \{\phi, \{\phi\}\}$

$P(A) = \{ \{\phi\}, \{\{\phi\}\}, \{A\} \}$

$A \cap P(A) = \{\phi\}$ VERDADERO

99.-



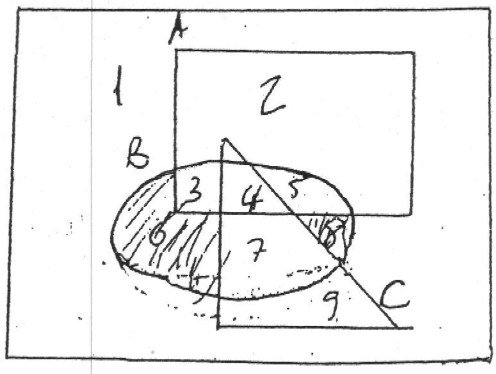
$A = \{2, 3, 4\}$
$C = \{3, 4, 5, 6\}$
$B = \{4, 6, 7\}$

$A - B = \{2, 3\}$

d) $(A-B) \cap C = \{3\}$

CORRECTO

100.–



$A = \{2, 3, 4, 5\}$

$B = \{3, 4, 5, 6, 7, 8\}$

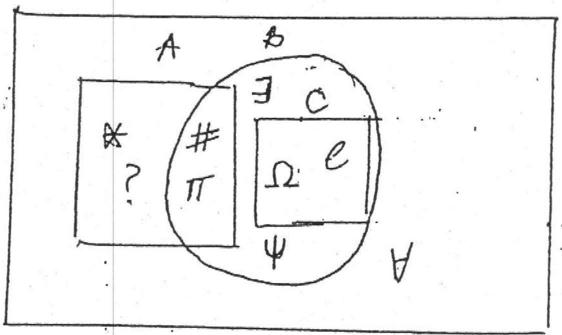
$C = \{4, 7, 9\}$

$A \cup C = \{2, 3, 4, 5, 7, 9\}$

$B - (A \cup C) = \{6, 8\}$

VERDADERO

101.–



$A = \{*, ?, \#, \pi\}$

$B = \{\#, \pi, \exists, \psi, \Omega, e\}$

$C = \{\Omega, e\}$

102.– $Re = \{*, !, \#, \$, \%, \&, ?\}$

$A = \{*, !, \#, \$\}$

$B = \{!, \%, \&, ?\}$

$C = \{\%, \&, ?\}$

$[(A-B)^c \cup C]^c$

$(A-B) = \{*, \#, \$\}$

$(A-B)^c = \{!, \%, \&, ?\}$

$(A-B)^c \cup C = \{!, \%, \&, ?\}$

$[(A-B)^c \cup C]^c = \{*, \#, \$\}$ e) correcto

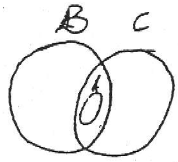
03.- $S=\{\{3\},\{1,4\}\}$

$P(S)=\{\{\{3\}\};\{\{1,4\}\};\{S\}\}$

b) correcto

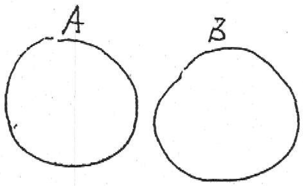
4.-

a)



VERDADERO

b)



NO HAY ELEMENTOS EN COMUN

VERDADERO

c)

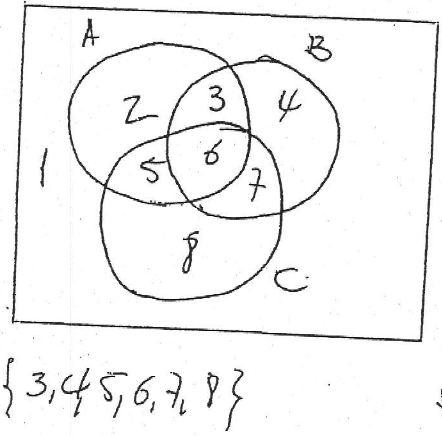
$N(A)=2$

$N(P(A))=2^2$

$=\underline{4}$

FALSO

$A-(B\cup C)^c=(A-B^c)\cup(A-C^c)$



$A=\{2,3,5,6\}$

$B=\{3,4,6,7\}$

$C=\{5,6,7,8\}$

$(B\cup C)=\{3,4,5,6,7,8\}$ $\qquad B^c=\{1,2,5,8\}$ $\qquad C^c=\{1,2,3,4\}$

$(B\cup C)^c=\{1,2\}$ $\qquad (A-B^c)=\{3,6\}$ $\qquad A-C^c=\{5,6\}$

$A-(B\cup C)^c=\{3,5,6\}$

$\cup$

$\{3,5,6\}$ CORRECTO

e) $N(A) = 4$

$N(B) = 3$

$N(A \cap B) = 2$

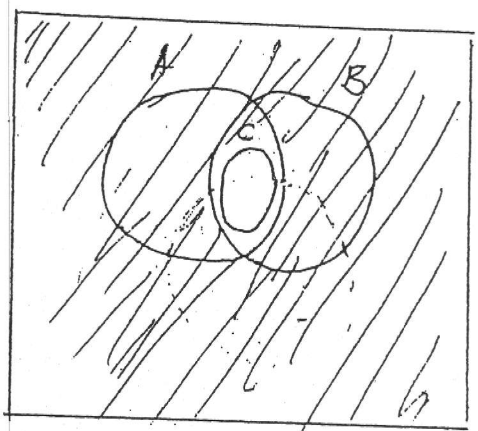
$N(A \cup B) = 4 + 3 - 2$

$\Rightarrow \underline{\underline{5}}$

$N(P(A \cup B)) = 2^5$ FALSO

$\Rightarrow \underline{\underline{32}}$

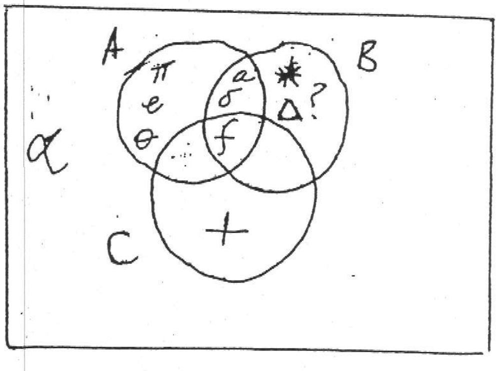
f)



FALSO

$C^c \cap (A \cup B) \neq \emptyset$

105.-



a) $A = \{\pi, e, \theta, a, \delta, f\}$

$B = \{a, \delta, f, *, \Delta, ?\}$

$C = \{f, +\}$

b) $N(C) \doteq N(B) - N(A)$

$2 = 6 - 6$

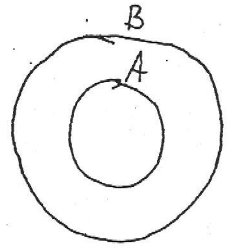
$2 \neq 0$ FALSO

$A \cup B = \{\theta, \pi, e, *, ?, \delta\}$

FALSO; faltan: Δ, a, f

6.-

$A \cup B = B$



a) $A \cap B = A$; VERDADERO

b) $A - B = A$; VERDADERO $\therefore A - B = \phi$

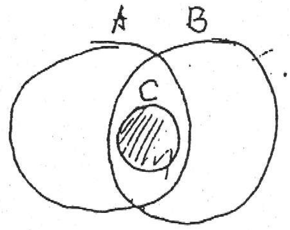
c) $B - A = A$; FALSO

d) $B^c = A$; FALSO

e) $B \subseteq A$; FALSO $\therefore A \subseteq B$

7.-

$C \subset (A \cap B)$



a) $(A \cup C) \subset (B \cap C)$; FALSO

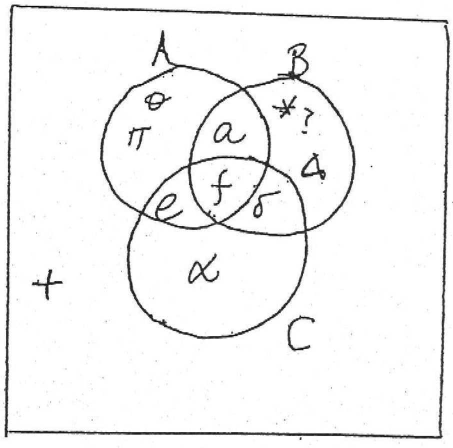
b) $A - B = \phi$; FALSO

c) $(A - B) \subseteq C$; FALSO

d) $(A - B) \cup (B - A) = C$; FALSO

e) $(C - A) = \phi$; VERDADERO

8.-



a) $A\cup B = \{\theta, \pi, e, *, ?, \delta, \Delta, f, a\}$; VERDADERO

b) $(B\cap C) - (A\cap B\cap C) = \emptyset$; FALSO ; RESPUESTA . δ.

109.— $A = \{\phi,\ 1,\ \{1\},\ \{2\},\ \{\phi\},\ \{1,2\}\}$

a) $\underbrace{(1 \cup A)}_{0} \veebar \underbrace{(\{1\}\in A)}_{1} \equiv 1$

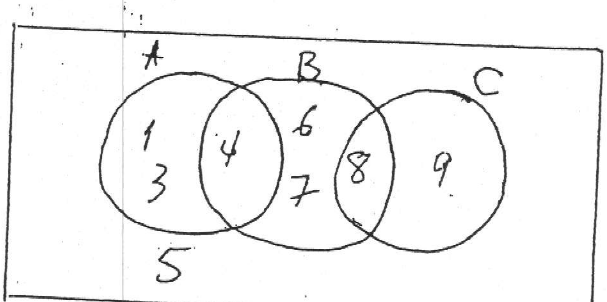
b) $\underbrace{(\phi \subseteq A)}_{1} \wedge \underbrace{(\phi\in A)}_{1} \equiv 1$

c) $\underbrace{(1\in A)}_{1} \Rightarrow \underbrace{(2\in A)}_{0} \equiv 0$

d) $\underbrace{(\underbrace{\{1,2\}}_{1} \in A \vee \underbrace{(\phi\subseteq A)}_{1})}_{1} \equiv 1$

e) $\underbrace{(x\in A)}_{1} \Rightarrow \underbrace{(2\in A)}_{0} \equiv 0$

110.—

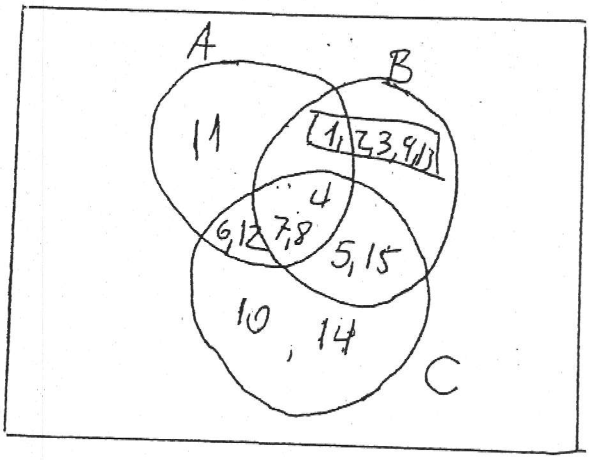


$A = \{1,3,4\}$

$B = \{4,6,7,8\}$

$C = \{8,9\}$

11.—

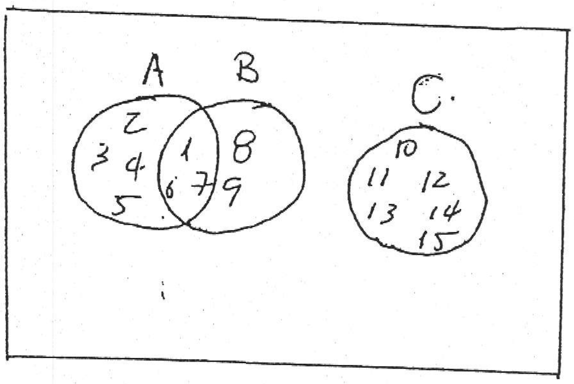


$A = \{3, 4, 6, 7, 8, 11, 12\}$

$B = \{1, 2, 3, 4, 5, 7, 8, 9, 13, 15\}$

$C = \{4, 5, 6, 7, 8, 10, 12, 14, 15\}$

12.—



$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{1, 6, 7, 8, 9\}$

$C = \{10, 11, 12, 13, 14, 15\}$

13.— $A = \{a, i, o, u\}$

$B = \{a, i\}$

$C = \{o, u\}$

a) FALSO; $n(C) = 2$

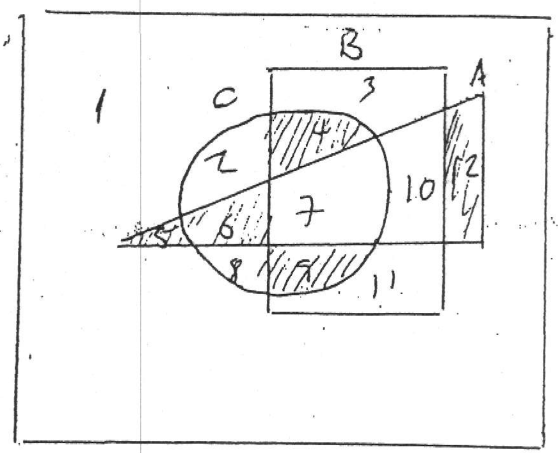
b) FALSO; $C - A = \phi$

c) FALSO; $N(P(B)) = 2^2 \Rightarrow 4$

d) FALSO;

e) $A - B = \{o, u\} = C$; FALSO

114.—



$A=\{5,6,7,10,12\}$

$B=\{3,4,7,10,9,12\}$

$C=\{2,4,6,7,8,9\}$

a) $(A-B)=\{5,6\}$

$(C\cap B)=\{4,7,9\}$ $\Big\rangle \cap = \emptyset$ FALSO

b) $A\cap B\cap C=\{7\}$

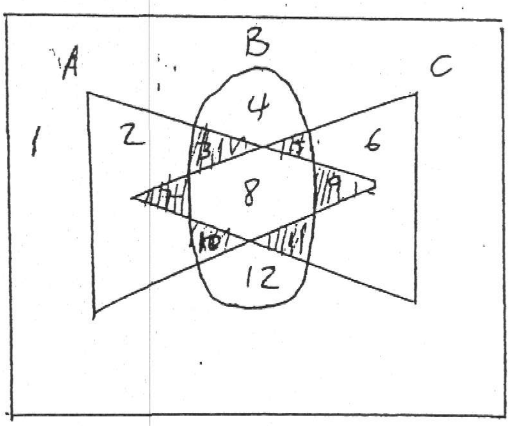
$(A\cap B\cap C)^c=\{1,2,3,4,5,6,8,9,10,11,12\}$ FALSO

c) $(C-A)=\{2,4,8,9\}$

$(C-A)\cap B=\{4,9\}$

$(A-B)=\{5,6,12\}$ $\Big\rangle \cup = \{4,5,6,9,12\}$ VERDADERO

115.—



$A=\{2,3,7,8,9,10\}$

$B=\{3,4,5,8,10,11,12\}$

$C=\{5,6,7,8,9,11\}$

b) $(A\cap B)=\{3,8,10\}$

$(A\cap B)-C=\{3,10\}$

$(A-B)=\{2,7,9\}$

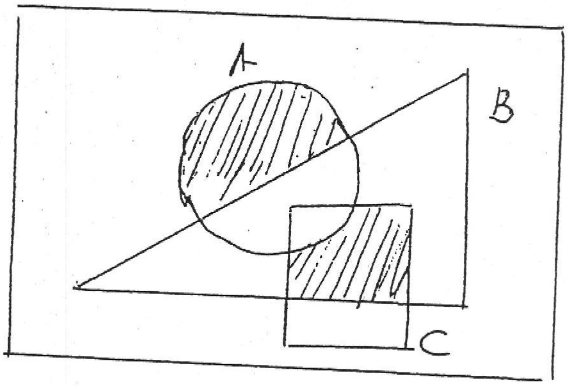
$(A-B)\cap C=\{7,9\}$

$(B-A)=\{4,5,11,12\}$

$(B-A)\cap C=\{5,11\}$

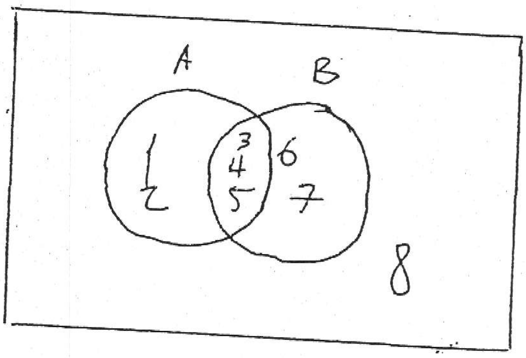
$\cup=\{3,5,7,9,10,11\}$

26.-



$(A-B)\cup[(B\cap C)-A]$

2.-



$A=\{1,2,3,4,5\}$

$B=\{3,4,5,6,7\}$

a) $A-B=\{1,2\}$; FALSO

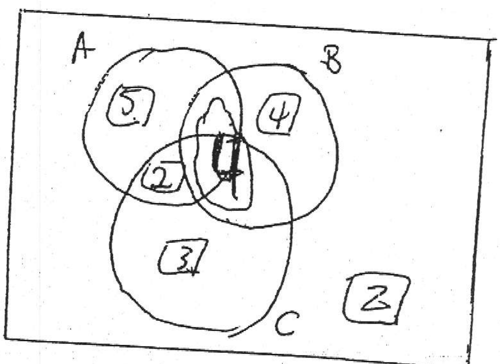
b) $B=\{3,4,5,6,7\}$; FALSO

c) $(A-B)\cap(B\cup A)$

$\{1,2\}$ $\{1,2,3,4,5,6,7\}$

$\cap=\{1,2\}$ VERDADERO

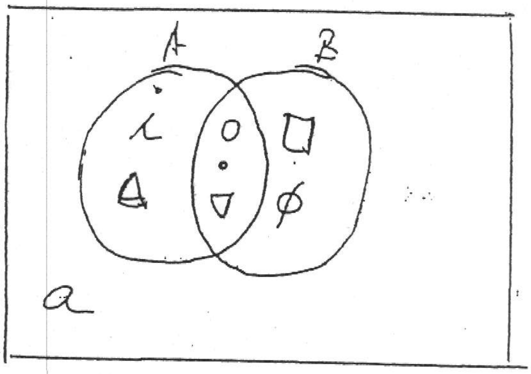
-



$N[\underbrace{(A\cap B)}_{4}\cup\underbrace{(A\cap C)}_{2}\cup\underbrace{(B\cap C)}]$

$4+2=\underline{6}$

119.—



$A=\{i, \Delta, O, \circ, \nabla\}$

$B=\{O, \circ, \nabla, \square, \phi\}$

a) $A-B=\{i, \Delta\}$; FALSO

b) $B=\{O, \circ, \nabla, \square, \phi\}$

c) $(A-B) \cap (A \cup B)$

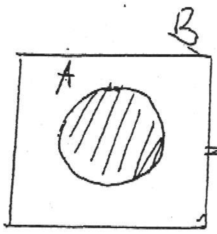
$\{i, \Delta\}$ $\quad$ $\{i, \Delta, O, \circ, \nabla, \square, \phi\}$

$\cup$

$\cap$

$\{i, \Delta\}$ VERDADERO

20.- $A \subseteq B \therefore (A\cap B)=B$



$\Rightarrow A\cap B=A.$

FALSO

21.- $P(A) \Rightarrow$ Son todos los subconjuntos que existen en A.

a) correcto

22.- $[\underbrace{(A\cap(B\cup A))}]\cap A^c = Re$

$$\underbrace{A\cap A^c}_{\phi}$$

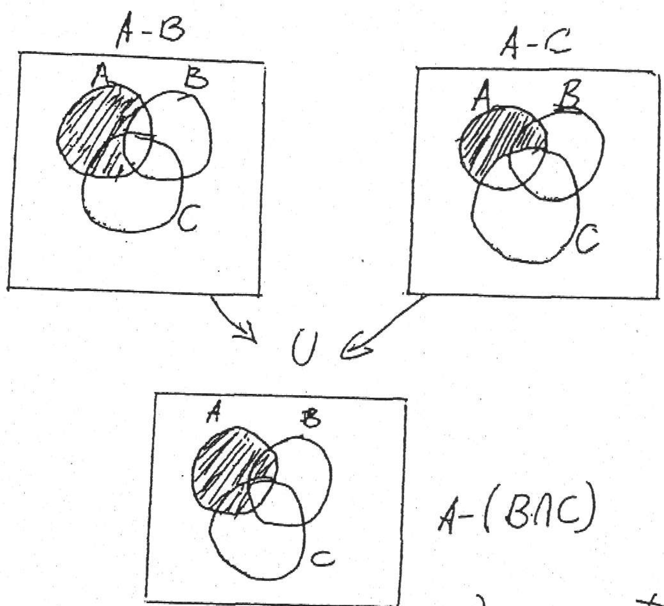
b) correcto

23.- $A=\phi \wedge B=\phi$

$A\cup B=\phi$

a) correcto

25.-

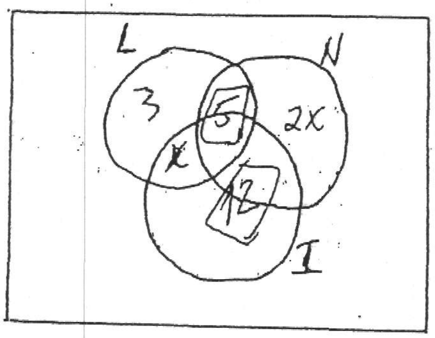


$A-(B\cap C)$

a) correcto

126.-

26 : Total
23 : N∪I ×
5 : L∩N
12 : I no L
Solo N : 2 solo I ∧ L no N



$3+x+5+2x+12=26$

$3x+20=26$

$3x=6$

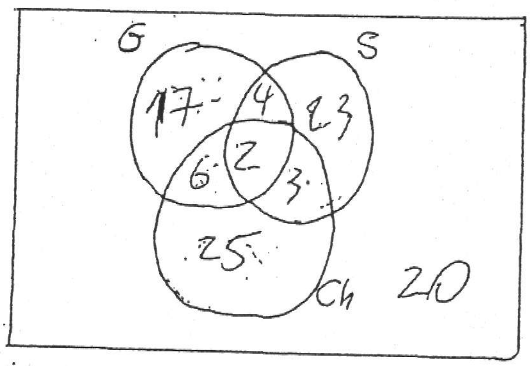
$x=2$

locales: $3+5+x$

: 10

127.-

2% G∩S∩C × (a)
6% G∩S ×
5% S∩C ×
29% G ×
32% S ×
36% C ×
8% G∩C ×



$29+26+25+x=100$

$80+x=100$

$x=20$

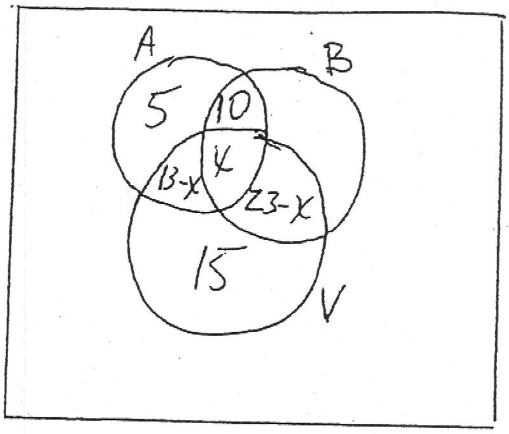
b) 20%

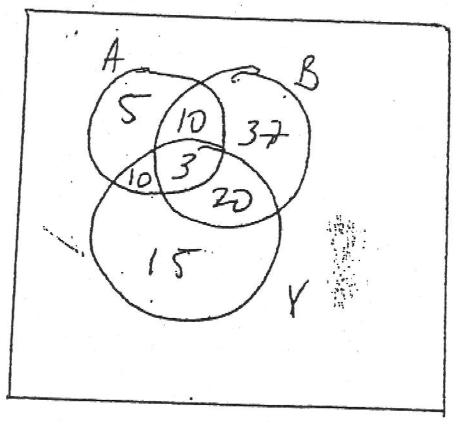
c) 2%

d) 40 personas

8.-

: total





$5+13-x+15=30$

$33-x=30$

$-x=30-33$

$x=3$

a) 3

b) 5+37+15 => 57

c) 100

d) 97

9.-

300 : total

110 > 20 años

120 : mujeres

50 : mujeres > 20 años

	Mujeres	Hombres
>20	50	60
<20	70	120

a) 180

b) 60

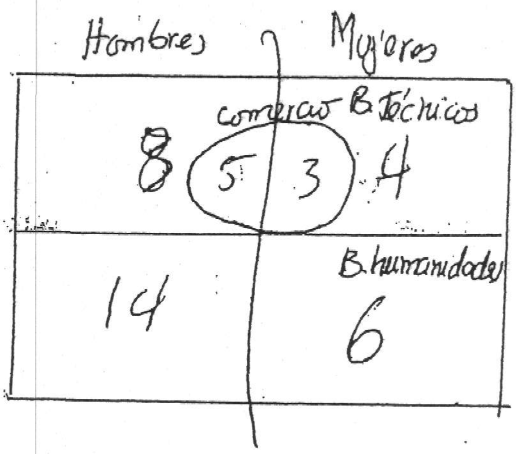
c) 70

d) 120

e) 190

130.-

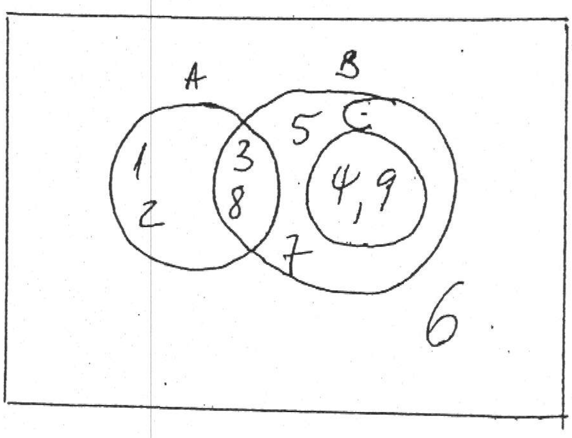
40: total



a) 4

b) 14

131.-



$A = \{1, 2, 3, 8\}$

$B = \{3, 4, 5, 7, 8, 9\}$

$C = \{4, 9\}$

132.-

a) $A^c \cap B^c = (A \cup B)^c$

$x \notin A \wedge x \notin B$

$(A \cup B)^c = \neg(x \in A \vee x \in B)$

$\Rightarrow x \notin A \wedge x \notin B$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap (B \cup C) = x \in A \wedge (x \in B \vee x \in C)$

$= (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C)$

$(A \cap B) \cup (A \cap C) = (x \in B \wedge x \in A) \vee (x \in A \wedge x \in C)$

c) $(A \cap B)^c = A^c \cup B^c$

$A^c \cup B^c = x \notin A \vee x \notin B$

$(A \cap B)^c = \neg(x \in A \wedge x \in B)$

$= x \notin A \vee x \notin B$

d) $A \cup \phi = A$

$x \in A \vee x \in \phi$

$x \in A \vee \phi$

$x \in A$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) = x \in A \vee (x \in B \wedge x \in C)$

$\Rightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$(A \cup B) \wedge (A \cup C) \Rightarrow (x \in A \vee x \in B) \wedge (x \in A \vee x \in C)$

$A - B = A \cap B^c$

$A - B = x \in A \wedge x \notin B$

$A \cap B^c = x \in A \wedge x \notin B$

$A - B = (A^c \cup B)^c$

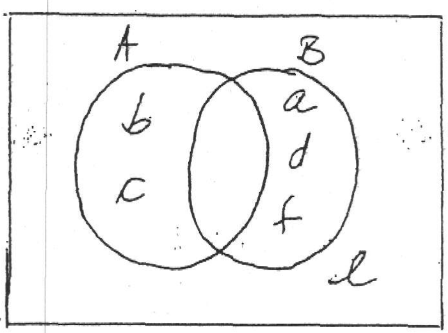
$A - B = x \in A \wedge x \notin B$

$A^c \cup B \Rightarrow x \notin A \vee x \in B$

$(A^c \cup B)^c \Rightarrow \neg(x \notin A \vee x \in B)$

$\Rightarrow x \in A \wedge x \notin B$

134.– Re:= {a, b, c, d, e, f}



a) $N(B-A)=3$; incorrecto

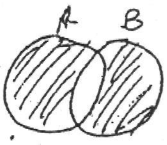
b) $N(A\cap B)=\phi$; incorrecto

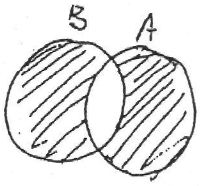
c) $A=\{b, c\}$; incorrecto

d) $N(B)=3$; incorrecto

e) $N(A^c \cup B)=\{a, d, f, e\}=\underline{4}$

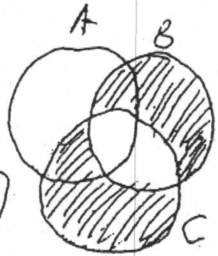
135.– a) $A \Delta B$ $= B \Delta A$





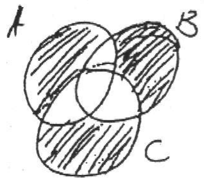
b)

$\boxed{B \Delta C}$

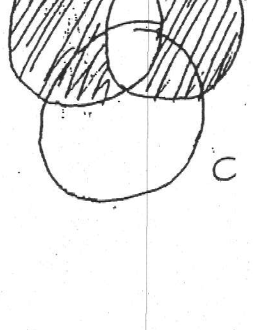


$\Rightarrow$

$A \Delta (B \Delta C)$

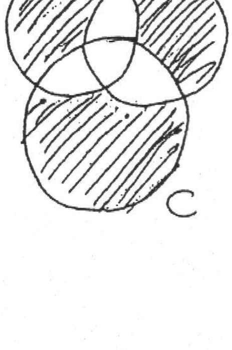


$(A \Delta B)$

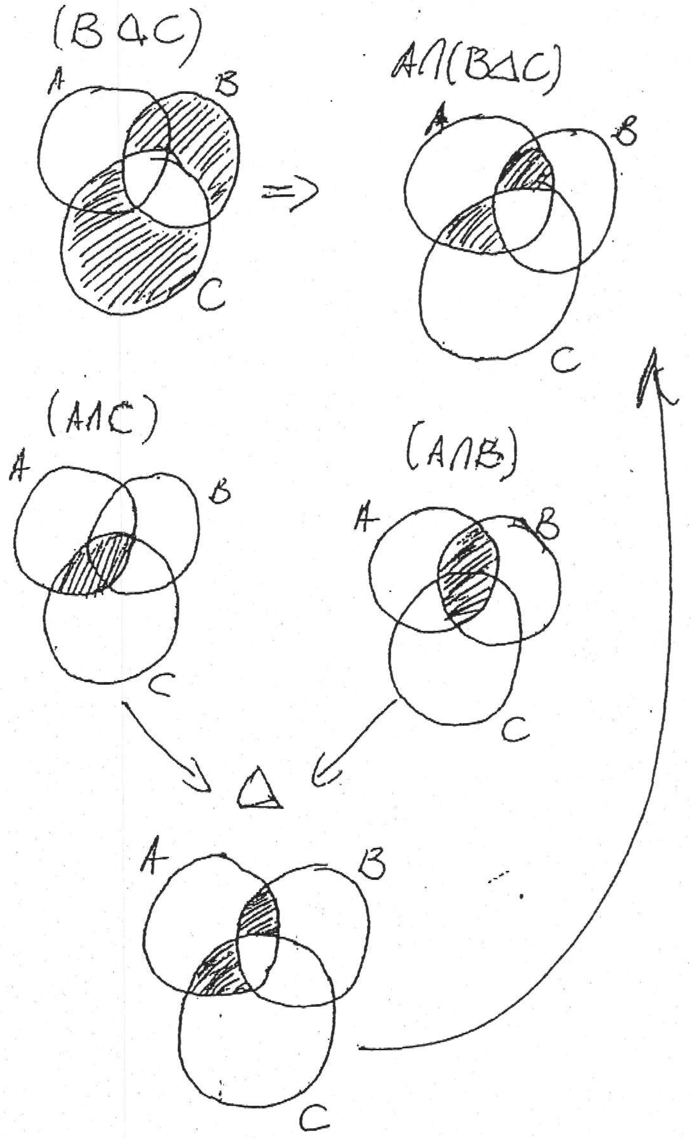


$\Rightarrow$

$(A \Delta B) \Delta C$



36.— $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$



—

total 335

C : 215

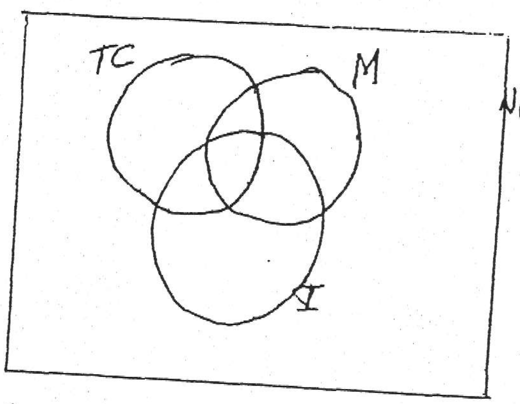
: 190

: x/omnus M

: TC∩I

: I∩M

: TC∩M



$N(A \cup B \cup C) = N(A \cup B) + N(A \cup C) + N(B \cup C) - N(A \cap B) - N(A \cap C) - N(B \cap C) + x$

$x = 335 - 215 - 190 - 255 + 70 + 110 + 145$

$x = 335 + 70 + 110 + 145 - (215 + 190 + 255)$

$x = \underline{\underline{0}}$

138.-

total: 100

B: 50

F: 40

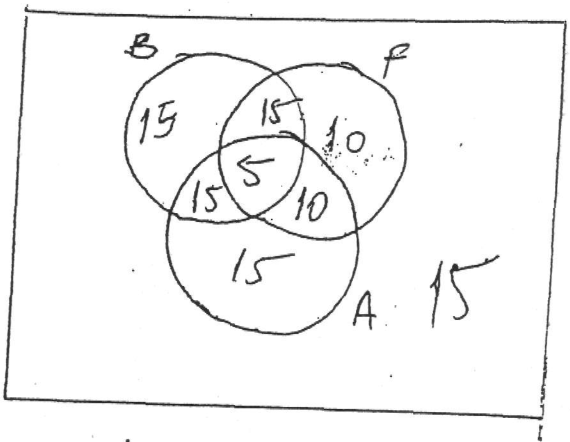
A: 45

$B \cap F$: 20

$B \cap A$: 20

$F \cap A$: 15

$F \cap A \cap B$: 5



a) Verdadero

b) Verdadero

c) Verdadero

d) Verdadero

e) Falso

139.- $p(x)$: x es impar $\Rightarrow A_{p(x)}: \{1,3,5,7,9...\}$

$q(x)$: x es par $\Rightarrow A_{q(x)}: \{2,4,6,8...\}$

a) $A(p(x) \Rightarrow q(x)) \subseteq A_{q(x)}$

$A^c_{p(x)} \cup A_{q(x)}$

$A_{q(x)} \cup A_{q(x)}$

$A_{q(x)} \subseteq A_{q(x)}$

Verdadero

b) $Re = A_{p(x)} \cup A_{q(x)}$

Verdadero

c) $A_{p(x)} = A^c_{q(x)}$

$A_{p(x)} = A_{p(x)}$

Verdadero

d) $A_{q(x)} - A_{p(x)} = \phi$

$A_{q(x)} \neq \phi$

Falso

140.– $p(x)$: x es divisor de 12 $\Rightarrow Ap(x): \{1, 2, 3, 4\}$

$q(x)$: x es primo $\Rightarrow Aq(x): \{2, 3, 5\}$

a) $Ap(x) \cup Aq(x) = Re$

Verdadero

b) $\exists x \in Re\ [p(x) \wedge q(x)] = \{2, 3\}$

Verdadero

c) $\exists x \in Re\ [\underbrace{\neg p(x)}_{5} \wedge q(x)]$

$5 \wedge \{2, 3, 5\} \Rightarrow \{5\}$

Verdadero

141.– $Re = \{-3, -2, -1, 1, 2, 3\}$

$p(x): x(x+2) = 0 \Rightarrow Ap(x): \{-2\}$

$q(x): x^2 > 0 \Rightarrow Aq(x) \{-3, -2, -1, 1, 2, 3\}$

a) $A[p(x) \wedge q(x)] = \{-2\}$; incorrecto

b) $A[p(x) \cup q(x)] = \{-3, -2, -1, 1, 2, 3\} = Re$

c) $A[p(x) \Rightarrow q(x)] = [\neg p(x) \cup q(x)]$

$\therefore \{-3, -1, 1, 2, 3\} \cup \{-3, -2, -1, 1, 2, 3\}$

$\therefore Re$; correcto

d) $A^c[q(x)] = \phi$

e) $A[q(x) \Rightarrow p(x)] = A[\underbrace{\neg q(x)}_{\phi} \cup p(x)]$

$\phi \cup \{-2\}$

$\therefore \{-2\}$

142 — $Re = \{0,1,2,3,4,5,6\}$

$p(x)$: x es número par $\Rightarrow Ap(x): \{2,4,6\}$

$q(x)$: x es mayor que 7 $\Rightarrow Aq(x): \emptyset$

$r(x)$: x es menor que 10 $\Rightarrow Ar(x): \{0,1,2,3,4,5,6\}$

$s(x)$: x es un número impar $\Rightarrow As(x): \{1,3,5\}$

a) $Ap(x) \cup A\,q(x) : \{2,4,6\}$

b) $A(s) \cap Ar(x) : \{1,3,5\}$

c) $Ap(x) \cup As(x) : \{1,2,3,4,5,6\}$

d) $A(p(x) \Rightarrow q(x))$

$A(\underbrace{\neg p(x)} \cup q(x))$

$\Downarrow$

$\{0,1,3,5\} \cup \emptyset$

$\therefore \{0,1,3,5\}$

e) $A[(p(x) \Rightarrow s(x)) \Rightarrow (q(x) \Rightarrow r(x))]$

$A[(\neg p(x) \cup s(x)) \cup (\neg q(x) \cup r(x))]$

$A[(\underbrace{p(x)} \cap \neg s(x)) \cup (\underbrace{\neg q(x) \cup r(x)})]$

$[\{2,4,6\} \cap \{0,2,4,6\}] \cup [Re \cup Re]$

$\{0,2,4,6\} \cup Re$

$\therefore Re$

f.-) $\underbrace{A^c r(x)}_{\phi} \cap \underbrace{A s(x)}_{\square}$

$\phi \cap \square$

ϕ

g.-) $\underbrace{(Re - Ap(x))}_{\{0,1,3,5\}} \cap \underbrace{(Aq(x) \cup As(x))}_{\{1,3,5\}}$

$\{0,1,3,5\} \cap \{1,3,5\}$

$\{1,3,5\}$

143.- $Re = \{1,2,3,4,5\}$

$p(x): x^2 - x + 41$ es primo

a) $Ap(x): \{1,2,3,4,5\}$

$(1)^2 - 1 + 41 \Rightarrow 41$ ✓

$(2)^2 - 2 + 41 \Rightarrow 43$ ✓

$(3)^2 - 3 + 41 \Rightarrow 47$ ✓

$(4)^2 - 4 + 41 \Rightarrow 53$ ✓

$(5)^2 - 5 + 41 \Rightarrow 61$ ✓

b1) $\exists x \in Re\ \neg p(x)$; Falso

b2) $\forall x \in Re\ p(x)$; verdadero

144.- $Re = \{1,2,3,4,5,6,7,8,9\}$

$p(x)$: x es divisor de 284 — $Ap(x): \{1,2,4\}$

$q(x): x + 3 < 9$ — $Aq(x): \{1,2,3,4,5\}$

$r(x): x + 2 = 8$ — $Ar(x): \{6\}$

$m(x)$: x es primo — $Am(x): \{1,2,3,5,7\}$

a) $A\,p(x): \{1,2,4\}$

b) $A\,q(x): \{1,2,3,4,5\}$

c) $A\,r(x): \{6\}$

d) $A\,m(x): \{1,2,3,5,7\}$

e) $A\,[p(x) \vee r(x)]$

$\{1,2,4,6\}$

f) $A\,[q(x) \wedge m(x)]: \{1,2,3,5\}$

g) $A\,[m(x) \Rightarrow \neg r(x)]$

$\therefore A\,[\underbrace{\neg m(x)} \vee \underbrace{\neg r(x)}]$

$:\{4,6,8,9\} \cup \{1,2,3,4,5,7,8,9\} = \{1,2,3,4,5,6,7,8,9\}$

h) $A\,[\underbrace{\neg r(x)} \wedge q(x)]$

$\rightarrow \{1,2,3,4,5\}$

$\{1,2,3,4,5,7,8,9\} \longrightarrow \{1,2,3,4,5\}$

145.— Para Todo x

Negación: existe un x.

Para todo número natural n; $n+2>8$

Negación Existe algún número natural tal que:

$n+2 \leq 8$

148.- $Re = \{1, 2, 3, 4, 5\}$

a) $x-3=1 \Rightarrow x=4 \quad \therefore \exists x\,(x-3=1)$; correcto

b) $x+3<5 \Rightarrow x=1 \quad \therefore \exists x\,(x+3<5)$; FALSO

c) $x>1 \Rightarrow x=2,3,4,5 \quad \therefore \exists x\,(x>1)$; FALSO

d) $x+3<4 \Rightarrow x=\phi \quad \therefore$ FALSO

e) $x^2-4x+3=0$; es una ecuación cuadrática, tendrá 2 raíces y no es el referencial

149.- $N(A)=2$

$N(B)=1$

$N(P \times A) = 2 \times 1$

$\Rightarrow 2$

$N(P(A \times B)) = 2^2$

$\Rightarrow 4$ // b) Falso

150.-

$$\underbrace{A \times (B \cap C)}_{\phi} \neq \underbrace{(A \times B)}_{No\ \phi} \cup \underbrace{(A \times C)}_{No\ \phi}$$

b) Falso

151.- $A = \{1, \{1\}, \phi\}$

$N(A) = 3$ b) Falso

152.- (a,b) ; (c,d)

$(a,b) = (c,d) \quad \therefore a=c \wedge b=d$

a) correcto

146.- $\forall x\,(x+2 = 5 \;\wedge\; x-1 \leq 3)$

Negación: $\neg\forall x\,(x+2 \neq 5 \wedge x-1 \leq 3)$

$\therefore\ \exists x\,(x+2 \neq 5 \vee x-1 > 3)$

b) Correcto

147.- $\forall x\,[(a(x) \wedge (\neg a(x) \Rightarrow \neg b(x)))]$

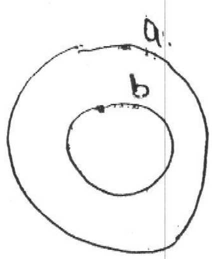
Negación: $\neg\forall x\,[(a(x) \wedge (b(x) \Rightarrow a(x)))]$

$\therefore\ \exists x\,\neg[(a(x) \wedge a(x)]$

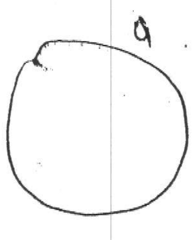
$\exists x\,\neg[(a(x))]$

$\exists x\,[\neg a(x)]$

$\forall x\,(b(x) \Rightarrow a(x)]$



$\forall x\,(a(x) \wedge (b(x) \Rightarrow a(x)))$



c) Correcto

153.- $A \times B \neq B \times A$.

$(a,3) \neq (3,a)$

b) Falso

154.- $(a,6) = (6,a)$

Condición $a=6$.

155.- $N(A) = 2$ $N(A \times B) = 2 * 4$

$N(B) = 4$ $\Rightarrow 8$

$N(P(A \times B)) = 2^8$

$\Rightarrow 256$

$$\underbrace{N(A \times B)}_{8} + \underbrace{N(P(A \times B))}_{256}$$

$\Rightarrow 264$ b) Falso

156.- $C - B = \{ \Diamond, \bullet, \nabla \}$

$N((A \times (C - B))) = 3 \times 3$

$\Rightarrow 9$ b) Falso

157.- a) verdadero

b) verdadero

c) verdadero

d) verdadero

e) $N(A \times B) = N(B \times A)$; Falso

158.- $N(A)=2$

$N(B)=3$

$N(C)=3$

$N(B\cap C)=2$

$B\times C \Rightarrow$ No toma en cuenta si hay elementos repetidos en los conjuntos

$N(B\times C)=3\times 3$

$\Rightarrow 9$

$N(A\times(B\times C))=2*9$

$\Rightarrow 18$ b) correcto

RELACIONES

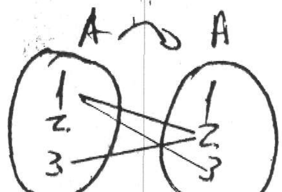
159.- En una FUNCION, el dominio debe ser el conjunto de partida b) Falso

160.- b) FALSO

161.- b) FALSO

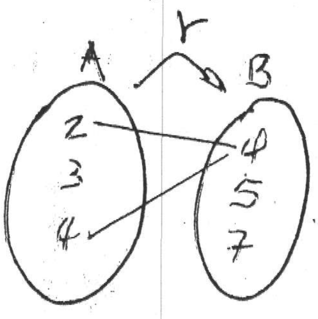
162.- a) verdadero

163.



a) verdadero

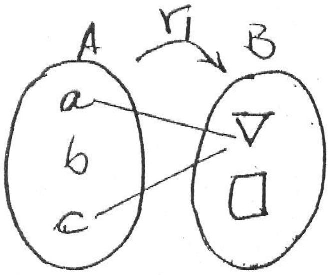
164 -

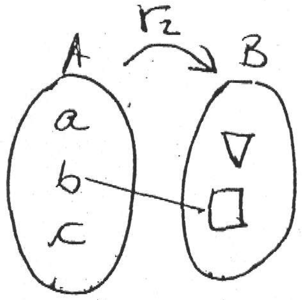


$N(r)=2$

b) Falso

65.—



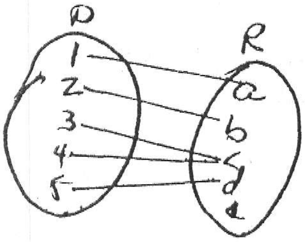


$r_1 \cup r_2$ es un función

a) verdadero; todo los elementos del dominio se relacionan 1 sola vez.

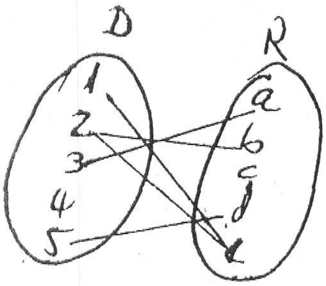
66.—

a)



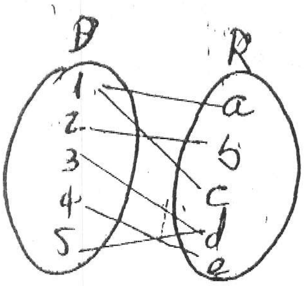
FUNCION

b)



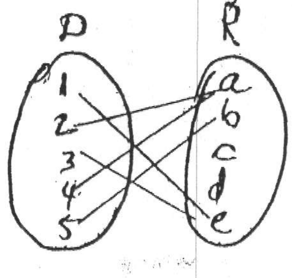
NO ES FUNCION

c)



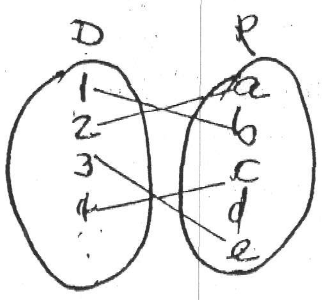
NO ES FUNCION

d)



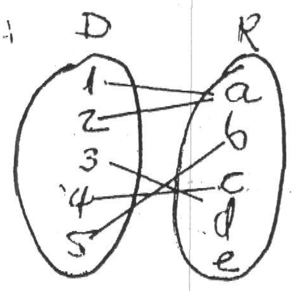
FUNCION

e)



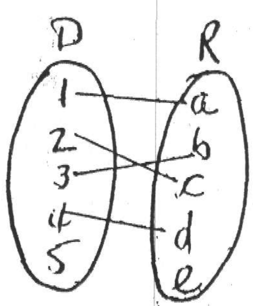
FUNCION

f)



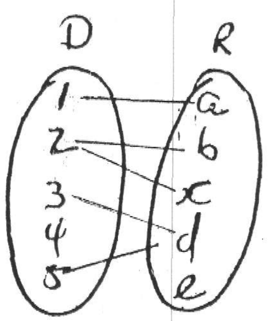
FUNCION

g)



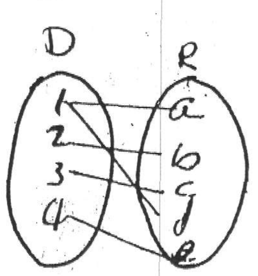
NO ES FUNCION

h)



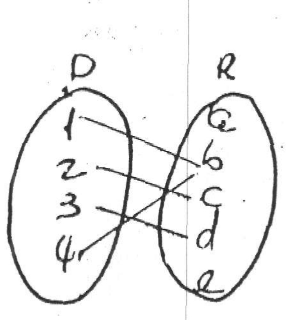
NO ES FUNCION

i)



NO ES FUNCION

j)



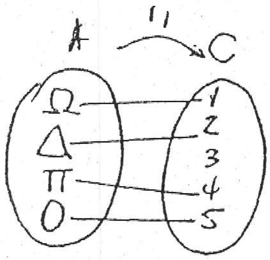
FUNCION

a) Función
d) Función
e) Función Inyectiva.
f) Función
g) Función Inyectiva.

No hay garantia que se relacionen los elementos uno a uno

b) Falso.

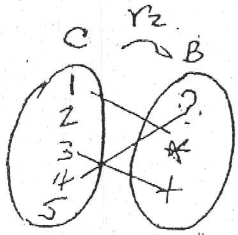
a)



$rg(r_1) = \{1, 2, 4, 5\}$

FALSO

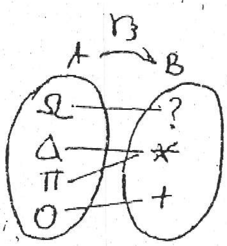
b)



$dom(r_2) = \{1, 3, 4\}$

FALSO

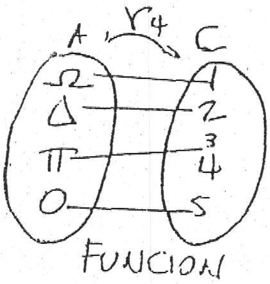
c)



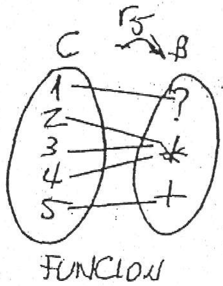
NO INYECTIVA
SÍ SOBREYECTIVA } No es biyectiva

FALSO

d)



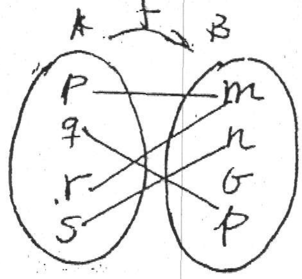
FUNCION

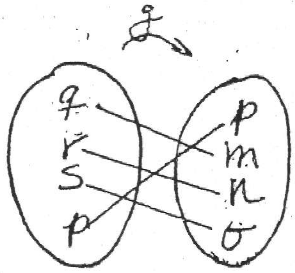


FUNCION

Si $(r_4 \wedge r_5)$ son funciones, entonces $(r_4 \vee r_5)$ son sobreyectivos

170.—



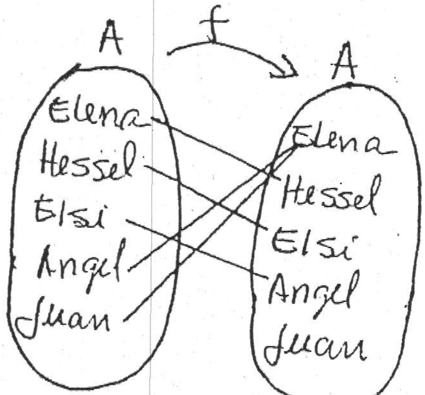


F. inyectiva

F. sobreyectiva

d) correcto

171.—



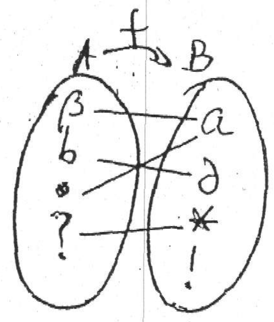
$f(\text{Juan}) = \text{Elena}$

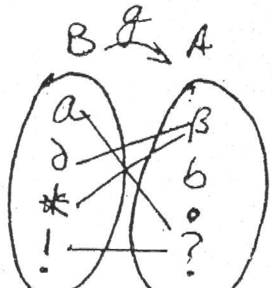
$f(\text{Elena}) = \text{Hessel}$

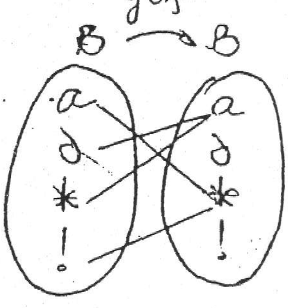
$\left.\right\}$ $f \circ f(\text{Juan}) = \text{Hessel}$

b) correcto

172—

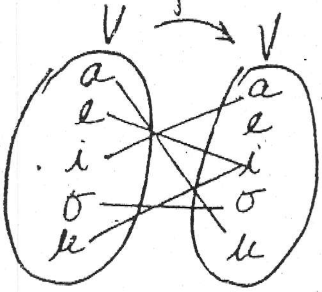


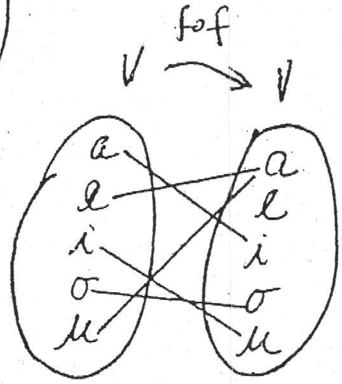




b) correcto

3.-

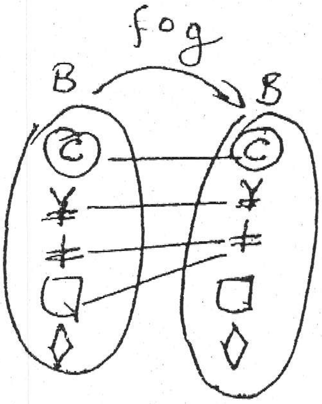


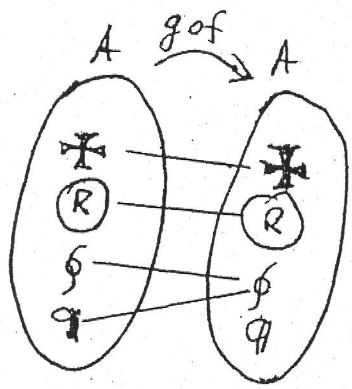


$rg(f\circ f)=\{a, i, o, u\}$

b) correcto

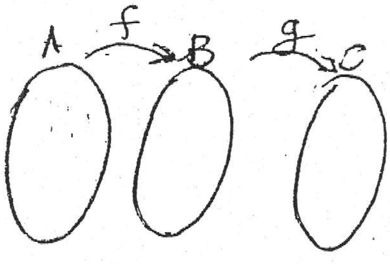
4.-





e) correcto

5.-



a) Falso

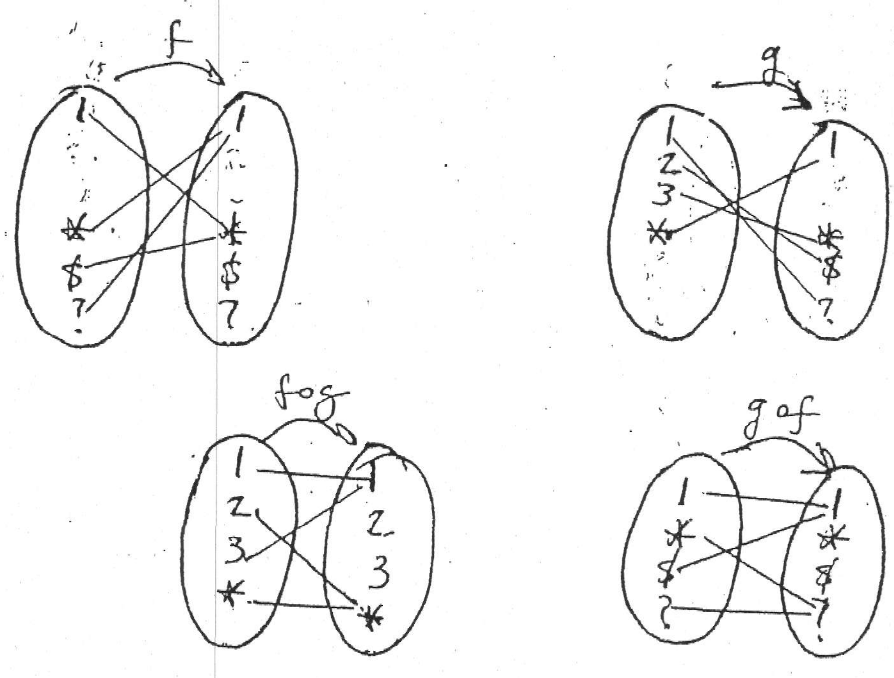
b) Falso

c) Verdadero

d) Falso

e) Falso

17.6.-

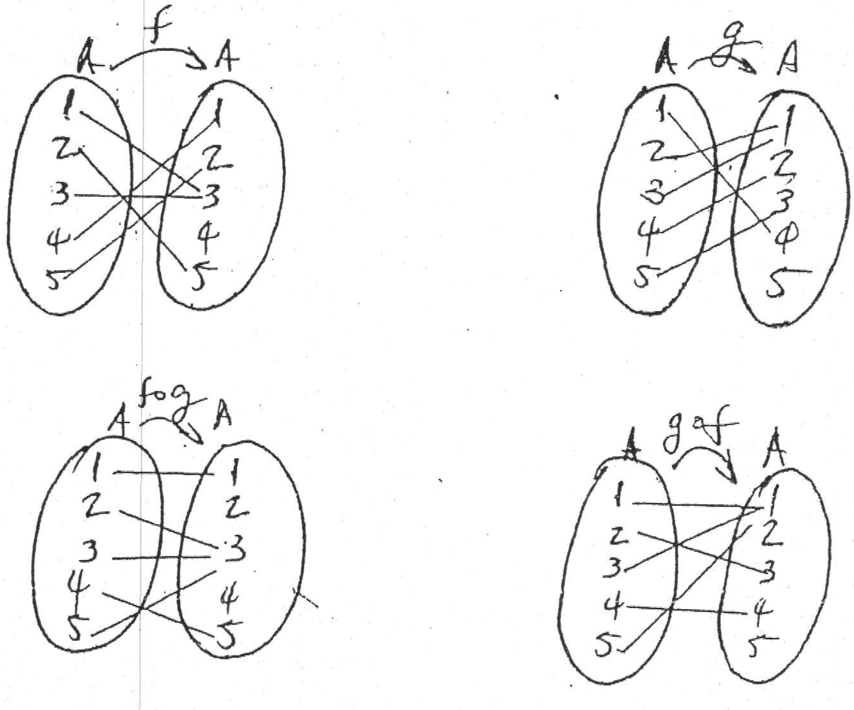


e) Falso

17.7.-

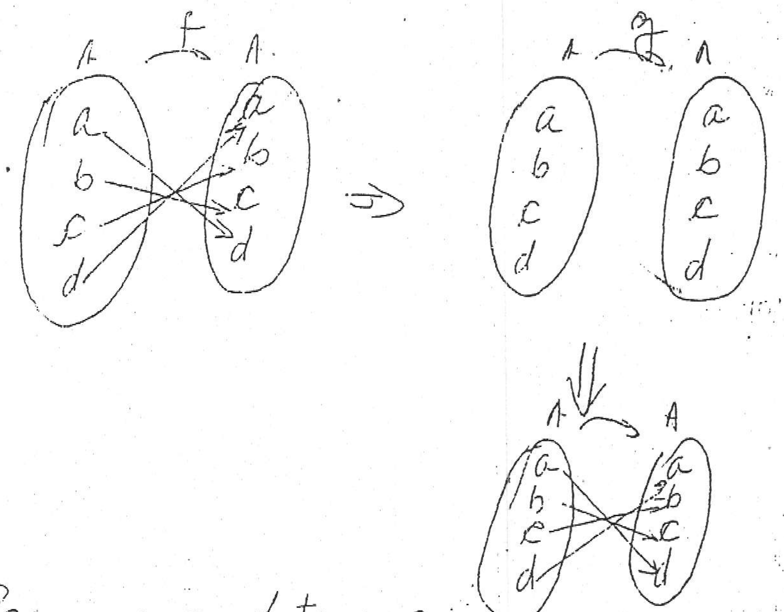
$$\left.\begin{aligned} g(1) &= 2 \\ h(2) &= 3 \end{aligned}\right\} \quad h \circ g(1) = 3$$

17.8.-



e) $0 \vee 0 \equiv 0$; Falso

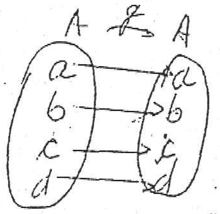
79



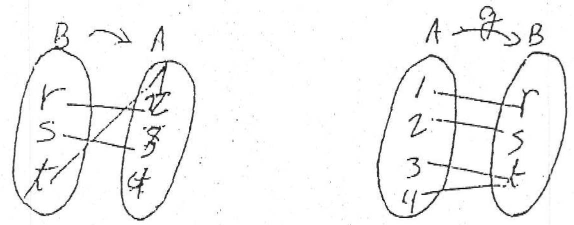
Para que se obteng $g \circ f$

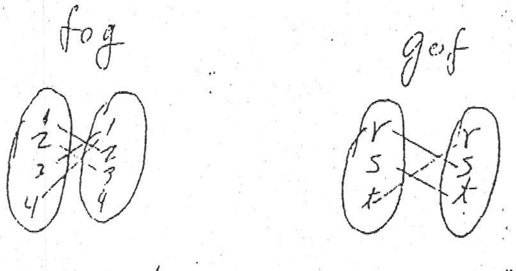
$$f = g \circ f$$

entonces g debe ser



80





a) Falso; no lo es

b) Falso; rango $\{1, 2, 3\}$

c) Falso; (s, t)

d) Verdadero

e) Falso; es f. biyectiva

SOLUCIÓN
EJERCICIOS PROPUESTOS
CAPÍTULO DOS

$3^{\underline{90}}$

MATEMÁTICAS
$\boxed{\text{NÚMEROS REALES}}$

1.- $\pi * \sqrt{2} \Rightarrow$ el resultado sigue siendo un número irracional

b) Falso

2.- $\frac{\cancel{\pi}}{2\cancel{\pi}} + 4$

$\Rightarrow \frac{1}{2} + 4 = \frac{9}{2}$ (número racional)

b) Falso

3.-

a) $\{x/x \in \mathbb{Z};\ 3<x<4\}$ NO HAY NUMEROS ENTEROS ENTRE 3 ∧ 4.

Sol: ϕ

b) $\{x/x \in \mathbb{Q};\ x^2-2=0\}$

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2} \Rightarrow$ # irracional

Sol: ϕ

c) $\{x/x \in \mathbb{I};\ x+1 \geq 0\}$

$x+1 \geq 0$

$\boxed{x \geq -1} \Rightarrow$ # racional

Sol: ϕ

d) $\{x/x \in \mathbb{R} \ ; x^2+1 \geq 0\}$

$x^2 \geq -1$

$x = \pm i \Rightarrow$ # imaginario ; NO ES NUMERO REAL

Sol: ϕ

e) $\{x/x \in \mathbb{N} \ ; \ x^{-1} - 2 = 0\}$ $\quad x^{-1} - 2 = 0$

$\frac{1}{x} = 2$

$\boxed{x = \frac{1}{2}} \Rightarrow$ NO ES NUMERO NATURAL

Sol: ϕ

4.- a) $$\frac{(7-6{,}35) \div 6{,}5 + 9{,}9}{(1{,}2 \div 36 + 1{,}2 \div 0{,}25 - 1\frac{5}{16}) \div \frac{169}{24}}$$

$$\Rightarrow \frac{0{,}65 \div 6{,}5 + 9{,}9}{\left(\frac{1{,}2}{36} + \frac{1{,}2}{0{,}25} - \frac{21}{16}\right) \times \frac{24}{169}} \Rightarrow \frac{\frac{65}{650} + 9{,}9}{\left(\frac{\cancel{12}^{1}}{\cancel{360}_{3}} + \frac{\cancel{120}^{24}}{\cancel{25}_{5}} - \frac{21}{16}\right) \times}$$

$$\Rightarrow \frac{\frac{1}{10} + 9{,}9}{\left(\frac{1}{30} + \frac{24}{5} - \frac{21}{16}\right) \times \frac{24}{169}} \Rightarrow \frac{0{,}1 + 9{,}9}{\left(\frac{8 + 1152 - 315}{240}\right) \times \frac{24}{169}}$$

$$\Rightarrow \frac{10}{\frac{\cancel{845}^{169}}{\cancel{240}_{\cancel{10}_{2}}} \times \frac{\cancel{24}^{1}}{\cancel{169}_{1}}} \Rightarrow \underline{\underline{20}}$$

b) $3(6-1{,}333) + 6(1{,}333\ldots) - 16{,}666$

$\Rightarrow 3(4{,}66666\ldots) + 3(2{,}666666\ldots) - 16{,}6666\ldots$

$\Rightarrow 3\left(4+\frac{2}{3}\right) + 3\left(2+\frac{2}{3}\right) - \left(16+\frac{2}{3}\right)$

$\Rightarrow 3\left(\frac{14}{3}\right) + 3\left(\frac{8}{3}\right) - \left(\frac{50}{3}\right)$

$\frac{1}{3}\left(3(14) + 3(8) - 50\right)$

$\frac{1}{3}\left(42 + 24 - 50\right)$

$\boxed{\frac{16}{3}}$

$x = 0{,}6666$

$10x = 6{,}6666\ldots$

$-x - 0{,}6666$

$9x = 6$

$x = \frac{\cancel{6}^{2}}{\cancel{9}_{3}}$

$\boxed{x = \frac{2}{3}}$

c) $\dfrac{3}{1+\frac{2}{0{,}5}} - \dfrac{2}{1+\frac{3}{0{,}5}} + \dfrac{\frac{54}{3}+1{,}666\ldots}{9+2{,}6666}$

$\Rightarrow \dfrac{3}{1+\frac{\cancel{20}^{4}}{\cancel{5}_{1}}} - \dfrac{2}{1+\frac{\cancel{30}^{6}}{\cancel{5}_{1}}} + \dfrac{\frac{54}{3}+\left(1+\frac{2}{3}\right)}{9+\left(2+\frac{2}{3}\right)}$

$\Rightarrow \dfrac{3}{5} - \dfrac{2}{7} + \dfrac{\frac{54}{3}+\frac{5}{3}}{9+\frac{8}{3}} \Rightarrow \dfrac{3}{5} - \dfrac{2}{7} + \dfrac{\frac{59}{\cancel{3}}}{\frac{35}{\cancel{3}}}$

$\Rightarrow \frac{3}{5} - \frac{2}{7} + \frac{59}{35} \Rightarrow \dfrac{21-10+59}{35} \Rightarrow$

$\frac{70}{35} \Rightarrow 2$

5.- NO LO ES

$\sqrt{2}+2 \Rightarrow$ es un número irracional.

6.- $S=\{1,2,3,4\}$

$$a*b=\begin{cases} a & ; a\geq b \\ b & ; a<b \end{cases}$$

a) $1*2=2 \quad 1*3=3 \quad 1*4=4 \quad 2*3=3 \quad 3*4=4$

$2*1=2 \quad 3*1=3 \quad 4*1=4 \quad 3*2=3 \quad 4*3=4$

SI ES CONMUTATIVA. ; FALSO

b) Si $a>b>c$

$$\underbrace{\underbrace{(a*b)}_{a}*c}_{a} \qquad \underbrace{a*\underbrace{(b*c)}_{b}}_{a}$$

Si es asociativa

FALSO

c) $1*2=2$

$1*3=3$

$1*4=4$

$1*5=5$

$1*1=1$

1 es el elemento neutro ; VERDADERO

d) FALSO ; $\underbrace{(4*a)}_{4}=\underbrace{(b*4)}_{4}$

e) $\underbrace{\underbrace{(1*3)}_{3}*2}_{3} = \underbrace{\underbrace{(2*1)}_{2}*\underbrace{(3*4)}_{4}}_{4}$ FALSO

$3 \neq 4$

a) $(a \nabla b) = a$; $(b \nabla a) = b$ } NO ES CONMUTATIVA;

FALSO

a) Módulo 7 $S = \{0,1,2,3,4,5,6\}$

i) $\overline{4} + \overline{5} = 2$

ii) $\overline{5} \cdot \overline{3} = 1$

iii) $\overline{2} \cdot \overline{4} = 1$

iv) $\overline{5} \cdot \overline{6} = 2$

v) $\overline{6} \cdot \overline{1} = 6$

vi) $\overline{1} + \overline{0} = 1$

vii) $\overline{2} \cdot \overline{0} = 0$

viii) $\overline{5} + \overline{3} = 1$

ix) $\overline{4} \cdot \overline{5} = 6$

x) $\overline{2} + \overline{5} = 0$

b) Módulo 6 $S = \{0,1,2,3,4,5\}$

i) $= \overline{4} + \overline{5} = 3$

ii) $\overline{5} \cdot \overline{3} = 3$

iii) $\overline{2} \cdot \overline{4} = 2$

iv) $\overline{5} \cdot \overline{6} = 0$

v) $\overline{6} \cdot \overline{1} = 0$

vi) $\overline{1} \cdot \overline{0} = 0$

vii) $\overline{2} \cdot \overline{0} = 0$

viii) $\overline{5} + \overline{3} = 2$

ix) $\overline{4} \cdot \overline{5} = 2$

x) $\overline{2} + \overline{5} = 1$

9.- $2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}$ $3/2$

$\Rightarrow 2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{3/2}}} \Rightarrow 2+\cfrac{1}{3+\cfrac{1}{1+\frac{2}{3}}}$ $5/3$

$\Rightarrow 2+\cfrac{1}{3+\cfrac{1}{5/3}} \Rightarrow 2+\cfrac{1}{3+\frac{3}{5}}$ $\frac{18}{5}$

$\Rightarrow 2+\cfrac{1}{\frac{18}{5}} \Rightarrow 2+\frac{5}{18} \Rightarrow \frac{41}{18}$ c) correcto

10.- $a*0=0$; porque 0 veces se repite a.

11.- $(a+b)\cdot c = a\cdot c + b\cdot c$

(a+b) se repite c veces

$(a+b)+(a+b)+(a+b)+ \cdots\cdots a+b$

$\therefore a+b+a+b+a+b+ \cdots\cdots a+b$

$\therefore$ habrá c veces a y c veces b

$\therefore c.a+cb$

$\therefore (a+b)\cdot c = ac+bc$

7.-

a) $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$ FALSO

$\therefore \frac{ac+ab}{bc}$

$\therefore \frac{a(b+c)}{bc}$

b) $(a+b) \div c = \frac{a}{c} + \frac{b}{c}$ VERDADERO

$\therefore \frac{a+b}{c}$

-

$\forall a \in \mathbb{R} \; (a+c) = a$

$a+c=a$

$c=a-a$

$\boxed{c=0}$

-

$\sqrt[7]{(1-\sqrt{3})^7} = \left(\sqrt[8]{1-\sqrt{3}}\right)^8$

$1-\sqrt{3} = 1-\sqrt{3}$

b) FALSO

15.– a) $e^{\pi} < 81$

$\pi \ln e < \ln 81$

$\pi < \ln 81$ verdadero

$8 < e^{\pi}$

$\ln 8 < \ln e^{\pi}$

$\ln 8 < \pi \cdot \cancel{\ln e}^{1}$

$\ln 8 < \pi \cdot$ verdadero

b) $\sqrt{3}+\sqrt{2} > \frac{1}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$; FALSO

$\Rightarrow \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^{2}-(\sqrt{2})^{2}}$

$\Rightarrow \sqrt{3}+\sqrt{2}$

$\sqrt{3}+\sqrt{2} = \sqrt{3}+\sqrt{2}$

16.–

a) $\frac{69}{200}$; $\frac{19}{100}$; 0.8 ; $\frac{1}{5}$ FALSO

0,8 ; 0,2

b) $\frac{19}{100}$; $\frac{1}{5}$; $\frac{69}{200}$; 0,8

0,19 ; 0,2 ; 0,345 ; 0,8 CORRECTO

$\frac{0,69}{2} = 0,345$

17.–

$\frac{6-1}{\frac{3}{4} \times 60}$

$\Rightarrow 45$

$\frac{6-2}{\frac{2}{3} \times 60}$

$\Rightarrow 40$

c) correcto

- total: 18 cuadros

rayados: 10 cuadros

área R $\frac{\cancel{10}^{5}}{\cancel{18}9}$

área R: $\frac{5}{9}$ (un poco más de la mitad)

d) correcto

—

a) $\frac{4}{5} = 0,8$

b) $\frac{3}{4} = 0,75$

c) $\frac{5}{8} = 0,625$

d) $\frac{7}{10} = 0,7$

e) $\frac{31}{40} = 0,775$

50 (8
20 0,625
40
(0)

310 (40
300 0,775
200
(00)

a) CORRECTO

Re = Naturalez

$M = a^3 \cdot b^2$

$\therefore M = a \cdot a \cdot a \cdot b \cdot b$

divisores

$1, a, b, ab, a^2b, a^3b, a^2b^2, a^3b^2, ab^2, a^2, a^3, b^2$

VERDADERO

21.-

72	108	90	2
36	54	45	3
12	18	15	3
4	6	5	

MCD = 18

b) FALSO

22.- a) VERDADERO

23.-

72	2
36	2
18	2
9	3
3	3
1	

$2^3 \cdot 3^2$

$\Rightarrow 12$

24.- Divisores: 1, 2, 3, 4, 8, 9, 6, 12, 24, 18, 36

$1+2+3+4+8+9+6+12+24+18+36$

$\Rightarrow 123$ b) correcto

25.-

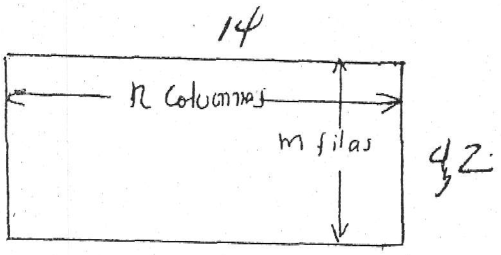
15	25	5
3	5	3
1	5	5
	1	

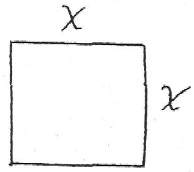
75.

b) correcto

6.-

	2	3	4	5	9	11
56	V	F	V	F	F	F
261	F	V	F	F	V	F
660	V	V	V	V	F	V
1455	F	V	F	V	F	F





$nx = 14 \Rightarrow x = \frac{14}{n}$

$mx = 4,2 \Rightarrow x = \frac{4,2}{m}$

$\frac{14}{n} = \frac{4,2}{m}$

$14m = 4,2n$

$n = \frac{14}{4,2} m$

$n = \frac{\cancel{140}^{\cancel{20}\,10}}{\cancel{42}_{\cancel{6}\,3}} m$

$\boxed{n = \frac{10}{3} m}$

Si $m = 3$

$\therefore n = 10$

$área = 14 * 4,2$

$área = 58,8$

$3 * 10 * x^2 = 58,8$

$x^2 = \frac{58,8}{30}$

$x^2 = \frac{\cancel{588}^{196}}{\cancel{300}_{100}}$

$x^2 = \frac{196}{100}$

$x = \frac{14}{10}$

$\boxed{x = 1,4 \text{ m}}$

28.— Bombones: 24
caramelos: 42

$\frac{24}{x}$; $\frac{42}{x}$

$x = 1, 2, 3, 6$ mayor

29.— $2x = 3y$ $\quad 3y = 5z$ $\quad 5z = 6w$

$\boxed{y = \frac{2}{3}x}$

$\cancel{3}\left(\frac{2}{\cancel{3}}x\right) = 5z$

$\boxed{z = \frac{2}{5}x}$

$\cancel{5}\left(\frac{2}{\cancel{5}}x\right) = 6w$

$w = \frac{2}{6}x$

$\boxed{w = \frac{1}{3}x}$

$x \Rightarrow$ debe ser entero

3	5	15, 30, 45 ......

$x = 15, 30, 45 \ldots$

$2x = 30$ ✗
$2x = 60$ ✓
$2x = 90$ ✗

30.— $$\frac{\frac{2}{1-a} + \frac{2}{1+a}}{\frac{2}{1+a} - \frac{2}{1-a}} \Rightarrow \frac{\frac{2(1+a)+2(1-a)}{\cancel{(1-a)(1+a)}}}{\frac{2(1-a)-2(1+a)}{\cancel{(1+a)(1-a)}}} \Rightarrow \frac{2+\cancel{2a}+2-\cancel{2a}}{\cancel{2}-2a-\cancel{2}-2a}$$

$$\Rightarrow \frac{4}{-4a} \Rightarrow -\frac{1}{a}$$

$$3x^2+7x-6$$

$$\Rightarrow \frac{(3x+9)(3x-2)}{3}$$

$$\Rightarrow (x+3)(3x-2).$$

c) Correcto

2.- $$\frac{x^3-1}{x^2-1}-(x+1)$$

$$\Rightarrow \frac{\cancel{(x-1)}(x^2+x+1)}{(x+1)\cancel{(x-1)}}-(x+1).$$

$$\Rightarrow \frac{(x^2+x+1)}{(x+1)}-(x+1) \Rightarrow \frac{x^2+x+1-(x+1)(x+1)}{(x+1)}.$$

$$\Rightarrow \frac{x^2+x+1-(x^2+2x+1)}{x+1} \Rightarrow \frac{\cancel{x^2}+\cancel{x}+\cancel{1}-\cancel{x^2}-\cancel{2x}-\cancel{1}}{x+1}.$$

$$\Rightarrow \boxed{-\frac{x}{x+1}}$$

e) Correcto.

3.- $$\frac{\sqrt{1-x}-\dfrac{x}{2\sqrt{1+x}}}{1+x} \Rightarrow \frac{\dfrac{2(\sqrt{1+x})^2-x}{2\sqrt{1+x}}}{} \Rightarrow$$

$$\Rightarrow \frac{\dfrac{2+2x-x}{2\sqrt{1+x}}}{1+x} \Rightarrow \frac{x+2}{2\sqrt{1+x}\,(1+x)} \Rightarrow \frac{x+2}{2(x+1)^{3/2}}$$

d) Correcto.

4.– a) $a^4+3a^3+4a^2-6a-12$

$$(a^4+4a^2-12)+(3a^3-6a)$$

$$(a^2+6)(a^2-2)+(3a^3-6a)$$

$$(a^2+6)(a^2-2)+3a(a^2-2)$$

$$(a^2-2)(a^2+3a+6)$$

$$(a^2-2)(a^2+3a+6)$$

$$(a+\sqrt{2})(a-\sqrt{2})(a^2+3a+6)$$

b) $(ab+ac+bc)(a+b+c)-abc$

$$a^2b+a^2c+3bac+ab^2+b^2c+ac^2+bc^2-abc$$

$$a^2b+a^2c+2abc+ab^2+b^2c+ac^2+bc^2$$

$$(a^2b+a^2c)+(abc+ac^2)+(abc+ab^2)+(b^2c+bc^2)$$

$$a^2(b+c)+ac(b+c)+ab(c+b)+bc(b+c)$$

$$(b+c)(a^2+ac+ab+bc)$$

$$(b+c)(a(a+c)+b(a+c))$$

$$(b+c)(a+b)(a+c)$$

c) x^3-5x^2-x+5

$\Rightarrow (x^3-5x^2)-(x-5)$

$\Rightarrow x^2(x-5)-(x-5)$

$$(x-5)(x^2-1)$$

$$(x-5)(x+1)(x-1)$$

1) $x^4-3x^3+4x^2-6x+4$

$(x^4+4x^2+4)-(3x^3+6x)$

$(x^2+2)^2-3x(x^2+2)$

$(x^2+2)(x^2-3x+2)$

$(x^2+2)(x-2)(x-1)$

) x^3-7x-6

$x^3+x^2-x^2-7x-6$

$(x^3+x^2)-(x^2+7x+6)$

$(x+1)-(x+6)(x+1)$

$x+1)(x^2-x-6)$

$x+1)(x-3)(x+2)$

c) $\frac{4}{x^2+xy}+\frac{4}{xy+y^2}$

$\Rightarrow \frac{4}{x(x+y)}+\frac{4}{y(x+y)} \Rightarrow \frac{4y+4x}{xy(x+y)} \Rightarrow$

$\Rightarrow \frac{4\cancel{(x+y)}}{x\,y\cancel{(x+y)}} \Rightarrow \frac{4}{x}$

b) $$\frac{2x}{x^2+3x+2} - \frac{x}{(x^2-4)}$$

$$\Rightarrow \frac{2x}{(x+2)(x+1)} - \frac{x}{(x+2)(x-2)} \Rightarrow \frac{2x(x-2)-x(x+1)}{(x+1)(x+2)(x-2)}$$

$$\Rightarrow \frac{2x^2-4x-x^2-x}{(x+1)(x+2)(x-2)} \Rightarrow \frac{x^2-5x}{(x+1)(x+2)(x-2)}$$

$$\Rightarrow \underline{\frac{x(x-5)}{(x+1)(x+2)(x-2)}}$$

c) $$\frac{3x}{x-1} + \frac{2}{x} - \frac{2}{x+1}$$

$$\Rightarrow \frac{3x(x+1)x+2(x-1)(x+1)-2x(x-1)}{x(x+1)(x-1)} \Rightarrow \frac{3x^3+3x^2+\cancel{2x^2}-\cancel{2}-\cancel{2x^2}+\cancel{2}}{x(x+1)(x-1)}$$

$$\Rightarrow \frac{3x^3+3x^2}{x(x+1)(x-1)} \Rightarrow \frac{3\cancel{x^2}\cancel{(x+1)}}{\cancel{x}\cancel{(x+1)}(x-1)}$$

$$\Rightarrow \frac{3x}{x-1}$$ //

d) $$\frac{4}{x^2-3x-4} + \frac{3}{x^2-16} - \frac{7}{x^2+5x+4}$$

$$\Rightarrow \frac{4}{(x-4)(x+1)} + \frac{3}{(x+4)(x-4)} - \frac{7}{(x+4)(x+1)}$$

$$\Rightarrow \frac{4(x+4)+3(x+1)-7(x-4)}{(x+1)(x+4)(x-4)} \Rightarrow \frac{\cancel{4x}+16+\cancel{3x}+3-\cancel{7x}+28}{(x+1)(x+4)(x-4)}$$

$$\Rightarrow \frac{47}{(x+1)(x+4)(x-4)}$$

$$) \quad \frac{x^2+6x+9}{x^2-9} \times \frac{x-3}{4}$$

$$\Rightarrow \frac{(\cancel{x+3})(x+3)}{(\cancel{x+3})(\cancel{x-3})} \times \frac{(\cancel{x-3})}{4}$$

$$\Rightarrow \frac{x+3}{4}$$

$$) \quad \frac{xy-x}{y^2-1} \times \frac{y+1}{x+2} \times \frac{2x+4}{5x}$$

$$\Rightarrow \frac{\cancel{x(y-1)}}{(\cancel{y+1})(\cancel{y-1})} \times \frac{(\cancel{y+1})}{(\cancel{x+2})} \times \frac{2(\cancel{x+2})}{5\cancel{x}}$$

$$\Rightarrow 2/5$$

$$) \quad \left(\frac{2}{x-3} - \frac{3}{x-2}\right) \cdot \left(\frac{3x}{x-5}\right)$$

$$\Rightarrow \frac{2(x-2)-3(x-3)}{(x-2)(x-3)} \times \frac{3x}{(x-5)} \Rightarrow \frac{2x-4-3x+9}{(x-2)(x-3)} \times \frac{3x}{x-5}$$

$$\Rightarrow \frac{-x+5}{(x-2)(x-3)} \times \frac{3x}{x-5} \Rightarrow \frac{-(\cancel{x-5})}{(x-2)(x-3)} \times \frac{3x}{(\cancel{x-5})}$$

$$\Rightarrow \frac{3x}{(x-2)(3-x)}$$

h) $\dfrac{5a^2-a-4}{a^3-1}$

$\Rightarrow \dfrac{(\cancel{5}a-\cancel{5})(5a+4)}{\cancel{5}(a-1)(a^2+a+1)} \Rightarrow \dfrac{\cancel{(a-1)}(5a+4)}{\cancel{(a-1)}(a^2+a+1)}$

$\Rightarrow \dfrac{5a+4}{a^2+a+1}$

i) $\left(\dfrac{\dfrac{x}{x+y}-\dfrac{y}{x-y}}{\dfrac{x}{x-y}+\dfrac{y}{x+y}}\right)$

$\Rightarrow \dfrac{\dfrac{x(x-y)-y(x+y)}{\cancel{(x+y)(x-y)}}}{\dfrac{x(x+y)+y(x-y)}{\cancel{(x+y)(x-y)}}} \Rightarrow \dfrac{x(x-y)-y(x+y)}{x(x+y)+y(x-y)}$

$\Rightarrow \dfrac{x^2-xy-xy-y^2}{x^2+xy+xy-y^2} \Rightarrow \dfrac{x^2-2xy-y^2}{x^2+2xy-y^2}$

j) $\dfrac{a^6+a^4+a^2+1}{a^3+a^2+a+1} \Rightarrow$

$\Rightarrow \dfrac{(a^6+a^4)+(a^2+1)}{(a^3+a^2)+(a+1)} \Rightarrow \dfrac{a^4(a^2+1)+(a^2+1)}{a^2(a+1)+(a+1)} \Rightarrow \dfrac{\cancel{(a^2+1)}(a^4+1)}{\cancel{(a^2+1)}(a+1)}$

$\Rightarrow \dfrac{a^4+1}{a+1}$

k) $\left(\frac{b}{a+b}+a\right)\left(\frac{a}{a-b}-b\right)-\left(\frac{a}{a+b}+b\right)\left(\frac{b}{a-b}-a\right)$

$\Rightarrow \left(\frac{b+a(a+b)}{a+b}\right)\left(\frac{a-b(a-b)}{a-b}\right)-\left(\frac{a+b(a+b)}{a+b}\right)\left(\frac{b-a(a-b)}{a-b}\right)$

$\Rightarrow \left(\frac{b+a^2+ab}{a+b}\right)\left(\frac{a-ab+b^2}{a-b}\right)-\left(\frac{a+ab+b^2}{a+b}\right)\left(\frac{b-a^2+ab}{a-b}\right)$

$\Rightarrow \frac{ab-ab^2+b^3+a^3-a^3b+\cancel{a^2b^2}+a^2b-\cancel{a^2b^2}+ab^3}{(a+b)(a-b)}-\frac{ab-a^3+a^2b+ab^2-a^3b+\cancel{a^2b^2}+b^3-\cancel{a^2b^2}+ab}{(a+b)(a-b)}$

$\Rightarrow \frac{\cancel{ab}-ab^2+\cancel{b^3}+a^3-\cancel{a^3b}+\cancel{a^2b}+\cancel{ab^3}-\cancel{ab}+a^3-\cancel{a^2b}-ab^2+\cancel{a^3b}-\cancel{b^3}-\cancel{ab^3}}{(a+b)(a-b)}$

$\Rightarrow \frac{2a^3-2ab^2}{a^2-b^2} \Rightarrow \frac{2a\cancel{(a^2-b^2)}}{\cancel{(a^2-b^2)}}$

$\Rightarrow 2a$

l.) $\frac{1}{1-a}-\frac{1}{1+a}-\frac{2a}{1+a^2}-\frac{4a^3}{1+a^4}-\frac{8a^7}{1+a^8}$

$\Rightarrow \frac{8a^7}{(1+a^8)}\times\frac{(1-a^8)}{(1-a^8)}=\frac{8a^7(1-a^8)}{(1+a^8)(1+a^4)(1+a^2)(1+a)(1-a)}$

$\Rightarrow \frac{(1+a^8)(1+a^4)(1+a^2)(1+a)-(1+a^8)(1+a^4)(1+a^2)(1-a)-2a(1+a^8)(1+a^4)(1-a)(1+a)-4a^3}{(1+a^8)(1-a^8)}$

$\frac{(1+a^8)(1+a^2)(1+a)(1-a)-8a^7(1-a^8)}{}$

36.-

a) $a^2b^2(b-a) + b^2c^2(c-b) + a^2c^2(a-c)$

$\Rightarrow a^2b^3 - a^3b^2 + b^2c^3 - b^3c^2 + a^3c^2 - a^2c^3$

$\Rightarrow (a^2b^3 - b^3c^2) - (a^3b^2 - b^2c^3) + (a^3c^2 - a^2c^3)$

$\Rightarrow b^3(a^2-c^2) - b^2(a^3-c^3) + a^2c^2(a-c)$

$\Rightarrow b^3(a+c)(a-c) - b^2(a-c)(a^2+ac+c^2) + a^2c^2(a-c)$

$\Rightarrow (a-c)[b^3(a+c) - b^2(a^2+ac+c^2) + a^2c^2]$

$\Rightarrow (a-c)[ab^3 + b^3c - b^2a^2 - ab^2c - b^2c^2 + a^2c^2]$

$\Rightarrow (a-c)[(ab^3+b^3c) - (a^2b^2+ab^2c) - (a^2c^2+b^2c^2)]$

$\Rightarrow (a-c)[b^3(a+c) - ab^2(a+c) - c^2(a^2-b^2)]$

$\Rightarrow (a-c)[(a+c)b^2\{b-a\} - c^2(a+b)(a-b)]$

$\Rightarrow (a-c)[b^2(a+c)(b-a) + (a+b)(b-a)]$

$\Rightarrow (a-c)(b-a)[b^2(a+c) + (a+b)]$

b) $(a-b)^3 - (a-c)^3 + (b-c)^3$

$\Rightarrow a^3 - 3a^2b + 3ab^2 - b^3 - (a^3 - 3a^2c + 3ac^2 - c^3) + b^3 - 3b^2c + 3bc^2 - c^3$

$\Rightarrow \cancel{a^3} - 3a^2b + 3ab^2 \cancel{-b^3} \cancel{-a^3} + 3a^2c - 3ac^2 \cancel{+c^3} \cancel{+b^3} - 3b^2c + 3bc^2 \cancel{-c^3}$

$3(-a^2b + ab^2 + a^2c - ac^2 - b^2c + bc^2)$

$3[-(a^2b - ab^2) + (a^2c - b^2c) - (ac^2 - bc^2)]$

$3[-ab(a-b) + c(a^2-b^2) - c^2(a-b)]$

$\Rightarrow 3(a-b)[-ab + c(a+b) - c^2]$

$\Rightarrow 3(a-b)[-ab + ac + bc - c^2]$

$\Rightarrow 3(a-b)[-(ab-ac) + (bc-c^2)]$

$\Rightarrow 3(a-b)[-a(b-c) + c(b-c)]$

$\Rightarrow 3(a-b)(b-c)[c-a]$

$a^4 - 18a^2 + 81$

$\Rightarrow (a^2-9)a^2-9)$

$\Rightarrow (a+3)(a-3)(a+3)(a-3)$

$\Rightarrow (a+3)^2(a-3)^2$

$3x^4 - 10x^3 + 10x - 3$

$\Rightarrow (3x^4 - 3) - (10x^3 - 10x)$

$\Rightarrow 3(x^4-1) - 10x(x^2-1)$

$\Rightarrow 3(x^2+1)(x^2-1) - 10x(x^2-1))$

$3(x^2-1)(3(x^2+1) - 10x)$

$3(x+1)(x-1)(3x^2 - 10x + 3)$

$3(x+1)(x-1)\,\frac{(\cancel{3}x-\cancel{3})(3x-3)}{\cancel{3}}$

$(x+1)(x-1)(x-1)(3x-3)$

$\rightarrow 3(x+1)(x-1)(x-1)(x-1)$

$3(x+1)(x-1)^3$

e) $(3x-6)(x^2-1)-(5x-10)(x-1)^2$

$\Rightarrow 3(x-2)(x+1)(x-1)-5(x-2)(x-1)^2$

$\Rightarrow (x-2)(x-1)[3(x+1)-5(x-1)]$

$\Rightarrow (x-1)(x-2)[3x+3-5x+5]$

$\Rightarrow (x-1)(x-2)[-2x+8]$

$\Rightarrow 2(x-1)(x-2)(4-x)$

37. a) $\dfrac{1}{(a-b)(a-c)}+\dfrac{1}{(b-c)(b-a)}+\dfrac{1}{(c-a)(c-b)}$

$\Rightarrow \dfrac{1}{(a-b)(a-c)}-\dfrac{1}{(a-b)(b-c)}+\dfrac{1}{(b-c)(a-c)}$

$\Rightarrow \dfrac{b-c-(a-c)+a-b}{(a-b)(b-c)(a-c)}$

$\Rightarrow \dfrac{\cancel{b-c-a+c+a-b}}{(a-b)(b-c)(a-c)} \Rightarrow \underline{\underline{0}}$

b) $\dfrac{a}{a^2-1}+\dfrac{a^2+a-1}{a^3-a^2+a-1}+\dfrac{a^2-a-1}{a^3+a^2+a+1}-\dfrac{2a^2}{a^4-1}$

$\Rightarrow \dfrac{a}{(a+1)(a-1)}+\dfrac{a^2+a-1}{a^2(a-1)+(a-1)}+\dfrac{a^2-a-1}{a^2(a+1)+(a+1)}-\dfrac{2a^2}{(a^2+1)(a+1)(a-1)}$

$\Rightarrow \dfrac{a}{(a+1)(a-1)}+\dfrac{a^2+a-1}{(a-1)(a^2+1)}+\dfrac{a^2-a-1}{(a+1)(a^2+1)}-\dfrac{2a^2}{(a^2+1)(a+1)(a-1)}$

$$\Rightarrow \frac{a(a^2+1)+(a+1)(a^2+a-1)+(a-1)(a^2-a-1)-2a^2}{(a+1)(a-1)(a^2+1)}$$

$$\Rightarrow \frac{a^3+\cancel{a}+a^3+\cancel{a^2}-\cancel{a}+\cancel{a^2}+\cancel{a}-\cancel{1}+a^3-\cancel{a^2}-\cancel{a}-\cancel{a^2}+\cancel{a}+\cancel{1}-2a^2}{(a+1)(a-1)(a^2+1)}$$

$$\frac{3a^3-2a^2+a}{(a+1)(a-1)(a^2+1)} \Rightarrow \frac{a(3a^2-2a+1)}{(a+1)(a-1)(a^2+1)}$$

$$\frac{a-c}{a^2+ac+c^2}\times\frac{a^3-c^3}{a^2b-bc^2}\left(1+\frac{c}{a-c}-\frac{1+c}{c}\right)\div\frac{c(1+c)-a}{bc}$$

$$\frac{a-c}{\cancel{(a^2+ac+c^2)}}\times\frac{(a-c)\cancel{(a^2+ac+c^2)}}{b(a^2-c^2)}\left(\frac{c(a-c)+c^2-(1+c)(a-c)}{c(a-c)}\right)\times\frac{bc}{c+c^2-a}$$

$$\frac{\cancel{a-c}}{b(a+c)\cancel{(a-c)}}\left(\frac{ac-\cancel{c^2}+\cancel{c^2}-(a-c+ac-c^2}{\cancel{c}(a-c)}\right)\frac{b\cancel{c}}{c+c^2-a}$$

$$\frac{1}{\cancel{b}(a+c)}\left(\frac{\cancel{ac}-a+c-\cancel{ac}+c^2}{a-c}\right)\times\frac{\cancel{b}}{c+c^2-a}$$

$$\frac{1}{a+c}\times\frac{\cancel{c+c^2-a}}{a-c}\times\frac{1}{\cancel{c+c^2-a}}$$

$$\Rightarrow \frac{1}{a^2-c^2}$$

d) $\dfrac{a+b}{(b-c)(c-a)} + \dfrac{b+c}{(c-a)(a-b)} + \dfrac{c+a}{(a-c)(b-c)}$

$\Rightarrow -\dfrac{a+b}{(b-c)(a-c)} - \dfrac{b+c}{(a-b)(a-c)} + \dfrac{c+a}{(a-c)(b-c)}$

$\Rightarrow \dfrac{-(a+b)(a-b)-(b+c)(b-c)+(c+a)(a-b)}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{-(a^2-b^2)-(b^2-c^2)+ac-bc+a^2-ab}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{\cancel{-a^2}\cancel{+b^2}\cancel{-b^2}+c^2+ac-bc\cancel{+a^2}-ab}{(a-b)(a-c)(b-c)} \Rightarrow \dfrac{c^2+ac-bc-ab}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{c(c+a)-(bc+ab)}{(a-b)(a-c)(b-c)} \Rightarrow \dfrac{c(c+a)-b(c+a)}{(a-b)(a-c)(b-c)}$

$\Rightarrow \dfrac{(c+a)(c-b)}{(a-b)(a-c)(b-c)} \Rightarrow -\dfrac{(c+a)\cancel{(c-b)}}{(a-b)(a-c)\cancel{(c-b)}} \Rightarrow \dfrac{c+a}{(a-b)(c-a)}$

e) $a^3+b^3+c^3 = 3abc \quad \therefore \quad a+b+c=0$

$(a+b+c)^3 = 0^3$

$a^3+b^3+c^3+3a^2b+3a^2c+3b^2c+3ab^2+3ac^2+3bc^2+6abc=0$

$+ \quad +3a^3-3a^3 \quad +3b^3-3b^3+3c^3-3c^3$

$a^3+b^3+c^3+3a^2\overset{0}{(a+b+c)}-3a^3+3b^2\overset{0}{\cancel{(a+b+c)}}-3b^3+3c^2\cancel{(a+b+c)}-3c^3+6abc=0$

$-2a^3-2b^3-2c^3=-6abc$

$-2(a^3+b^3+c^3)=-6abc$

$a^3+b^3+c^3=3abc$

38.- $\dfrac{5x+2}{x+1}$ NO DEFINIDA

$x+1=0$

$x=-1$

39.- $\dfrac{(x^3-y^3)(x^2+2xy+y^2)}{(x^2-y^2)(x^2+xy+y^2)} - x - y$

$\Rightarrow \dfrac{\cancel{(x-y)}\cancel{(x^2+xy+y^2)}(x+y)^{\cancel{2}}}{\cancel{(x+y)}\cancel{(x-y)}\cancel{(x^2+xy+y^2)}} - x - y$

$\Rightarrow x+y-x-y$

$\Rightarrow 0$

Valor Absoluto

40.- $S = \{-2, -1, 0, 1, 2\}$

$a \# b = |a-b| - 2$

a) $1 \# 0 = -1$

$|1-0| = 2$

$1-2 = -1$ Correcto

b) $\underbrace{(-2 \# -1)} \# 1 = 0$

$\underbrace{|-2-(-1)|} - 2$

$\underbrace{1 - 2}$

$\underbrace{-1 \# 1}$

$\underbrace{|-1-1|} - 2$

$2 - 2 = 0$

Correcto

c) Verdadero ; $|a-b| = |b-a|$

d) $a \# 0 = |a| - 2$

$|a-0| - 2$

$|a| - 2$

Verdadero

e) $a \# a = a$

$|a-a| - 2$

$\underbrace{0 - 2}_{-2} \neq$

Falso

41.-

a) $\underbrace{(\sqrt{4} = -2)}_{0} \Rightarrow (-2\pi \in I)$

$\underbrace{\Rightarrow}_{1}$

b) $\underbrace{6\div(10\div5)=3}_{1} \vee \underbrace{(-15)^{-1}\in \text{Negativo}}_{0}$; VERDADERO

$$\underbrace{1 \vee 0}_{1}$$

c) $\underbrace{\frac{2e}{e}\in I}_{0} \wedge \underbrace{|x-e|=|e-x|}_{1}$; FALSO

$$\underbrace{0 \wedge 1}_{0}$$

d) $\underbrace{(\sqrt{2}\in I)}_{1} \Rightarrow \underbrace{(-3=1-4)}_{1}$; VERDADERO

$1 \Rightarrow 1 \equiv 1$

e) VERDADERO

a) $\underbrace{|-7|+|3|}_{10}-|-5| = 5$

$\underbrace{10-5}_{5}$

b) $\underbrace{|6-9|}+\underbrace{|10-4|}+\underbrace{|5|}-\underbrace{|-5|} = 9$

$3+6+\cancel{5}-\cancel{5}$

$\Rightarrow 9$

c) $\underbrace{|4-8|}-\underbrace{|-6|}+\underbrace{|14-11|}-\underbrace{|-8|}_{8} = -7$

$4-6+3-8$

$7-14$

$\Rightarrow -7$

d) $|3(-1)-(-1)|-|2(-1)-(-1)|-|3-(-1)|$

$\Rightarrow |-3+1|-|-2+1|-|3+1|$

$2-1-4 \Rightarrow -3$//

e) $2(1) - 3 - |3(1) - 2(2)| + |4 - 5(-2)|$

$\underbrace{2 - 3} - |3-4| + |4+10|$

$-1 - 1 + 14$

$\Rightarrow 12$

f) $\underbrace{|3(0) - 1|} - [3(-4) + 6] - |-3 - 2(-4)|$

$1 - [-12 + 6] - |-3 + 8|$

$1 + 6 - 5$

$\Rightarrow \underline{\underline{2}}$

43.-

a) $|x - a| \Rightarrow x - a \geq 0 \wedge -x + a < 0 \Rightarrow \sqrt{(x-a)^2}$

b) $|1 - x| \Rightarrow 1 - x \geq 0 \wedge -1 + x < 0 \Rightarrow \sqrt{(1-x)^2}$

c) $|x - a + b| \Rightarrow x - a + b \geq 0 \wedge -x + a - b < 0 \Rightarrow$

$\sqrt{(x-a+b)^2}$

4— $x \Rightarrow$ número de canicas original

	regala	sobra
①	$\frac{x}{2}$	$\frac{x}{2}$
②	$\frac{1}{3}\left(\frac{x}{2}\right)$	$\frac{2}{3}\left(\frac{x}{2}\right) = 6$

$$\Rightarrow \quad \frac{x}{3} = 6$$

$\boxed{x = 18 \text{ canicas}}$ a) correcto

5—

$x \Rightarrow$ pastel

Elena: $\frac{x}{2}$

Antonio: $\frac{x}{4}$

mamá: $\frac{x}{4}$

$$\frac{x}{2} + \frac{x}{4} + \frac{x}{4} \Rightarrow x \text{ (todo el pastel)}$$

sobra: $\underline{0}$

d) correcto

6—

1er número: x

2do número: $3x+1$

3er número: $2(3x+1)-1$

$$x + (3x+1) + (2(3x+1)-1) = 12$$

$$x + 3x + 1 + 6x + \cancel{2}\,\cancel{1} = 12$$

$$10x = 12 - 2$$

$$10x = 10$$

$$\boxed{x = 1}$$

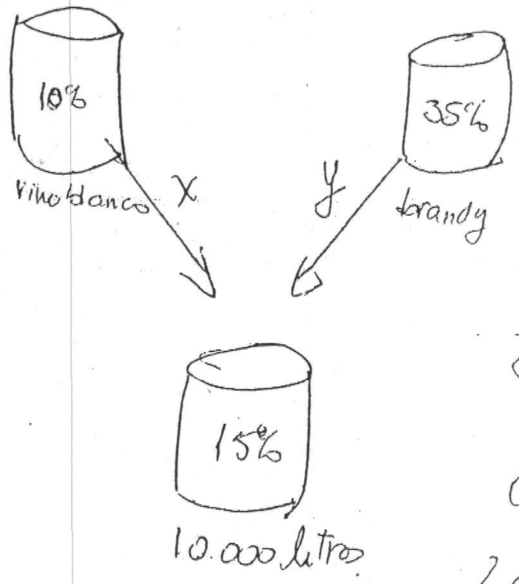
1er número $\Rightarrow 1$

2º núm. $\Rightarrow 4$

3º número $\Rightarrow 7$

d) correcto

47.-



$$\left\{ \boxed{x+y=10.000} \right. \quad (-0,1)$$

$$0,1x+0,35x=0,15(10.000)$$

$$\left\{ \boxed{0,1x+0,35x=1500} \right.$$

$$\begin{array}{r} -0,1x-0,1y=-1000 \\ 0,1x+0,35y=1500 \\ \hline // \quad 0,25y=500 \end{array}$$

$$y=\frac{500}{0,25}$$

$$\boxed{y=2000 \text{ litros}} \quad \boxed{x=8000 \text{ litros}}$$

a) correcto

48.- $\dfrac{2}{x+5}+\dfrac{1}{x-5}+\dfrac{20}{(x+5)(x-5)}=0$

$$\Rightarrow \frac{2(x-5)+x+5+20}{(x+5)(x-5)}=0 \Rightarrow 2x-10+x+25=0$$

$$3x=-15$$

$$x=-\frac{15}{3}$$

$$\boxed{x=-5}$$

Apcx) : $\{-5\}$

49.— $kx^2+4kx+3=x^2$

Suma de raices $=10$
$(x_1+x_2=10)$

Suma de raices: $-\frac{b}{a}=10$

$kx^2-x^2+4kx+3=0$

$\underbrace{x^2(k-1)}_{a}+\underbrace{4k}_{b}x+3=0$

$-\frac{4k}{k-1}=10$

$\frac{-4k}{k-1}-10=0$

$\frac{-4k-10(k-1)}{k-1}=0$

$\underbrace{-4k-10k}+10=0$

$-14k=-10$

$k=\frac{-10}{-14}$

$\boxed{k=\frac{5}{7}}$ c) correcto

50.— $p(x): \sqrt{x}+\sqrt{x+2}=3$

$(\sqrt{x}+\sqrt{x+2})^2=3^2$

$(\sqrt{x})^2+2\sqrt{x}\sqrt{x+2}+(\sqrt{x+2})^2=9$

$x+2\sqrt{x(x+2)}+x+2=9$

$2\sqrt{x^2+2x}=9-2-2x$

$(2\sqrt{x^2+2x})^2=(7-2x)^2$

$4(x^2+2x)=49-28x+4x^2$

$4x^2+8x=49-28x+4x^2$

$\cancel{4x^2}+8x+28x-\cancel{4x^2}=49$

$36x=49$

$\boxed{x=\frac{49}{36}}$

51.– $p(x): \frac{\cancel{x}}{x-2} + \frac{x}{x^2-4} = 0$

$\Rightarrow \frac{x(x+2)+x}{x^2-4} = 0 \Rightarrow x^2+2x+x=0$

$\Rightarrow x^2+3x=0$

$\Rightarrow x(x+3)=0$

$\Rightarrow \boxed{x=0}$ $\quad x+3=0$

$\boxed{x=-3}$

$x_1+x_2 = -\frac{b}{a}$

$\Rightarrow -\frac{3}{1}$

$\Rightarrow -3$

Suma: $0-3$

$\Rightarrow -3$

b) correcto

52.– $|9-x^2| - \frac{7x}{22} - \frac{48}{11} = 0$

$22\left(9-x^2-\frac{7x}{22}-\frac{48}{11}\right)=0 \cdot 22 \wedge 22\left(-9+x^2-\frac{7x}{22}-\frac{48}{11}\right)=0 \cdot 22$

$198-22x^2-7x-96=0 \quad \wedge \quad -198+22x^2-7x-96=0$

$-22x^2-7x+102=0$

$22x^2-7x-294=0$

$22x^2+7x-102=0$

Suma de soluciones: $-\frac{b}{a}$

$x_1+x_2 = \frac{-7}{22}$

$x_3+x_4 = -\frac{-7}{22}$

$\Rightarrow \frac{7}{22}$

Suma: $-\frac{7}{22}+\frac{7}{22}$

$\Rightarrow 0$ // a) correcto

3.- $x \rightarrow$ costo x hora normal

dinero $\rightarrow$ precio * cantidad

$435 = 40x + 12 * 1{,}5x$

$435 = 40x + 18x$

$58x = 435$

$x = \dfrac{435}{58}$

$\boxed{x = \$7{,}5}$ d) correcto

4.-

a) $\dfrac{x^2+17}{x^2-1} = \dfrac{x-2}{x+1} - \dfrac{5}{1-x}$

$\Rightarrow \dfrac{x^2+17}{x^2-1} - \dfrac{x-2}{x+1} - \dfrac{5}{x-1} = 0 \Rightarrow \dfrac{x^2+17-(x-2)(x-1)-5(x+1)}{x^2-1} = 0$

$\Rightarrow x^2+17-(x^2-x-2x+2)-5x-5=0 \Rightarrow \cancel{x^2}+17-\cancel{x^2}+3x-2-5x-5=0$

$-2x+10=0 \Rightarrow -2x=-10$

$\boxed{x=5}$

b) $\dfrac{ax+b}{ax-b} - \dfrac{ax-b}{ax+b} = \dfrac{4b}{a^2x^2-b^2}$

$\Rightarrow \dfrac{ax+b}{ax-b} - \dfrac{ax-b}{ax+b} - \dfrac{4b}{(ax+b)(ax-b)} = 0 \Rightarrow \dfrac{(ax+b)^2-(ax-b)^2-4b}{(ax+b)(ax-b)} = 0$

$\Rightarrow a^2x^2+2abx+b^2-(a^2x^2-2abx+b^2)-4b=0 \Rightarrow \cancel{a^2x^2}+2abx+\cancel{b^2}-\cancel{a^2x^2}+2abx-\cancel{b^2}-4b=0$

$4abx-4b=0 \Rightarrow 4b(ax-1)=0 \quad ax=1 \Rightarrow \boxed{x=1/a}$

c) $1-|x|=5$

$-|x|=5-1$

$-|x|=4$

$|x|=-4$

$\boxed{x=-4} \wedge -x=-4$

$\boxed{x=4}$

d) $|5-x|=13-x$

$5-x=13-x \wedge -5+x=13-x$

$-x+x=13-5$ | $x+x=13+5$

$0=8$ | $2x=18$

FALSO | $\boxed{x=9}$

e) $|3-x|-|x+2|=5$

$3-x-|x+2|=5 \wedge -3+x-|x+2|=5$

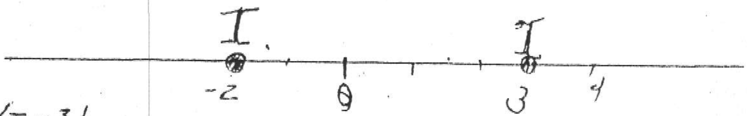
$3-x-(x+2)=5 \wedge 3-x-(-x-2)=5$ | $-3+x-(x+2)=5 \wedge -3+x-(-x-2)=5$

$3-x-x-2=5 \wedge 3-\cancel{x}+\cancel{x}+2=5$ | $-3+\cancel{x}-\cancel{x}-2=5 \wedge -3+x+x+2=5$

$-2x=5+2-3$ | $5=5$ | $-5\neq 5$ | $2x=5-2+3$

$-2x=4$ | Todo los números | FALSO, NINGUN NUMERO | $2x=6$

$\boxed{x=-2}$ | $\boxed{x=3}$



$\boxed{x=-3}$ | $\boxed{x=0}$ | $x=4$

$|3-(-3)|-|-3+2|=5$ | $|3-0|-|0+2|=5$ | $|3-4|-|4+2|=5$

$5=5$ | $1\neq 5$ | $-5\neq 5$

SI | NO | NO

Sol: $(-\infty,-2]$

) $|2x-3| = |x+7|$

$2x-3 = |x+7|$ $\wedge$ $-2x+3 = |x+7|$

$2x-3 = x+7 \wedge 2x-3 = -x-7$ $\wedge$ $-2x+3 = x+7 \wedge -2x+3 = -x-7$

$2x-x = 7+3 \wedge 2x+x = -7+3$ $\quad$ $-2x-x = 7-3$ $\quad$ $-2x+x = -7-3$

$x = 10$ $\quad$ $3x = -4$ $\quad$ $-3x = 4$ $\quad$ $-x = -10$

$\boxed{x = -4/3}$ $\quad$ $\boxed{x = -4/3}$ $\quad$ $\boxed{x = 10}$

Sol: $\{-4/3, 10\}$

) $|2x-3| = 5$

$2x-3 = 5 \wedge -2x+3 = 5$

$2x = 5+3 \wedge -2x = 5-3$

$2x = 8$ $\quad$ $-2x = 2$

$\boxed{x = 4}$ $\quad$ $\boxed{x = -1}$

Sol: $\{-1, 4\}$

) $|x-2| + |x-1| = x-3$

$x-2+|x-1| = x-3$ $\wedge$ $\wedge$ $-x+2+|x-1| = x-3$

$x-2+x-1 = x-3 \wedge \cancel{x}-2-\cancel{x}+1 = x-3$ $\wedge$ $-\cancel{x}+2+\cancel{x}-1 = x-3 \wedge -x+2-x+1 = x-3$

$x-x = -3+3$ $\wedge$ $-x = -3-1+2$ $\wedge$ $-x = -3+1-2$ $\wedge$ $-2x-x = -3-3$

$\boxed{x = 0}$ $\quad$ $-x = -2$ $\quad$ $-x = -4$ $\quad$ $-3x = -6$

$\boxed{x = 2}$ $\quad$ $\boxed{x = 4}$ $\quad$ $\boxed{x = 2}$

$|0-2|+|0-1| \neq 0-3$ $\quad$ $|2-2|+|2-1| \neq 2-3$ $\quad$ $|4-2|+|4-1| \neq 4-3$

NO $\quad$ NO $\quad$ NO $\quad$ NO

Sol: ϕ

35

i) $\underbrace{4x + 6x}_{10x} = 9x^2 - 15x$

$9x^2 - 15x - 10x = 0$

$9x^2 - 25x = 0$

$x(9x - 25) = 0$

$\boxed{x = 0}$

$9x - 25 = 0$

$9x = 25$

$\boxed{x = \frac{25}{9}}$

j) $(x+1)^2 |x+1| + 1 = 0$

$(x^2 + 2x + 1)(x+1) + 1 = 0 \quad \wedge \quad (x^2 + 2x + 1)(-x - 1) + 1 = 0$

$x^3 + x^2 + 2x^2 + 2x + x + 1 + 1 = 0 \quad \wedge \quad -x^3 - x^2 - 2x^2 - 2x - x - \cancel{1} + \cancel{1} = 0$

$x^3 + 3x^2 + 3x + 2 = 0 \quad \wedge \quad -x^3 - 3x^2 - 3x = 0$

$(x+1)^3 + 1 = 0$

$-x(x^2 + 3x + 3) = 0$

$\sqrt[3]{(x+1)^3} = \sqrt[3]{-1}$

$\boxed{x = 0}$

$x + 1 = -1$

No.

$\boxed{x = -2}$

No.

$x^2 + 3x + 3 = 0$

Tiene raíces imaginarias

Solución: ϕ

5°

1) $x \in \mathbb{N}$

2) $n-1$

3) $n+1$

4) $2n$; $n \in \mathbb{N}$

5) $2n+1$; $n \in \mathbb{N}$

6) $(n+1)^2$

7) $(n+1)^2$

8) $2n+1$, $2(n+1)+1$; $n \in \mathbb{N}$

9) $(n+1)^2 - (n)^2$; $n \in \mathbb{N}$

10) $(2(n+1))^3 - (2n)^3$; $n \in \mathbb{N}$

11) $n - r = 0$; n es inverso aditivo de r

12) $n \times r = 1$; n es inverso multiplicativo de r

13) $\frac{1}{s+1}$

14) $3x$

15) $(a+b)^2$

16) a^2+b^2

17) abc

18) $100c+10d+u$

19) $\frac{p}{q}$

20) $\frac{m}{2}$

$\frac{n}{4}$

22) $|x|$

23) $\frac{m+n}{2}$

24) $\sqrt{x}$

25) $\sqrt{ab}$

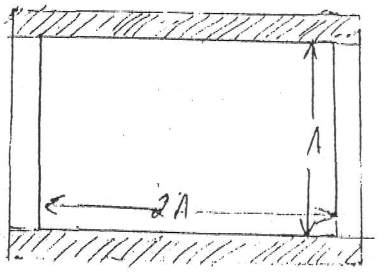
26) $MG = \sqrt{ab}$

27) $x \propto y$

28) $x \infty y$

29) $\overline{ab} < \delta$

56.- $L = 2A$



$$\text{área marco} = 2(2A+4)2 + 2A\cdot 2$$

$$244 = 4(2A+4) + 4A$$

$$244 = 8A + 16 + 4A$$

$$228 = 12A$$

$$A = \frac{228}{12} = 19$$

$$L = 2(19) = 38$$

57.- $x \to$ número

$$x - 4 + 2\tfrac{1}{2} = \tfrac{1}{3}x$$

$$x - \tfrac{1}{3}x = 4 - \tfrac{5}{2}$$

$$\frac{3x - x}{3} = \frac{8-5}{2}$$

$$\frac{2x}{3} = \frac{3}{2}$$

$$x = \frac{9}{4}$$

58.-

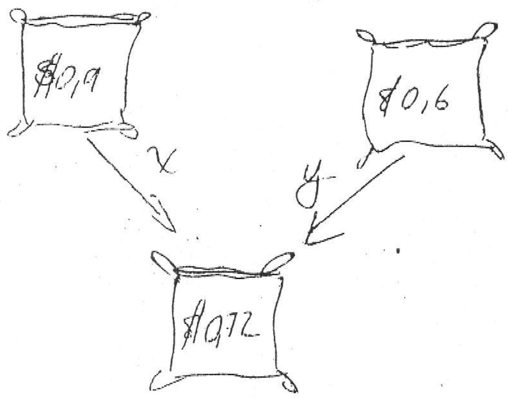
$$\frac{1}{t} = \frac{1}{15} + \frac{1}{20} + \frac{1}{30}$$

$$\frac{1}{t} = \frac{4+3+2}{60}$$

$$\frac{1}{t} = \frac{\cancel{9}^{\,3}}{\cancel{60}_{\,20}}$$

$$\boxed{t = \frac{20}{3}\ h}$$

59.-



$$0,9x + 0,6y = 0,72(x+y)$$

$$\cancel{90}x + \cancel{60}y = \cancel{72}(x+y)$$

$$30x + \cancel{20}y = \cancel{24}(x+y)$$

$$15x + 10y = 12(x+y)$$

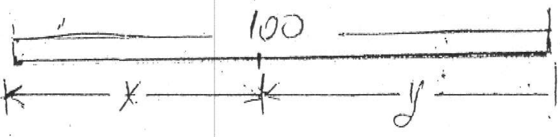
$$15x + 10y = 12x + 12y$$

$$15x - 12x = 12y - 10y$$

$$3x = 2y$$

$$\boxed{x = \frac{2}{3}y}$$

60.-



$$\boxed{x+y=100} \Rightarrow x=100-y$$

$$\boxed{\left(\frac{x}{4}\right)^2}\ x/4 + \boxed{\left(\frac{y}{4}\right)^2}\ y/4 = 397$$

$$\frac{x}{4} \qquad \frac{y}{4}$$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 397$$

$$\frac{x^2}{16} + \frac{y^2}{16} = 397$$

$$x^2+y^2 = 397 \times 16$$

$$\boxed{x^2+y^2 = 6352}$$

$$(100-y)^2+y^2 = 6352$$

$$10.000 - 200y + y^2 + y^2 = 6352$$

$$2y^2 - 200y + 3648 = 0$$

$$y^2 - 100y + 1824 = 0$$

$$y = \frac{100 \pm \sqrt{100^2 - 4(1)(1824)}}{2(1)}$$

$$y = \frac{100 \pm 52}{2} \rightarrow 76 \quad \rightarrow 24$$

61.-

$$\text{Ingreso} = 120 \times 40$$

$$\Rightarrow \$4800$$

$$(120+5x)\cdot(40-x) = 4800$$

$$\cancel{4800} - 120x + 200x - 5x^2 = \cancel{4800}$$

$$5x^2 - 80x = 0$$

$$5x^2(x-16) = 0$$

$$x=0 \quad \boxed{x=16}$$

$$\text{Precio} = 120+5x$$

$$\Rightarrow 120+5(16)$$

$$\Rightarrow \$200$$

c) correcta

2.-

Ingreso = Precio * cantidad

$11475 = (180+5x)(60-x)$

$11475 = 10800 - 180x + 300x - 5x^2$

$5x^2 - 120x + 11475 - 10800 = 0$

$5x^2 - 120x + 675 = 0$

$5(x^2 - 24x + 135) = 0$

$x^2 - 24x + 135 = 0$

$(x-15)(x-9) = 0$

$x = 15 \wedge x = 9$

$Precio_1 = 180 + 5(15)$

$= 180 + 75$

$P_1 = \$255$

$P_2 = 180 + 5(9)$

$P_2 = 180 + 45$

$P_2 = \$225$

3.- $I \rightarrow$ tasa de interés en 1er año

$2I \rightarrow$ tasa de interés en 2do año

al final 1er año $= C(1 + \frac{I}{100})$

$= 100(1 + \frac{I}{100})$

$= 100(\frac{100+I}{100})$

al final 2do año $= C(1 + \frac{2I}{100})$

$11232 = (100+I)(\frac{100+2I}{100})$

$11232 = (100+I)(100+2I)$

$11232 = 10000 + 200I + 100I + 2I^2$

$2I^2 + 300I - 1232 = 0$

$I = \frac{-300 \pm \sqrt{300^2 - 4(2)(1232)}}{2(2)}$

$I = \frac{-300 \pm 316}{4} \rightarrow 4$

$I_1 = 4\%$

$I_2 = 8\%$

41

64.-

a) $17.500 = (600 - 5x) \cdot x$

$17.500 = 600x - 5x^2$

$5x^2 - 600x + 17.500 = 0$

$5(x^2 - 120x + 3500) = 0$

$x^2 - 120x + 3500 = 0$

$(x - 70)(x - 50) = 0$

$x = 70 \vee x = 50$

b) $18.000 = (600 - 5x) \cdot x$

$18000 = 600x - 5x^2$

$5x^2 - 600x + 18000 = 0$

$5(x^2 - 120x + 3600) = 0$

$x^2 - 120x + 3600 = 0$ Precio $= 600 - 5(60)$

$(x - 60)(x - 60) = 0$ $\Rightarrow \$300$

$\boxed{x = 60}$

c) $U = I - C$

$5500 = (600 - 5x) \cdot x - (8000 + 75x)$

$5500 = 600x - 5x^2 - 8000 - 75x$

$5x^2 - 525x + 13500 = 0$

$5(x^2 - 105x + 2700) = 0$

$(x - 60)(x - 45) = 0$

$\boxed{x = 60} \vee \boxed{x = 45}$

d) $U = I - C$

$5750 = (600 - 5x) \cdot x - (8000 + 75x)$

$5750 = 600x - 5x^2 - 8000 - 75x$

$5x^2 - 525x + 13750 = 0$

$5(x^2 - 105x + 2750) = 0$

$x^2 - 105x + 2750 = 0$

$(x - 55)(x - 50) = 0$

$x = 55 \lor x = 50$

Precio $= 600 - 5x$

$P_1 = 600 - 5(55)$

$P_1 = \$325$

$P_2 = 600 - 5(50)$

$P_2 = 600 - 250$

$P_2 = \$350$

5.- $\underbrace{L = A + 2}$

área $= A \cdot L$

$= A \cdot (A+2)$

$\Rightarrow A^2 + 2A$

incremento

$L' = A + 2 + 3$

$L' = A + 5$

$A' = A + 3$

área' $= L' \cdot A'$

$\Rightarrow (A+5) * (A+3) = A^2 + 2A + 51$

$\cancel{A^2} + 3A + 5A + 15 = \cancel{A^2} + 2A + 51$

$8A - 2A = 51 - 15$

$6A = 36$

$\boxed{A = 6}$

$\boxed{L = 8}$

66.—

$$\frac{1}{T}=\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{3}}+\frac{1}{5}$$

$$\Rightarrow \frac{1}{T}=\frac{1}{\frac{5}{2}}+\frac{1}{\frac{10}{3}}+\frac{1}{5} \Rightarrow \frac{1}{T}=\frac{2}{5}+\frac{3}{10}+\frac{1}{5}$$

$$\Rightarrow \frac{1}{T}=\frac{4+3+2}{10} \Rightarrow \frac{1}{T}=\frac{9}{10} \Rightarrow \boxed{T=\frac{10}{9}h}$$

67.—

$$\frac{1}{T}=\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$$

$$\Rightarrow \frac{1}{T}=\frac{4+3+2}{12} \Rightarrow \frac{1}{T}=\frac{\cancel{9}^{3}}{\cancel{12}_{4}}$$

$$\boxed{T=\frac{4}{3}\text{ día}}$$

68.— $T_y = 8h$

$T_p = 10h$

$T_c = 12h$

$X \Rightarrow$ obra

$$X = V_1(1) + V_2 T$$

$$X = (V_y+V_p)(1)+(V_y+V_c)T$$

$$X = \left(\frac{X}{8}+\frac{X}{10}\right)(1)+\left(\frac{X}{8}+\frac{X}{12}\right)T$$

$$X-\frac{X}{8}-\frac{X}{10}=\left(\frac{3X+2X}{24}\right)T$$

$$\frac{40X-5X-4X}{40}=\frac{5X}{24}T$$

$$\frac{31X}{\cancel{40}_{5}}=\frac{5X}{\cancel{24}_{3}}T$$

$$T=\frac{93}{25}\text{ horas}$$

$$T_{total}=T+1h.$$

$$T_{tot}=\frac{93}{25}+1$$

$$\boxed{T_{total}=\frac{118}{25}\text{ horas}}$$

$$I_2 - I_1 = 110$$

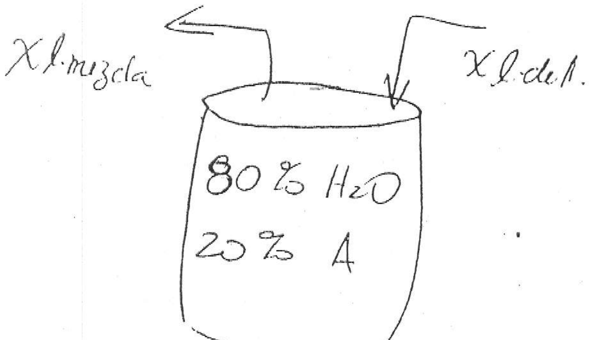
$$5000\left(\frac{i+1}{100}\right) - 4000\left(\frac{i}{100}\right) = 110$$

$$50\cancel{00}\left(\frac{i}{1\cancel{00}}\right) - 40\cancel{00}\left(\frac{i}{1\cancel{00}}\right) = 11\cancel{0}$$

$$5i + 5 - 4i = 11$$

$$5i - 4i = 11 - 5$$

$$\boxed{i = 6\,\%} \quad \boxed{i = 7\,\%}$$



Concentración: $0{,}2(10-x) + x = 10(0{,}5)$

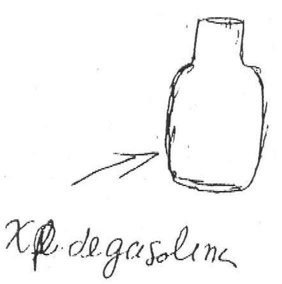
$$2 - 0{,}2x + x = 5$$

$$0{,}8x = 3$$

$$x = \frac{3}{0{,}8} \times \frac{5}{5} = \boxed{\frac{15}{4}\ l}$$

combustible

1 litro (6 partes)



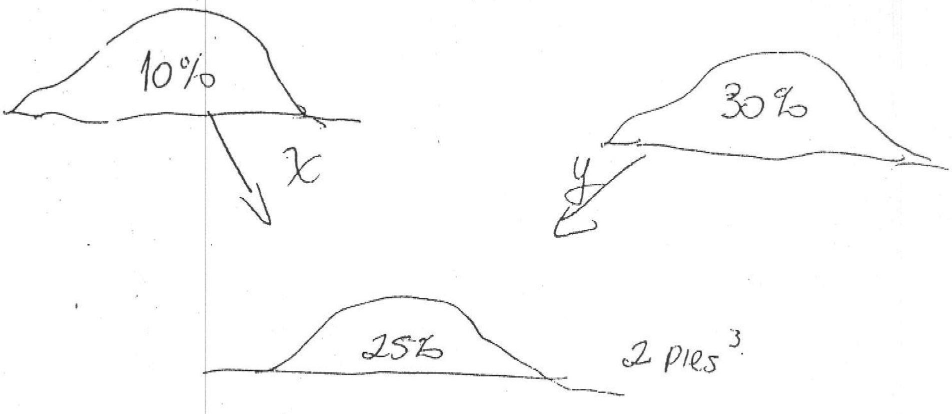
5P. gasolina
1P aceite ⇒

$$5 + x = 23$$

$$x = 18\ P \times \frac{1\ litro}{6P}$$

$$\boxed{x = 3\ litros\ gasolina}$$

72.-



$$\boxed{x+y=2} \quad (-1)$$

$$0{,}1x+0{,}3y=2(0{,}25)$$

$$0{,}1x+0{,}3y=0{,}5$$

$$\boxed{x+3y=5}$$

$$-x-y=-2$$
$$x+3y=5$$

$$2y=3$$

$$\boxed{y=\frac{3}{2}\text{ pies}^3}$$

$$\boxed{x=\frac{1}{2}\text{ pies}^3}$$

73.-

$x \rightarrow$ galones de extra

$y \rightarrow$ galones de super $\quad \boxed{x+y=15000} \quad (-11)$

$$0{,}55x+0{,}6y=8550$$

$$55x+60y=855000$$

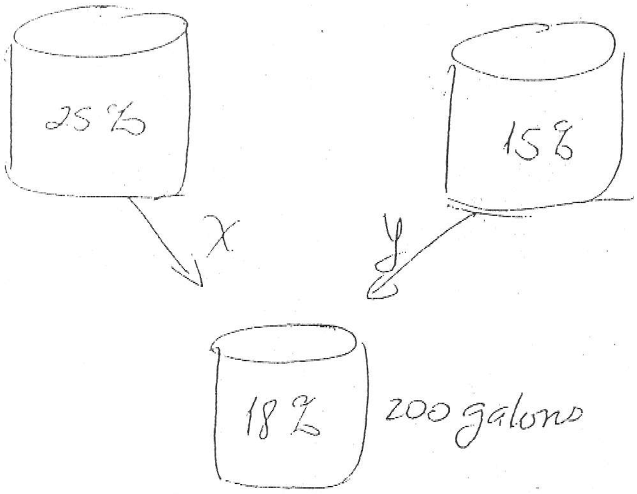
$$11x+12y=171000$$

$$-11x-11y=-165000$$

$$\boxed{y=6000\text{ galones}}$$

$$x=15000-6000$$

$$\boxed{x=9000\text{ galons}}$$



$\boxed{x+y=200}\ |(-3)$

$0,25x+0,15y=0,18(200)$

$0,25x+0,15y=36$

$25x+15y=3600$

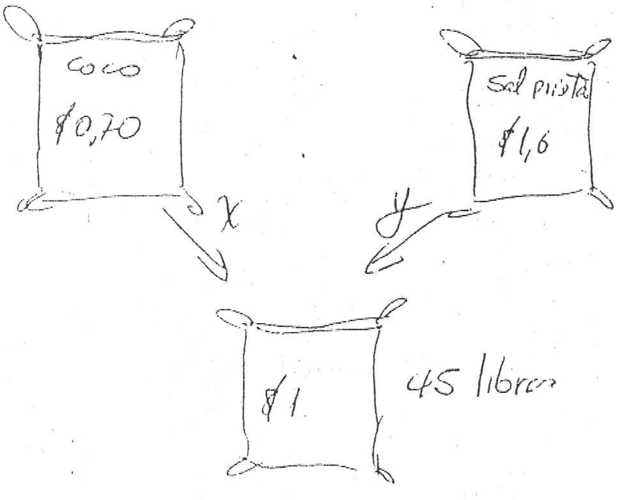
$\boxed{5x+3y=720}$

$$\begin{array}{r} -3x-3y=-600 \\ 5x+3y=720 \\ \hline 2x \quad =120 \end{array}$$

$\boxed{x=60 \text{ galons}}$

$y=200-60$

$\boxed{y=140 \text{ galo}}$



$\boxed{x+y=45}\ (-7)$

$0,7x+1,6y=1(45)$

$\boxed{7x+16y=450}$

$$\begin{array}{r} -7x-7y=-315 \\ 7x+16y=450 \\ \hline 9y=135 \end{array}$$

$\boxed{y=15 \text{ libras}}$

$x=45-15$

$\boxed{x=30 \text{ libras}}$

76.-

$x \to$ capital al 9%

$y \to$ capital al 6%

$I_{total} = 18000(0{,}08)$

$I_{total} = \$1440$

$$\boxed{x + y = 18000} \;(-2)$$

$$0{,}09x + 0{,}06y = 1440$$

$$9x + 6y = 144000$$

$$\boxed{3x + 2y = 48.000}$$

$$-2x - 2y = -36000$$
$$3x + 2y = 48.000$$

$$\boxed{x = 12000}$$

$$y = 18.000 - 12000$$

$$\boxed{y = 6000}$$

77.- $4x^2 - 4xy - y^2 = 1$

(x) $4x^2 - 4xy - (y^2 + 1) = 0$

$$x = \frac{4y \pm \sqrt{(4y)^2 - 4(4)(-(y^2+1))}}{2(4)}$$

$$x = \frac{4y \pm \sqrt{16y^2 + 16y^2 + 16}}{8}$$

$$x = \frac{4y \pm \sqrt{16(2y^2+1)}}{8}$$

$$x = \frac{\cancel{4}y \pm \cancel{4}\sqrt{2y^2+1}}{\cancel{8}_2}$$

$$\boxed{x = \frac{y \pm \sqrt{2y^2+1}}{2}}$$

(y) $y^2 + 4xy + (1 - 4x^2) = 0$

$$y = \frac{-4x \pm \sqrt{(-4x)^2 - 4(1)(1-4x^2)}}{2(1)}$$

$$y = \frac{-4x \pm \sqrt{16x^2 - 4 + 16x^2}}{2}$$

$$y = \frac{-4x \pm \sqrt{4(8x^2-1)}}{2}$$

$$y = \frac{-\cancel{4}x \pm \cancel{2}\sqrt{8x^2-1}}{\cancel{2}}$$

$$\boxed{y = -2x \pm \sqrt{8x^2-1}}$$

78.-

a) $p(x) = (x+1)(x+2)(x+3) = x(x+4)(x+5)$

$(x^2+2x+x+2)(x+3) = x(x^2+5x+4x+20)$

$(x^2+3x+2)(x+3) = x^3+9x^2+20x$

$\cancel{x^3}+3x^2+2x+3x^2+9x+6 = x^3+9x^2+20x$

$\cancel{x^3}+6x^2+11x+6-\cancel{x^3}-9x^2-20x=0$

$-3x^2-9x+6=0$

$-3(x^2+3x-2)=0$

$x^2+3x-2=0$

$x = \frac{-3 \pm \sqrt{3^2-4(1)(-2)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{17}}{2}$

b) $(x+1)^3+(x-1)^2 = x^3+x+1$

$x^3+3x^2+3x+1+x^2-2x+1 = x^3+x+1$

$\cancel{x^3}+4x^2+\cancel{x}+2-\cancel{x^3}-\cancel{x}-1=0$

$4x^2+1=0$

$4x^2=-1$

$\sqrt{x^2} = \frac{-1}{4}$ No hay raíces reales, sino imaginarias

$\boxed{x = \phi}$

c) $p(x)$: $x^2-4ax+4a^2-c^2=0$

$x = \frac{4a \pm \sqrt{(4a)^2-4(1)(4a^2-c^2)}}{2(1)}$

$x = \frac{4a \pm \sqrt{16a^2-\cancel{16a^2}+4c^2}}{2}$

$x = \frac{\cancel{4}a \pm \cancel{2}c}{\cancel{2}}$

$\boxed{x = 2a \pm c}$

d) $q(x) = a^2x^2 + a(b-c)x - bc = 0$

$$x = \frac{-a(b-c) \pm \sqrt{[a(b-c)]^2 - 4a^2(-bc)}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{a^2b^2 - 2a^2bc + a^2c^2 + 4a^2bc}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{a^2b^2 + 2a^2bc + a^2c^2}}{2a^2}$$

$$x = \frac{-a(b-c) \pm \sqrt{(a^2(b+c)^2)}}{2a^2} \Rightarrow \frac{-\cancel{a}(b-c) \pm \cancel{a}(b+c)}{2a^{\cancel{2}}}$$

$$x = \frac{-(b-c) \pm (b+c)}{2a} \Rightarrow \frac{-\cancel{b}+c+\cancel{b}+c}{2a} \Rightarrow \frac{\cancel{2}c}{\cancel{2}a} \Rightarrow \frac{c}{a}$$

$$\Rightarrow \frac{-b+\cancel{c}-b-\cancel{c}}{2a} \Rightarrow -\frac{\cancel{2}b}{\cancel{2}a} \Rightarrow -\frac{b}{a}$$

e) $\frac{2x+a}{b} - \frac{x-b}{a} = \frac{3ax+(a-b)^2}{ab}$

$$\frac{a(2x+a) - b(x-b)}{\cancel{ab}} = \frac{3ax + a^2 - 2ab + b^2}{\cancel{ab}}$$

$$2ax + a^2 - bx + b^2 = 3ax + a^2 - 2ab + b^2$$

$$2ax - 3ax - bx = \cancel{a^2} - 2ab + \cancel{b^2} - \cancel{a^2} - \cancel{b^2}$$

$$-ax - bx = -2ab$$

$$-x(a+b) = -2ab$$

$$x = \frac{2ab}{a+b}$$

f) $p(x): \frac{x+a}{a-b} + \frac{x+a}{a+b} = \frac{x+b}{a+b} + \frac{2(x-b)}{a-b}$

$\Rightarrow \frac{(x+a)(a+b)+(x+a)(a-b)}{\cancel{(a+b)(a-b)}} = \frac{(x+b)(a-b)+2(x-b)(a+b)}{\cancel{(a+b)(a-b)}}$

$ax+\cancel{bx}+a^2+\cancel{ab}+ax-\cancel{bx}+a^2-\cancel{ab} = ax-\cancel{bx}+\cancel{ab}-b^2+2ax+\cancel{2bx}-\cancel{2ab}-2b^2$

$2ax+2a^2 = 3ax+bx-ab-3b^2$

$-3ax+2ax-bx = -ab-3b^2-2a^2$

$-ax-bx = -ab-2a^2-3b^2$

$-x(a+b) = -(2a^2+3b^2+ab)$

$x = \frac{2a^2+3b^2+ab}{a+b}$

g) $p(x): \frac{(a+b)^2(x+1)-(a+b)(x+1)+(x+1)}{a+b+1} = (a+b)^2-(a+b)+1$

$\Rightarrow \frac{(x+1)\left(\cancel{(a+b)^2-(a+b)+1}\right)}{a+b+1} = \left[\cancel{(a+b)^2-(a+b)+1}\right]$

$\Rightarrow (x+1) = a+b+1 \Rightarrow x+1 = a+b+1 \Rightarrow x = a+b+\cancel{1}-\cancel{1}$

$\Rightarrow \boxed{x = a+b}$

h) $p(x): (a+x)(b+x)-a(b+c) = \frac{a^2c}{b} + x^2$

$\Rightarrow \cancel{ab}+ax+bx+\cancel{x^2}-\cancel{ab}-ac = \frac{a^2c}{b} + \cancel{x^2}$

$\Rightarrow ax+bx = \frac{a^2c}{b} + ac \Rightarrow x(a+b) = \frac{a^2c+abc}{b}$

$\Rightarrow \cancel{x}(a+b) = \frac{ac(\cancel{a+b})}{b} \Rightarrow x = \frac{ac}{b}$

l) $\frac{x}{ab}+\frac{x}{bc}+\frac{x}{ac}-1 = abc - x(a+b+c)$

$\Rightarrow \frac{cx+ax+bx-abc}{abc} = abc - x(a+b+c)$

$\Rightarrow \frac{x(a+b+c)-abc}{abc} = abc - x(a+b+c)$

$\Rightarrow x(a+b+c) - abc = abc(abc - x(a+b+c))$

$\Rightarrow \underline{x(a+b+c)} - abc = a^2b^2c^2 - \underline{xabc(a+b+c)}$

$\Rightarrow x(a+b+c) + xabc(a+b+c) = a^2b^2c^2 + abc$

$\Rightarrow x(a+b+c)(1+abc) = abc(abc+1)$

$\Rightarrow x = \frac{abc\cancel{(abc+1)}}{(a+b+c)\cancel{(abc+1)}} \Rightarrow \frac{abc}{a+b+c}$

79.- $g(x): \frac{4}{x+4}+\frac{1}{x+3}+\frac{3}{x+1}=0$

$\Rightarrow \frac{4(x+3)(x+1)+(x+4)(x+1)+3(x+4)(x+3)}{(x+4)(x+3)(x+1)}=0$

$\Rightarrow 4(x^2+x+3x+3)+x^2+x+4x+4+3(x^2+3x+4x+12)=0$

$\Rightarrow 4x^2+16x+12+x^2+5x+4+3x^2+21x+36=0$

$\Rightarrow 8x^2+42x+52=0$ $\quad 4x^2+21x+26=0$

$(4x+13)(4x+8)=0$

$4x-13=0$ $\quad 4x+8=0$

$4x=13$ $\quad 4x=-8$

$x=13/4$ $\quad x=-2$

) $p(x): \dfrac{3x}{2x+1} + \dfrac{x+5}{x+1} = \dfrac{x-19}{2x^2+3x+1}$

$$\Rightarrow \frac{3x(x+1)+(x+5)(2x+1)}{(2x+1)(x+1)} = \frac{x-19}{\dfrac{(\cancel{2}x+\cancel{2})(2x+1)}{\cancel{2}}}$$

$$\Rightarrow \frac{3x^2+3x+2x^2+x+10x+5}{(2x+1)\cancel{(x+1)}} = \frac{x-19}{(x+1)\cancel{(2x+1)}}$$

$$\Rightarrow 5x^2+14x+5-x+19=0$$

$$\Rightarrow 5x^2+13x+24=0$$

$$x = \frac{-13 \pm \sqrt{13^2-4(5)(24)}}{2(5)} \Rightarrow \cancel{\frac{-13 \pm \sqrt{169-480}}{10}}$$

tiene soluciones imaginarias

) $m(x): \dfrac{3a+x}{3a-x} + \dfrac{2a-3x}{2a+3x} = \dfrac{9}{2}$

$$\Rightarrow \frac{(3a+x)(2a+3x)+(2a-3x)(3a-x)}{(3a-x)(2a+3x)} \quad \frac{9}{2} = 0$$

$$\Rightarrow \frac{6a^2+9ax+2ax+3x^2+6a^2-2ax-9ax+3x^2}{(3a-x)(2a+3x)} - \frac{9}{2} = 0$$

$$\Rightarrow \frac{6x^2+12a^2}{(3a-x)(2a+3x)} - \frac{9}{2} = 0 \Rightarrow \frac{2(6x^2+12a^2)-9(3a-x)(2a+3x)}{2(3a-x)(2a+3x)} = 0$$

$$\Rightarrow 12x^2+24a^2-9(6a^2+9ax-2ax-3x^2)=0$$

$$\Rightarrow 12x^2+24a^2-54a^2-63ax+27x^2=0 \Rightarrow 39x^2-30a^2-63ax=0$$

$$\Rightarrow 3(13x^2-21ax-10a^2)=0$$

$$(13x-26a^2)(13x+5a^2)=0$$

$\boxed{x=2a^2}$ $\boxed{x=-\dfrac{5a^2}{13}}$

d) $p(x): \frac{1}{x-a} + \frac{1}{x-b} = \frac{1}{x-c}$

$$\Rightarrow \frac{1}{x-a} + \frac{1}{x-b} - \frac{1}{x-c} = 0 \Rightarrow \frac{(x-b)(x-c)+(x-a)(x-c)-(x-b)(x-a)}{(x-a)(x-b)(x-c)} = 0$$

$$\Rightarrow x^2 - cx - bx + bc + x^2 - cx - ax + ac - (x^2 - ax - bx + ab) = 0$$

$$\Rightarrow \cancel{2}x^2 - 2cx - \cancel{ax} - \cancel{bx} + ac + bc - \cancel{x^2} + \cancel{ax} + \cancel{bx} - ab = 0$$

$$\Rightarrow x^2 - 2cx - ab + ac + bc = 0$$

$$x = \frac{2c \pm \sqrt{(2c)^2 - 4(1)(-ab+ac+bc)}}{2(1)} \Rightarrow \frac{2c \pm \sqrt{4c^2 + 4ab - 4ac - 4bc}}{2}$$

$$\Rightarrow \frac{2c \pm \sqrt{4[(c^2 - ac) - (bc - ab)]}}{2} \Rightarrow \frac{\cancel{2}c \pm \cancel{2}\sqrt{c(c-a) - b(c-a)}}{\cancel{2}}$$

$$\Rightarrow c \pm \sqrt{(c-a)(c-b)}$$

e) $\frac{1}{a} + \frac{1}{b} - \frac{1}{x} = \frac{1}{a+b-x}$

$$\Rightarrow \frac{bx + ax - ab}{abx} - \frac{1}{a+b-x} = 0 \Rightarrow \frac{bx(a+b-x) + ax(a+b-x) - ab(a+b-x) - abx}{abx(a+b-x)}$$

$$\Rightarrow abx + b^2x - bx^2 + a^2x + abx - ax^2 - a^2b - ab^2 + \cancel{abx} - \cancel{abx} = 0$$

$$-ax^2 - bx^2 + a^2x + 2abx + b^2x - a^2b - ab^2 = 0$$

$$-x^2(a+b) + x(a+b)^2 - ab(a+b) = 0 \Rightarrow -(a+b)(x^2 - (a+b)x + ab) = 0$$

$$\Rightarrow (x-a)(x-b) = 0 \Rightarrow \boxed{x = a} \quad \boxed{x = b}$$

$$r(x):\ \frac{6}{x+4} - \frac{x+4}{x-4} + \frac{7x^2+50}{3(x^2-16)} = \frac{4}{3}$$

$$\Rightarrow \frac{6}{x+4} - \frac{x+4}{x-4} + \frac{7x^2+50}{3(x+4)(x-4)} - \frac{4}{3} = 0$$

$$\Rightarrow \frac{18(x-4) - 3(x+4)^2 + 7x^2 + 50 - 4(x+4)(x-4)}{3(x+4)(x-4)} = 0$$

$$\Rightarrow 18x - 72 - 3(x^2+8x+16) + 7x^2 + 50 - 4(x^2-16) = 0$$

$$18x - 72 - \cancel{3x^2} - 24x - 48 + \cancel{7x^2} + 50 - \cancel{4x^2} + 64 = 0$$

$$-6x = 72 + 48 - 50 - 64$$

$$-6x = 6$$

$$\boxed{x = -1}$$

$$\frac{x+a+b}{x+a} = \frac{x+a-b}{x-a} - \frac{a^2+b^2}{x^2-a^2}$$

$$\Rightarrow \frac{x+a+b}{x+a} - \frac{x+a-b}{x-a} + \frac{a^2+b^2}{(x+a)(x-a)} = 0$$

$$\Rightarrow \frac{(x+a+b)(x-a) - (x+a-b)(x+b) + a^2 + b^2}{(x+a)(x-a)} = 0$$

$$\Rightarrow x^2 - \cancel{ax} + \cancel{ax} - a^2 + bx - ab - (x^2 + \cancel{bx} + ax + ab - \cancel{bx} - b^2) + a^2 + b^2 = 0$$

$$\Rightarrow \cancel{x^2} - \cancel{a^2} + bx - ab - \cancel{x^2} - ax - ab + b^2 + \cancel{a^2} + b^2 = 0$$

$$\Rightarrow bx - ax = +2ab - 2b^2$$

$$\Rightarrow x(b-a) = 2b(a-b) \Rightarrow x = \frac{-2b\cancel{(b-a)}}{\cancel{b-a}} \Rightarrow x = -2b$$

h) $p(x): \dfrac{\dfrac{x+1}{x-1}}{1+\dfrac{x+x}{x-1}} = \dfrac{1}{2}$

$\Rightarrow \dfrac{\dfrac{x+1}{\cancel{x-1}}}{\dfrac{x-1+2x}{\cancel{x-1}}} = \dfrac{1}{2} \Rightarrow \dfrac{x+1}{3x-1} - \dfrac{1}{2} = 0$

$\Rightarrow \dfrac{2(x+1)-(3x-1)}{2(3x-1)} = 0 \Rightarrow 2x+2-3x+1=0$

$-x+3=0$

$\boxed{x=3}$

80.- a) $r(x): \sqrt{x^2-7} = 3$

$(\sqrt{x^2-7})^2 = 3^2$

$x^2-7=9$

$x^2 = 9+7$

$x^2=16$

$\boxed{x=\pm 4}$

b) $m(x): \sqrt{x+1} = x-1$

$(\sqrt{x+1})^2 = (x-1)^2$

$x+1 = x^2-2x+1$

$x^2-2x+\cancel{1}-x-\cancel{1}=0$

$x^2-3x=0$

$x(x-3)=0$

$\boxed{x=0}$ $\boxed{x=3}$

c) $p(x): \sqrt{x} + \sqrt{3x+4} = \sqrt{4-x}$

$(\sqrt{x}+\sqrt{3x+4})^2 = (\sqrt{4-x})^2$

$(\sqrt{x})^2 + 2\sqrt{x}\sqrt{3x+4} + (\sqrt{3x+4})^2 = 4-x$

$\Rightarrow x + 2\sqrt{x(3x+4)} + 3x+4 = 4-x$

$\Rightarrow 2\sqrt{3x^2+4x} = \cancel{4}-x-3x-\cancel{4}-x$

$\Rightarrow (2\sqrt{3x^2+4x})^2 = (-5x)^2$

$4(3x^2+4x) = 25x^2$

$12x^2+16x-25x^2=0$

$-13x^2+16x=0$

$-x(13x-16)=0$

$\boxed{x=0}$

$\boxed{x=16/13}$

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d.- $q(x): \sqrt{a+x}+\sqrt{a-x}=\sqrt{2a}$

$$(\sqrt{a+x})^{\cancel{2}}=(\sqrt{2a}-\sqrt{a-x})^2$$

$$a+x=(\sqrt{2a})^{\cancel{2}}-2\sqrt{2a}\sqrt{a-x}+(\sqrt{a-x})^{\cancel{2}}$$

$$\cancel{a}+x=2a-2\sqrt{2a(a-x)}+\cancel{a}-x$$

$$-2a+x+x=-2\sqrt{2a^2-2ax}$$

$$-2a+2x=-2\sqrt{2a^2-2ax}$$

$$-\cancel{2}(a-x)=-\cancel{2}\sqrt{2a^2-2ax}$$

$$(a-x)^2=(\sqrt{2a^2-2ax})^{\cancel{2}}$$

$$a^2-\cancel{2ax}+x^2=2a^2-\cancel{2ax}$$

$$x^2=2a^2-a^2$$

$$\sqrt{x^{\cancel{2}}}=\sqrt{a^{\cancel{2}}}$$

$$x=\pm a$$

e).- $r(x): \dfrac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}}=x-3$

$$\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}-\sqrt{x-4}}\times\frac{\sqrt{x+4}+\sqrt{x-4}}{\sqrt{x+4}+\sqrt{x-4}}=x-3$$

$$\frac{(\sqrt{x+4}+\sqrt{x-4})^2}{(\sqrt{x+4})^{\cancel{2}}-(\sqrt{x-4})^{\cancel{2}}}=x-3 \Rightarrow \frac{(\sqrt{x+4})^{\cancel{2}}+2\sqrt{x+4}\sqrt{x-4}+(\sqrt{x+4})^{\cancel{2}}}{x+4-(x-4)}$$

$$\frac{x\cancel{+4}+2\sqrt{(x+4)(x-4)}+x\cancel{-4}}{x+4-(x-4)}=x-3$$

$$\frac{2x+2\sqrt{x^2-16}}{\cancel{x}+4-\cancel{x}+4}=x-3$$

$$\frac{\overset{1}{\cancel{2}}(x+\sqrt{x^2-16})}{\cancel{8}\,4}=x-3$$

$$x+\sqrt{x^2-16}=4(x-3)$$

$$\sqrt{x^2-16}=4x-12-x$$

$$(\sqrt{x^2-16})^2=(3x-12)^2$$

$$x^2-16=9x^2-72x+144$$

$$9x^2-x^2-72x+144+16=0$$

$$8x^2-72x+160=0$$

$$8(x^2-9x+20)=0$$

$$x^2-9x+20=0$$

$$(x-5)(x-4)=0$$

$$x-5=0 \qquad x-4=0$$

$$x=5 \qquad x=4$$

f.- $$\frac{1}{\sqrt{3a+x}-\sqrt{a-x}}+\frac{1}{\sqrt{3a+x}+\sqrt{a-x}}=\frac{1}{\sqrt{a}}$$

$$\frac{\sqrt{3a+x}+\cancel{\sqrt{a-x}}+\sqrt{3a+x}-\cancel{\sqrt{a-x}}}{(\sqrt{3a+x}-\sqrt{a-x})(\sqrt{3a+x}+\sqrt{a-x})}=\frac{1}{\sqrt{a}}$$

$$\frac{2\sqrt{3a+x}}{(\cancel{\sqrt{3a-x}})^{\cancel{2}}-(\cancel{\sqrt{a-x}})^{\cancel{2}}}=\frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{2\sqrt{3a+x}}{3a-x-(a-x)}=\frac{1}{\sqrt{a}} \Rightarrow \frac{2\sqrt{3a+x}}{3a\cancel{-x}-a\cancel{+x}}=\frac{1}{\sqrt{a}}$$

$$\Rightarrow \frac{\cancel{2}\sqrt{3a+x}}{\cancel{2}a}=\frac{1}{\sqrt{a}} \Rightarrow \sqrt{3a+x}=\frac{a}{\sqrt{a}}$$

$$\Rightarrow (\cancel{\sqrt{3a+x}})^{\cancel{2}}=\left(\frac{a}{\sqrt{a}}\right)^2 \Rightarrow 3a+x=\frac{a^{\cancel{2}}}{\cancel{a}}$$

$$\Rightarrow 3a+x=a \Rightarrow x=a-3a$$

$$x=-2a$$

g.- $$\frac{1}{\sqrt{4a+x}+\sqrt{a}}+\frac{1}{\sqrt{4a+x}-\sqrt{a}}=\frac{4}{3\sqrt{a}}$$

$$\frac{\sqrt{4a+x}-\cancel{\sqrt{a}}+\sqrt{4a+x}+\cancel{\sqrt{a}}}{(\sqrt{4a+x}+\sqrt{a})(\sqrt{4a+x}-\sqrt{a})}=\frac{4}{3\sqrt{a}}$$

$$\frac{2\sqrt{4a+x}}{(\sqrt{4a+x})^2-(\sqrt{a})^2}=\frac{4}{3\sqrt{a}}$$

$$\frac{\overset{1}{\cancel{2}}\sqrt{4a+x}}{4a+x-a}=\frac{\cancel{4}^{2}}{3\sqrt{a}}$$

$$\frac{\sqrt{4a+x}}{4a+x-a}=\frac{2}{3\sqrt{a}} \Rightarrow \frac{\sqrt{4a+x}}{3a+x}=\frac{2}{3\sqrt{a}}$$

$$3\sqrt{a}\sqrt{4a+x}=2(3a+x)$$

$$\left(3\sqrt{a(4a+x)}\right)^2=(6a+2x)^2$$

$$9(a(4a+x))=36a^2+24ax+4x^2$$

$$\cancel{36a^2}+9ax=\cancel{36a^2}+24ax+4x^2$$

$$4x^2+24ax-9ax=0$$

$$4x^2+15ax=0$$

$$x(4x+15a)=0$$

$$x=0 \qquad 4x+15a=0$$

$$4x=-15a$$

$$x=-\frac{15a}{4}$$

h.-) $\varphi(x): \sqrt{x+3} - \sqrt{x-1} = \sqrt{2x+2}$

$$\left(\sqrt{x+3} - \sqrt{x-1}\right)^{2} = \left(\sqrt{2x+2}\right)^{2}$$

$$\left(\sqrt{x+3}\right)^{2} - 2\sqrt{x+3}\sqrt{x-1} + \left(\sqrt{x-1}\right)^{2} = 2x+2$$

$$x+3 - 2\sqrt{(x+3)(x-1)} + x - 1 = 2x+2$$

$$\cancel{2x+2} - 2\sqrt{x^2 - x + 3x -} = \cancel{2x+2}$$

$$-2\sqrt{x^2 - x + 3x} = 0$$

$$\left(\sqrt{x^2 - x + 3x}\right)^2 = 0^2$$

$$x^2 - x + 3x = 0.$$

$$(x+3)(x-1) = 0$$

$$(x+3) = 0 \qquad x-1=0$$

$$\cancel{x=-3} \qquad x = 1$$

Sol $: x=1$ //

i.-) $\left(\sqrt{4x+3} + 1\right)^2 = \left(\sqrt{2x-2}\right)^{2}$

$$\left(\sqrt{4x+3}\right)^{2} + 2\sqrt{4x+3} + 1 = 2x-2$$

$$4x+3 + 2\sqrt{4x+3} + 1 = 2x-2$$

61

$$2\sqrt{4x+3} = 2x-2-4x-4$$

$$2\sqrt{4x+3} = -2x-6$$

$$\cancel{2}\sqrt{4x+3} = \cancel{2}(-x-3) \Rightarrow (\sqrt{4x+3})^2 = (-x-3)^2$$

$$4x+3 = x^2+6x+9 \Rightarrow x^2+6x-4x+9-3$$

$$\Rightarrow x^2+2x+6=0$$

Sol: ϕ.

d.- $(\sqrt{x+4})^2 = (1+\sqrt{2x-2})^2$

$$x+4 = 1+2\sqrt{2x-2}+(\sqrt{2x-2})^2$$

$$x+4 = 1+2\sqrt{2x-2}+2x-2$$

$$\cancel{x}+4-1-\cancel{2x}+2 = 2\sqrt{2x-2}$$

$$(-x+5)^2 = (2\sqrt{2x-2})^2$$

$$x^2-10x+25 = 4(2x-2)$$

$$x^2-10x+25 = 8x-8$$

$$x^2-10x-8x+25+8=0$$

$$x^2-18x+33=0$$

$$x = \frac{18 \pm \sqrt{18^2-4(1)(33)}}{2(1)}$$

$$x = \begin{cases} 2{,}07 \checkmark \\ 15{,}92 \times \end{cases}$$

k.- $\sqrt{x}+\sqrt{x+2}=3$

$$(\sqrt{x+2})^2 = (3-\sqrt{x})^2$$

$$x+2 = 9-6\sqrt{x}+(\sqrt{x})^2$$

$$\cancel{x}+2 = 9-6\sqrt{x}+\cancel{x}$$

$$6\sqrt{x} = 9-2$$

$$(6\sqrt{x})^2 = (7)^2$$

$$36x = 49$$

$$x = \frac{49}{36}$$

81.- $\frac{x-2}{\sqrt{2x-7}} = \sqrt{x-4}$

$\Rightarrow \frac{x-2}{\sqrt{2x-7}} - \sqrt{x-4} = 0 \Rightarrow \frac{x-2-\sqrt{x-4}\,\sqrt{2x-7}}{\sqrt{2x-7}} \Rightarrow$

$\Rightarrow x-2-\sqrt{(x-4)(2x-7)}=0 \Rightarrow (x-2)^2 = \left(\sqrt{2x^2-7x-8x+28}\right)^2$

$\Rightarrow x^2-4x+4 = 2x^2-15x+28$

$\Rightarrow 2x^2-15x+28-x^2+4x-4=0$

$\Rightarrow x^2-11x+24=0 \Rightarrow (x-8)(x-3)=0$

$\boxed{x=8}$ $\cancel{x=3}$

82.- $P(x): kx^2+3x+1=0$

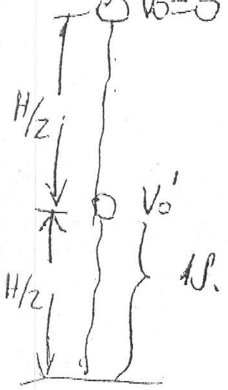
Solución única: $b^2-4ac=0$

$3^2-4k(1)=0$

$-4k=9 \Rightarrow \boxed{k=-9/4}$

83.- $H,$ $y=V_0t+\frac{1}{2}gt^2$ $V^2=V_0^2+2gy$

$\frac{H}{2}$; $t=1s.$ $\frac{H}{2}=V_0'(1)+\frac{1}{2}g(1)^2$ $(V_0')^2=0+2g\left(\frac{H}{2}\right)$



$H=2V_0'+g$ $(V_0')^2=gH$

$\frac{(V_0')^2}{g}=2V_0'+g$ $H=\frac{(V_0')^2}{g}$

$(V_0')^2=2gV_0'+g^2$

$(V_0')^2-2gV_0'-g^2=0 \Rightarrow V_0'=\frac{2g\pm\sqrt{(2g)^2-4(1)(-g^2)}}{2(1)}$

$V_0'=\frac{2g\pm\sqrt{8}g}{2} \Rightarrow g\left(\frac{2\pm 2\sqrt{2}}{2}\right) \Rightarrow g(1+\sqrt{2})$

$V_0' = g(1+\sqrt{2})$

$H = V_0' + g$

$H = g(1+\sqrt{2}) + g$

$\boxed{H = 33,46\,m.}$

$V = V_0 + gt$

$V_0' = 0 + gt$

$t = \frac{V_0'}{g}$

$t = \frac{g(1+\sqrt{2})}{g}$

$\boxed{t = 1+\sqrt{2}\,s}$

$T = 1+\sqrt{2}+1 = (2+\sqrt{2})\,s.$

84.- $a = 4\,m/s^2$

$V = 3\,m/s$

$x = 20\,m$

$t = ??$

$x = V_0 t + \frac{1}{2} a t^2$

$20 = 3t + \frac{1}{2}(4)t^2$

$20 = 3t + 2t^2$

$2t^2 + 3t - 20 = 0$

$(2t+8)(2t-5) = 0$

$2t = -8 \qquad 2t = 5$

$\cancel{t = -4\,s.} \qquad \boxed{t = \frac{5}{2}\,s.}$

85.- $P(x): \underbrace{||x-1|+1|}_{} \leq 0$

siempre será positivo

$\underbrace{|x-1|}_{}+1$

también siempre será positivo, por lo tanto nunca será 0

$A_{P(x)} = \phi$

a) Verdadero.

36.- $P(x): |x-a| < \delta$

$x - a < \delta \wedge -x + a < \delta$

$\boxed{x < a + \delta} \wedge -x < \delta - a$

$\boxed{x > a - \delta}$

a) verdadero

7.- $q(x): \dfrac{|x-2|-1}{x^2+1} \geq 0$

$|x-2|-1 \geq 0$

P. crítico

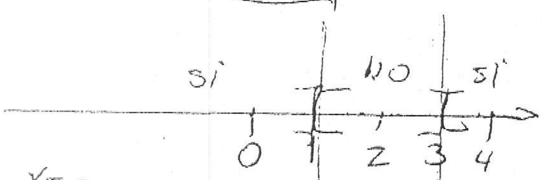
$x^2+1=0$

NO HAY

$x-2-1 \geq 0 \vee -x+2-1 \geq 0$

$\boxed{x \geq 3} \vee -x \geq -1$

$\boxed{x \leq 1}$



$x=0$

$\dfrac{|-2|-1}{1} \geq 0$

$x=2$

$\dfrac{0-1}{5} \geq 0$

NO

$x=4$

$\dfrac{2-1}{17} \geq 0$

si

$Aq(x): (-\infty, 1] \cup [3, +\infty)$

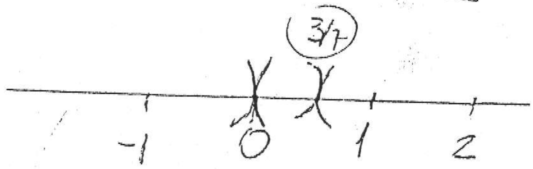
$P(x): 7x^2 < 3x$

$7x^2 - 3x < 0$

$x(7x-3) < 0$

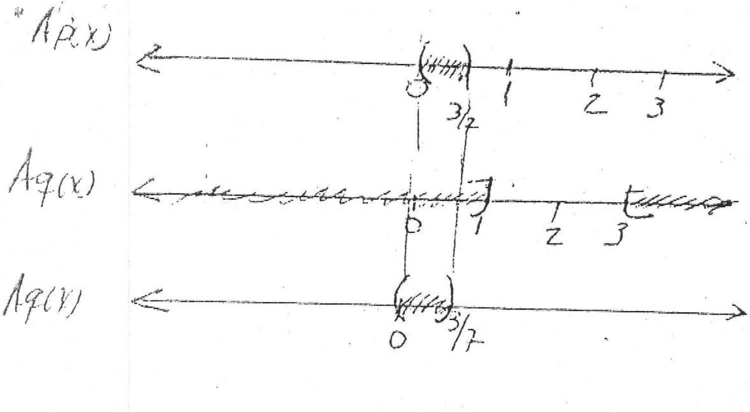
$\boxed{x < 0} \wedge 7x < 3$

$\boxed{x < 3/7}$



$x=-1$	$x=0,25$	$x=1$
$7(-1)^2-3(-1)<0$	$7(0,25)^2-3(0,25)<0$	$7(1)-3(1)<0$
NO	Si	NO

$Ap(x): (0, 3/7)$



d) correcto

88.- $p(x): |7x+4| \leq 10$

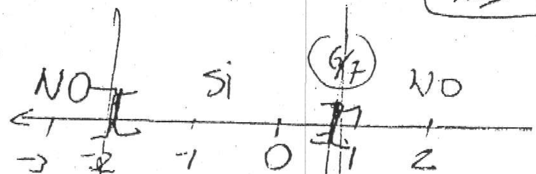
$7x+4 \leq 10 \wedge -7x-4 \leq 10$

$7x \leq 10-4 \qquad -7x \leq 10+4$

$7x \leq 6 \qquad -7x \leq 14$

$\boxed{x \leq \frac{6}{7}} \qquad 7x \geq -14$

$\boxed{x \geq -2}$



$x=-3 \qquad x=0 \qquad x=2$

$17 \leq 10 \qquad 4 \leq 10 \qquad 18 \leq 10$

NO $\qquad$ Si $\qquad$ NO

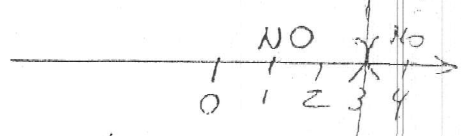
$Ap(x): [-2, 6/7]$.

$q(x): x^2 - 6x + 9 < 0$

$\sqrt{(x-3)^2} < \sqrt{0}$

$x - 3 < 0$

$\boxed{x < 3}$



$x=0 \qquad x=4$

$0+0+9<0 \qquad 16-24+9<0$

NO $\qquad$ NO

$\boxed{Aq(x): \phi}$

a) verdadera

89.- $1-x \geq 2x+6$

$-x-2x \geq 6-1$

$-3x \geq 5$

$3x \leq -5 \Rightarrow \boxed{x \leq -5/3}$ d) correcto

90.- $p(n): |2n-5| \leq 7$

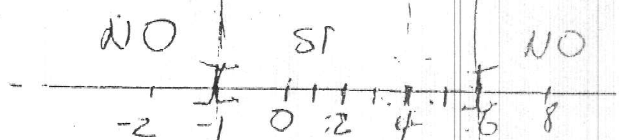
$2n-5 \leq 7 \wedge -2n+5 \leq 7$

$2n \leq 12 \wedge -2n \leq 7-5$

$\boxed{n \leq 6} \wedge -2n \leq 2$

$2n \geq -2$

$\boxed{n \geq -1}$



$n=-2 \qquad n=0 \qquad n=8$

$9 \leq 7 \qquad 5 \leq 7 \qquad 11 \leq 7$

NO $\qquad$ Si $\qquad$ NO

$Ap(x): [-1, 6]$

6 elementos: $\{1, 2, 3, 4, 5, 6\}$

91.- $[0,+\infty)$

92.- a) $(x \leq 14) \wedge (x > 2)$

$[2, 14]$

b) $[(x \leq -3) \wedge (x > 2)] \vee [(x \geq 3 \wedge (x \leq 5)]$

$\phi \cup [3,5]$

$\therefore [3,5]$

93.-

a) $5(x-1) - x(7-x) > x^2$

$5x - 5 - 7x \cancel{+x^2} \cancel{-x^2} > 0$

$2x > 5$

$\boxed{x > 5/2}$

b) $|2x+4| < 10$

$2x+4 < 10 \wedge -2x-4 < 10$

$2x < 6 \quad \wedge \quad -2x < 14$

$\boxed{x < 3} \qquad 2x > -14$

$\boxed{x > -7}$

Sol: $(-7, 3)$

c) $\left| \dfrac{x-3}{x-4} \right| < \dfrac{5}{2}$

$\Rightarrow \dfrac{x-3}{x-4} - \dfrac{5}{2} < 0 \quad \wedge \quad \dfrac{3-x}{x-4} - \dfrac{5}{2} < 0$

$\Rightarrow \dfrac{2(x-3) - 5(x-4)}{2(x-4)} < 0 \quad \wedge \quad \dfrac{2(3-x) - 5(x-4)}{2(x-4)} < 0$

$\Rightarrow \dfrac{2x - 6 - 5x + 20}{2(x-4)} < 0 \quad \wedge \quad \dfrac{6 - 2x - 5x + 20}{2(x-4)} < 0$

$$\Rightarrow \frac{-3x+14}{2(x-4)} > 0 \quad \wedge \quad \frac{-7x+26}{2(x-4)} > 0$$

P. crítico

$x-4=0$

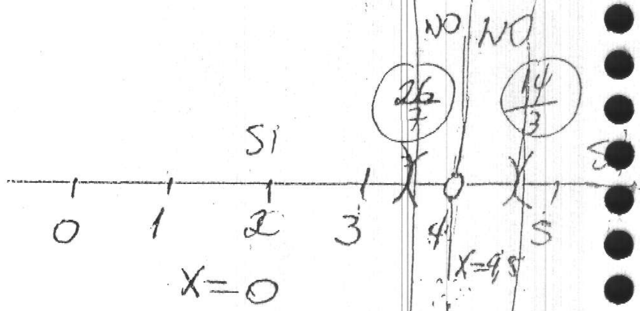
$\boxed{x=4}$

$-3x+14>0 \quad \wedge \quad -7x+26>0$

$-3x>-14 \qquad -7x>-26$

$3x<14 \qquad 7x<26$

$\boxed{x<\frac{14}{3}} \qquad \boxed{x<\frac{26}{7}}$



$x=0$

$\frac{3}{4}<\frac{5}{2}$

SI

$x=4,5$	$x=5$	$x=3,8$
$\frac{1,5}{0,5}<2,5$	$\frac{2}{1}<2,5$	$\left\lvert\frac{0,8}{0,2}\right\rvert<2,5$
NO		NO

Sol: $\left(-\infty, \frac{26}{7}\right) \cup \left(\frac{14}{3}, +\infty\right)$

d) $\frac{4}{x+1} - \frac{3}{x+2} > 1$

$$\Rightarrow \frac{4}{x+1} - \frac{3}{x+2} - 1 \geq 0 \Rightarrow \frac{4(x+2)-3(x+1)-(x+1)(x+2)}{(x+1)(x+2)} > 0$$

$$\Rightarrow \frac{\cancel{4x}+8-\cancel{3x}-3-x^2-2x-\cancel{x}-2}{(x+1)(x+2)} > 0 \Rightarrow \frac{-x^2-2x+3}{(x+1)(x+2)} > 0$$

$-x^2-2x+3>0$

$x^2+2x-3<0$

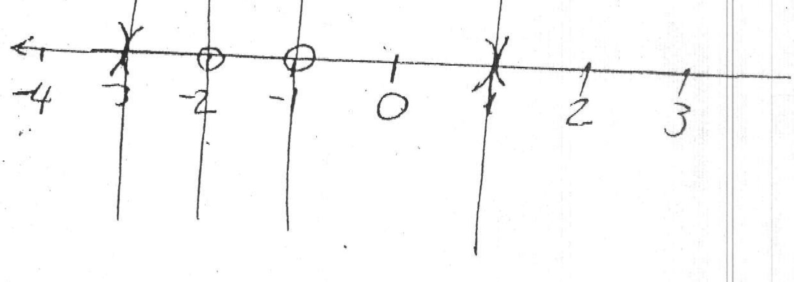
$(x+3)(x-1)<0$

$\boxed{x<-3} \; \boxed{x<1}$

P. crítico

$x+1=0 \qquad x+2=0$

$\boxed{x=-1} \qquad \boxed{x=-2}$



$x=-4$

$-\frac{4}{3}+\frac{3}{2}>1$

$-1,33+1,5>1$

NO

$x=-2,5$

$-\frac{4}{1,5}+\frac{3}{0,5}>1$

$-2,67+6>1$

SI

$x=-1,5$

$-\frac{4}{0,5}-\frac{3}{0,5}>1$

NO

$x=0$

$4-\frac{3}{2}>1$

SI

$x=2$

$\frac{4}{3}-\frac{3}{4}>1$

$1,33-0,75>1$

NO

Sol: $(-3,-2)\cup(-1,1)$

2) $|x-1|\geq\frac{x+1}{2}$

$x-1\geq\frac{x+1}{2}$ $\vee$ $-x+1\geq\frac{x+1}{2}$

$2(x-1)\geq x+1$ $\vee$ $2(-x+1)\geq x+1$

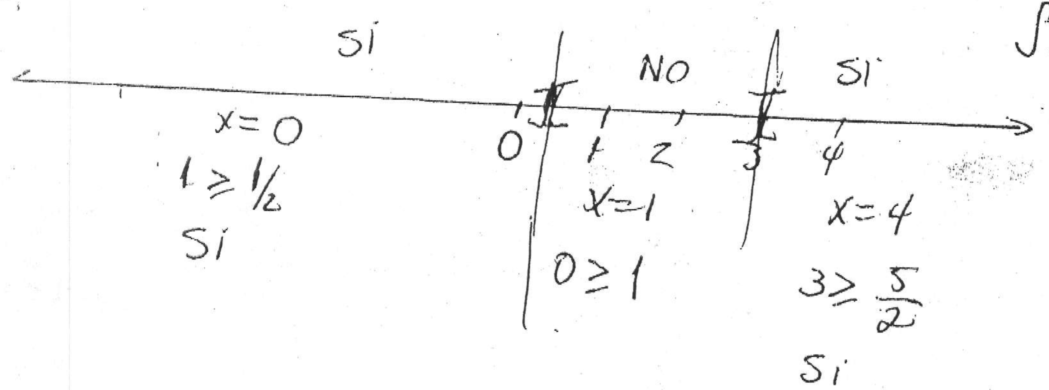
$2x-2\geq x+1$ $-2x+2\geq x+1$

$2x-x\geq 2+1$ $-2x-x\geq 1-2$

$x\geq 3$ $-3x\geq -1$

$3x\geq 1$

$x\geq 1/3$



Sol: $(-\infty,\frac{1}{3}]\cup[3,+\infty)$

f) $\frac{3x}{2} + 3|x-2| \leq 3$

$\Rightarrow 3x + 6|x-2| \leq 6$

$3x+6(x-2) \leq 6 \quad \wedge \quad 3x+6(-x+2) \leq 6$

$3x+6x-12 \leq 6 \quad \wedge \quad 3x-6x+12 \leq 6$

$9x \leq 6+12 \quad \wedge \quad -3x \leq 6-12$

$x \leq \frac{18}{9} \qquad +3x \geq 6$

$(x \leq 2) \qquad (x \geq 2)$

NO NO

0 1 2 3

Sol: ϕ

$x=0 \qquad x=3$

$6 \leq 3 \qquad \frac{9}{2}+3 \leq 3$

NO

94.- a) $p(x)$: $2+4x < 6x+7$

$-6x+4x < 7-2$

$-2x < 5$

$2x > -5$

$(x > -5/2)$

$A_{p(x)}: (-5/2, +\infty)$

b) $q(x)$: $2 < 2x-2 \leq 12$

$2 < 2x-2 \quad \wedge \quad 2x-2 \leq 12$

$-2x < -2-2 \quad \wedge \quad 2x \leq 12+2$

$-2x < -4 \qquad x < \frac{14}{2}$

$2x > 4$

$(x > 2) \qquad (x < 7)$

0 1 2 3 4 5 6 7 8

$x=0$	$x=3$	$x=8$
$2 < -2 \leq 12$	$2 < 4 < 12$	$2 < 14 < 12$
NO	SI	NO

Sol $(2, 7)$

c) $r(x): \; 8-3x \leq 2x-7 < x-13$

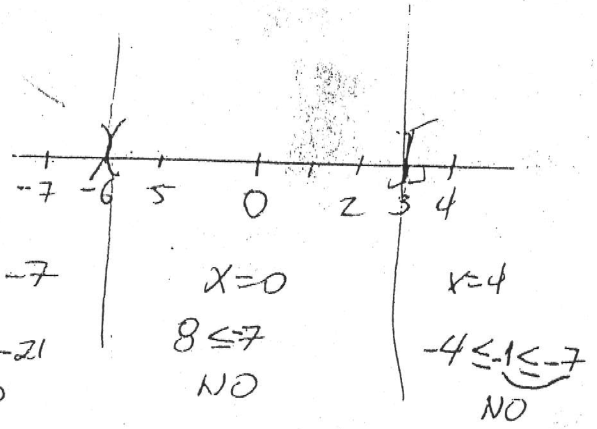
$8-3x \leq 2x-7 \quad \wedge \quad 2x-7 < x-13$

$-3x-2x \leq -7-8 \quad \wedge \quad 2x-x < -13+7$

$-5x \leq -15$

$5x \geq 15$

$\boxed{x \geq 3}$ $\qquad \boxed{x < -6}$



Sol: ϕ

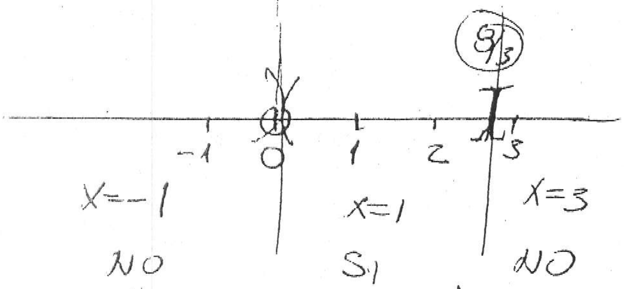
d) $m(x): \; \dfrac{8}{x} \geq 3$

$\dfrac{8}{x} - 3 \geq 0 \Rightarrow \dfrac{8-3x}{x} \geq 0$ $\qquad$ P. crítico $\boxed{x=0}$

$8-3x \geq 0$

$-3x \geq -8$

$\boxed{x \leq 8/3}$



Sol: $(0, \frac{8}{3}]$

2) $p(x): \; \dfrac{2x}{x-4} \leq 8$

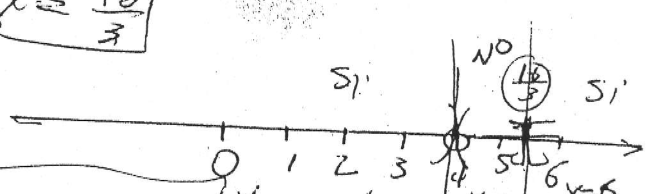
$\Rightarrow \dfrac{2x}{x-4} - 8 \leq 0 \Rightarrow \dfrac{2x-8(x-4)}{x-4} \leq 0 \Rightarrow \dfrac{2x-8x+32}{x-4} \leq 0$

$\dfrac{-6x+32}{x-4} \leq 0$ $\qquad$ P. crítico $\boxed{x=4}$ $\qquad \boxed{x \leq \frac{16}{3}}$

$-6x+32 \leq 0$

$-6x \leq -32$

$6x \leq 32$



Sol: $(-\infty, 4) \cup (\frac{16}{3}, +\infty)$ $\qquad$ x=0 $\;$ 0<8 (si) $\qquad$ x=5 $\;$ 10≤8 $\qquad$ si

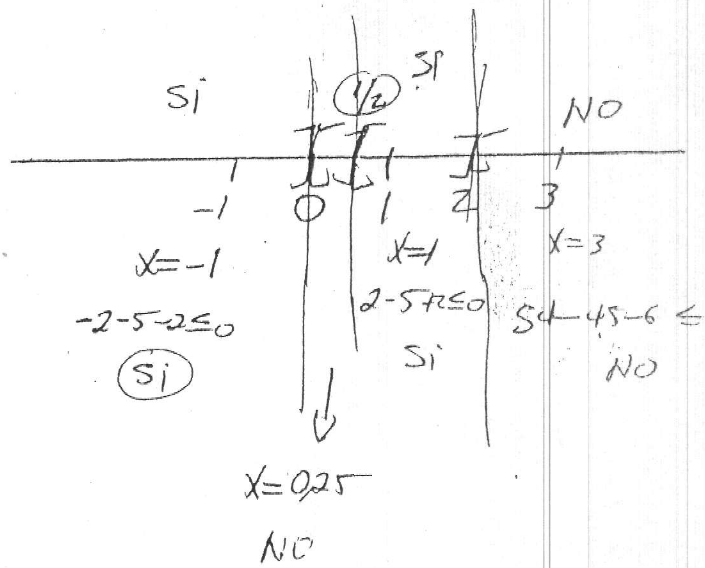
f) $q(x): 2x^3 - 5x^2 + 2x \leq 0$

$x(2x^2 - 5x + 2) \leq 0$

$x(2x-4)(2x-1) \leq 0$

$\boxed{x \leq 0}$ $\quad 2x \leq 4 \quad 2x \leq 1$

$\boxed{x \leq 2}$ $\quad \boxed{x \leq 1/2}$



Sol: $(-\infty, 0] \cup [1/2, 2]$.

g) $p(x): \dfrac{x^2 - 3x - 18}{13x - x^2 - 42} \geq 0$

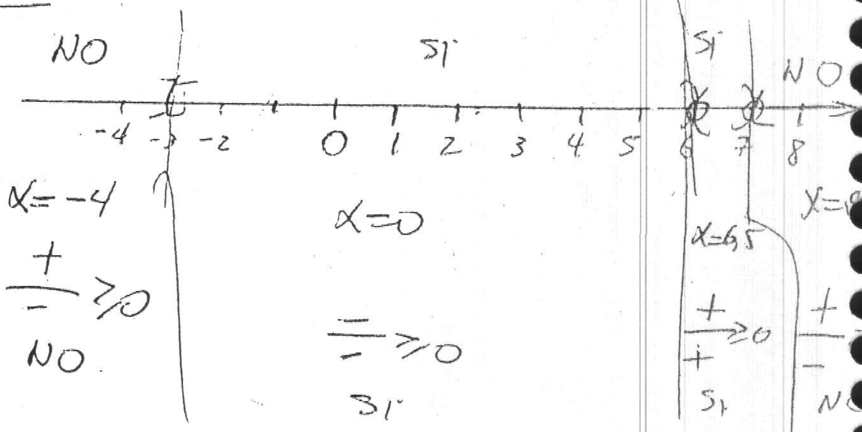
$\Rightarrow \dfrac{(x-6)(x+3)}{-(x^2 - 13x + 42)} \geq 0 \Rightarrow \dfrac{(x-6)(x+3)}{-(x-7)(x-6)} \geq 0$

P. critic

$\boxed{x=7}$ $\boxed{x=6}$

$(x-6)(x+3) \geq 0$

$\boxed{x \geq 6}$ $\boxed{x \geq -3}$



Sol: $[-3, 6) \cup (6, 7)$

h) $q(x): \dfrac{x^2 - 3x - 6}{x^2 - 1} \leq 1$

$\Rightarrow \dfrac{(x^2 - 3x - 6)}{(x+1)(x-1)} - 1 \leq 0 \Rightarrow \dfrac{x^2 - 3x - 6 - (x^2 - 1)}{(x+1)(x-1)} \leq 0$

$\Rightarrow \dfrac{\cancel{x^2} - 3x - 6 - \cancel{x^2} + 1}{x^2 - 1} \leq 0 \Rightarrow \dfrac{-3x - 5}{x^2 - 1} \leq 0$

P. critic

$x^2 = 1$

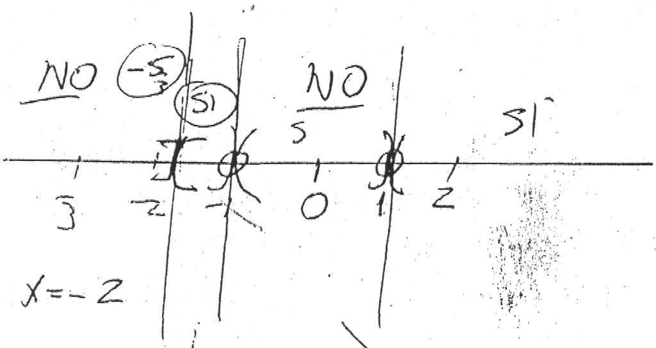
$\boxed{x = \pm 1}$

$-3x-5 \le 0$

$-3x \le 5$

$3x \ge -5$

$\boxed{x \ge -5/3}$



$x=-2$ | $x=-1{,}5$ | $x=0$ | $x\ge 2$

$\frac{4}{3} \le 1$ NO | $0{,}6 \le 1$ SI | $\frac{-6}{-1} \le 1$ NO | $\frac{-}{+} \le 1$

Sol: $\left[-\frac{5}{3}, -1\right) \cup (1, +\infty)$

95.- $a \ge 0 \wedge b \ge 0$

$$\left(\frac{a+b}{2}\right)^2 \ge (\sqrt{ab})^2$$

$$\frac{a^2+2ab+b^2}{4} \ge ab \Rightarrow a^2+2ab+b^2 \ge 4ab$$

$$\Rightarrow a^2+2ab-4ab+b^2 \ge 0 \Rightarrow a^2-2ab+b^2 \ge 0$$

$$\Rightarrow (a-b)^2 \ge 0$$ el cuadrado siempre dá como resultado un número positivo

96.- $ab > 0$

$$\frac{a}{b} + \frac{b}{a} \ge 2$$

$$\Rightarrow \frac{a^2+b^2}{ab} \ge 2$$

$$\Rightarrow a^2+b^2 \ge 2ab$$

$$\Rightarrow a^2-2ab+b^2 \ge 0$$

$$\underbrace{(a-b)^2}_{} \ge 0$$

siempre sera un número positivo

97.- $a^2+4b^2+3c^2+14 > 2a+12b+6c$

$(a^2-2a)+(4b^2-12b)+(3c^2+6c) > -14$

$a(a-2)+4b(b-3)+3c(c+2) > -14$

$(-)(-)+4(-)(-)+3(-)(-) > -14$

$(+)+(+)+(+) > -14$

$(+) > (-)$

98.- $a+b+c \geq 0 \therefore a^3+b^3+c^3 \geq 3abc$

$\therefore (a+b+c)^3 \geq 0^3$

$\therefore \underbrace{a^3+b^3+c^3}_{3abc}+3ab^2+3ac^2+3a^2b+3a^2c+3bc^2+3b^2c+6abc \geq 0$

$\therefore (3ab^2+3abc+3a^2b)+(3ac^2+3abc+3a^2c)+(3b^2c+3abc+3bc^2) \geq 0$

$\therefore 3ab(b+c+a)+3ac(c+b+a)+3bc(b+a+c) \geq 0$

$\therefore 3\underbrace{(a+b+c)}_{\geq 0}\underbrace{(ab+ac+bc)}_{\text{debe ser también positivo}} \geq 0$

99.- $a>0;\ b>0;\ c>0$

$(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$

$\Rightarrow (a+b+c)\left(\frac{bc+ac+ba}{abc}\right) \geq 9 \Rightarrow \frac{abc+a^2c+a^2b+b^2c+abc+ab^2+bc^2+ac^2}{abc}$

$\Rightarrow \frac{3abc+a^2c+a^2b+b^2c+b^2a+bc^2+ac^2}{abc} \geq 9 \Rightarrow 3abc+a^2c+a^2b+b^2c+ab^2+bc^2+ac^2 \geq 9abc$

$\Rightarrow a^2c+a^2b+ab^2+b^2c+ac^2+bc^2-6abc \geq 0$

100.- $a \geq 0;\ b \geq 0,\ c \geq 0,\ d \geq 0$

$$\left(\frac{a+b+c+d}{4}\right)^4 - \left(\sqrt[4]{abcd}\right)^4$$

$$\Rightarrow (a+b+c+d)^4 \geq 256\,abcd$$

$$[(a+b)+(c+d)]^4 \geq 4a \times 4b \times 4c \times 4d$$

$$\Rightarrow \underbrace{(a+b)^4}_{\substack{5\text{ término}\\(a^4 \ldots b^4)}} + \underbrace{4(a+b)^3(c+d)}_{4(a^3 \ldots b^3)} + 6\underbrace{(a+b)^2(c+d)^2}_{\substack{6(a^2+b^2+c^2+d^2)\\24abcd}} + 4(a+b)(c+d)^3 + \underbrace{(c+d)^4}_{\substack{5\text{ término}\\(c^4 \ldots d^4)}} \geq 4a \cdot 4b \cdot 4c \cdot 4d$$

101.-

① $p(1) = \frac{(1)^2(1+1)^2}{4} = 1^3$

$$\frac{1(2)^2}{4} = 1 \qquad \frac{4}{4} = 1$$

** Necesitamos llegar a.

$$\frac{(N+1)^2\,(N+2)^2}{4}$$

* Hay que demostrar que la fórmula se cumple para. $N+1$

$$1^3 + 2^3 + 3^3 + \ldots\ldots N^3 + (n+1)^3 = \frac{N^2(N+1)^2}{4} + (N+1)^3$$

$$\frac{N^2(N^2+2N+1)}{4} + \frac{4(N+1)^3}{4}$$

$$\frac{N^4 + 2N^3 + N^2 + 4(N^3 + 3N^2 + 3N + 1)}{4}$$

$$\frac{N^4 + 2N^3 + N^2 + 4N^3 + 12N^2 + 12N + 4}{4}$$

$$\frac{N^4 + 6N^3 + 13N^2 + 12N + 4}{4}$$

Se ha reemplazado N por $(N+1)$

$$\frac{(N+1)^2\,(N+2)^2}{4}$$

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102.

a) $2^{2n}+5$ divisible 3

$n=1 \Rightarrow \frac{9}{3}=3$

$n=2 \Rightarrow \frac{21}{3}=7$ 2^4+5

$n=3 \Rightarrow \frac{69}{3}=23$ 2^6+5

$n=4 \Rightarrow \frac{261}{3}=87$ 2^8+5

$\vdots$

$n \Rightarrow 2^{2(n-1)}+k$

$$\frac{2^{2n}+5}{3}=k+2^{2(n+1)}$$

$$\frac{2^{4(n+1)}+5}{3}=k+2^{2(n+1)}$$

$$\frac{2^{2n+2}+5}{3}=k+2^{2n}\cdot\frac{2^2}{2^2}$$

$$\frac{2^{2n}\cdot 4+5}{3}=k+4\cdot 2^{2n}\cdot 2^{-2}$$

$$\boxed{\frac{4\cdot 2^{2n}+5}{3}=k+4\cdot 2^{2(n-1)}}$$

b) $\underbrace{2^{2n}\ 13n-1}$ divisible 9

NO CUMPLE PARA $n \in N$

$n=1$ $\frac{6}{9}$ NO ES DIVISIBLE

$n=2$ $\frac{21}{9}$ NO ES DIVISIBLE

$n=3$ $\frac{72}{9}=8$

c) a^n-b^n es divisible $(a-b)$

$n \Rightarrow$ impar

$$(a^n-b^n)=\underline{(a-b)}(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ldots ab^{n-2}+b^{n-1})$$

$n \Rightarrow$ par

$$(a^n-b^n)=(a^{\frac{n}{2}}-b^{\frac{n}{2}})(a^{\frac{n}{2}}+b^{\frac{n}{2}})$$ impar

$$\Rightarrow (a^{\frac{n}{2}}+b^{\frac{n}{2}})\cdot\underline{(a-b)}(a^{\frac{n}{2}-1}+a^{\frac{n}{2}-2}b+\cdots ab^{\frac{n}{2}-2}+b^{n-1})$$

$)\ a^{2n}-1$ es divisible $(a+1)$

$a^{(2n)}-1$ → garantiza que sea siempre una diferencia de cuadrado

$$(a^{2n}-1)=(a^n-1)(a^n+1)$$

$$\Rightarrow (a^n-1)(a^n+1) \begin{cases} n \text{ es par} \Rightarrow (a^n-1) \text{ es nuevamente diferencia de cuadrado, obteniéndose } n \text{ impar.} \\ n \text{ es impar} \Rightarrow \underline{(a^n+1)}=(a+1)(a^{n-1}-a^{n-2}+a^{n-3}+\dots 1) \end{cases}$$

e) $a+ar+ar^2+\cdot\ ar^{n-2}+ar^{n-1} = \dfrac{a(1-r^n)}{1-r}$

$$+ar^{n-1}+ar^n = \frac{a(1-r^n)}{1-r}+ar^n$$

$$\Rightarrow \frac{a(1-r^n)+ar^n(1-r)}{1-r}$$

$$\Rightarrow \frac{a-\cancel{ar^n}+\cancel{ar^n}-ar^n r}{1-r}$$

$$\Rightarrow \frac{a-ar^{n+1}}{1-r} \Rightarrow \frac{a(1-r^{n+1})}{1-r}$$

Forma original.

$$\frac{a(1-r^n)}{1-r} \Rightarrow^{n=n+1} \frac{a(1-r^{n+1})}{1-r}$$

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f.) $a_1+(a_1+d)+(a_1+2d)+(a_1+3d)+\cdots(a_1+(n-1)d)=\frac{n}{2}(2a_1+(n-1)d)$

$$+(a_1+(n-1)d)+(a_1+nd)=\frac{(n+1)}{2}(2a_1+(n+1-1)d)$$

$$\Rightarrow \frac{(n+1)}{2}(2a_1+nd)$$

$$\Rightarrow \frac{(2a_1n+n^2d+2a_1+nd)}{2}$$

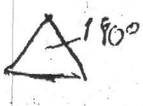
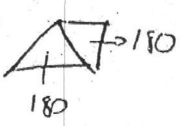
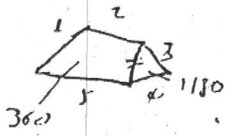
$$\Rightarrow \frac{(2a_1n+n^2d+2a_1+nd-nd+nd}{2}$$

$$\Rightarrow \frac{(2a_1+2nd)+(2a_1n+n^2d-nd}{2}$$

$$\Rightarrow \frac{\cancel{2}(a_1+nd)}{\cancel{2}}+\frac{n(2a_1+nd-d)}{2}$$

$$\Rightarrow (a_1+nd)+\frac{n}{2}(2a_1+(n-1)d)$$

103.—

n			
3		$\Rightarrow 180°$	
4		$\Rightarrow 360°$	
5		$\Rightarrow 540°$	

$a_1=180°$ $\quad S=2R(n-2)$

$d=180°$ $\quad S=180(n-2)$

$$a_n=S$$

$$a_1+(n-3)d=2R(n-2)$$

$$180+(n-3)180=180(n-2)$$

$$\cancel{180}(1+n-3)=\cancel{180}(n-2)$$

$$\cancel{n-2}=\cancel{n-2}$$

$0=0$ Son iguales

104.— $1+3+5+\cdots+(2n-1)=n^2$

$$\cdots+(2n-1)+(2n+2-1)=(n+1)^2$$

$$\cdots+(2n-1)+(2n+1)=n^2+2n+1$$

$$\Rightarrow n^2+(2n+1)$$

105.- $2^1 + 2^2 + 2^3 + \ldots\ldots + 2^n = 2(2^n - 1)$.

$\ldots\ldots + 2^n + 2^{n+1} = 2(2^{n+1} - 1)$

$\Rightarrow 2 \cdot 2^{n+1} - 2$.

$\Rightarrow 2^{n+1} + 2^{n+1} - 2$.

$\Rightarrow 2^{n+1} + 2 \cdot 2^n - 2$.

$\Rightarrow 2^{n+1} + 2(2^n - 1)$

106.- 1) $p(1) = \dfrac{1(1+1)(6+9+1-1)}{30} = (1)^4$

$\dfrac{1(2)(15)}{30} = 1 \qquad \dfrac{30}{30} = 1$

* Reemplazamos N por N+1.

$\dfrac{(N+1)(N+2)\ [6(N+1)^3 + 9(N+1)^2 + N+1-1]}{30}$

$\dfrac{(N+1)(N+2)(6N^3 + 18N^2 + 18N + 6 + 9N^2 + 18N + 9 + N)}{30}$

$\dfrac{(N+1)(N+2)(6N^3 + 27N^2 + 37N + 15)}{30}$

$1^4 + 2^4 + 3^4 + \ldots\ldots N^4 + (N+1)^4 = \dfrac{N(N+1)(6N^3 + 9N^2 + N - 1)}{30} + (N+1)^4$

$\dfrac{(N^2+N)(6N^3 + 9N^2 + N - 1) + 30N^4 + 120N^3 + 180N^2 + 120N + 30}{30}$

$\dfrac{6N^5 + 6N^4 + 9N^4 + 9N^3 + N^2 + N^2 - N^2 - N + 30N^4 + 120N^3 + 180N^2 + 120N + 30}{30}$

$\dfrac{6N^5 + 45N^4 + 130N^3 + 180N^2 + 119N + 30}{30}$

$$\boxed{\dfrac{(N+1)(N+2)(6N^3 + 27N^2 + 37N + 15)}{30}}$$

107.- $1 + \dfrac{1}{2} + \dfrac{1}{2^2} + \dfrac{1}{2^3} + \ldots\ldots + \dfrac{1}{2^n} = 2 - \dfrac{1}{2^n}$

$$+\frac{1}{2^n} + \frac{1}{2^{n+1}} = 2 - \frac{1}{2^{n+1}}$$

$$\Rightarrow 2 - \frac{1}{2 \cdot 2^n} \Rightarrow 2 - \frac{1}{2} \cdot \left(\frac{1}{2^n}\right)$$

$$\Rightarrow 2 - \left(1 - \frac{1}{2}\right)\frac{1}{2^n} \Rightarrow 2 - \frac{1}{2^n} + \frac{1}{2 \cdot 2^n}$$

$$\Rightarrow 2 - \frac{1}{2^n} + \frac{1}{2^{n+1}}$$

108.- $2+6+10+ \dots + (4n-2) = 2n^2$

$$\dots + (4n-2) + (4(n+1)-2) = 2(n+1)^2$$

$$\dots + (4n-2) + (4n+4-2) = 2(n^2+2n+1)$$

$$\dots + (4n-2) + (4n+2) = 2n^2+4n+2$$

$$\Rightarrow 2n^2 + (4n+2)$$

109.- $K + K^2 + K^3 + \dots + K^n = \frac{K}{K-1}(K^n - 1)$

$$\dots + K^n + K^{n+1} = \frac{K(K^n-1)}{K-1} + K^{n+1}$$

$$\Rightarrow \frac{K \cdot K^n - K + K^{n+1}(K-1)}{K-1}$$

(original) $\frac{K}{K-1}(K^n-1)$

$$\Rightarrow \frac{K^{n+1} - K + K \cdot K^{n+1} - K^{n+1}}{K-1}$$

$n = n+1 \quad \frac{K}{K-1}(K^{n+1}-1)$

$$\Rightarrow \frac{K \cdot K^{n+1} - K}{K-1} \Rightarrow \frac{K(K^{n+1}-1)}{K-1}$$

110.- 1, 2, 3, 4, 5

5	4

1, 2, 3, 4, 5 — 4 números ya está en el 1er Casillero

$\Rightarrow 5 * 4$
$\Rightarrow 20$

a) Verdadero

111.-

C	A
3	4

3	2	1

$\Rightarrow 3 * 4 * 3 * 2 * 1$

$\Rightarrow$ 72 formas.

112.-

a) $n = 6$
$x = 4$

$${}_6P_4 \Rightarrow \frac{6!}{2!} \Rightarrow \frac{6*5*4*3*\cancel{2!}}{\cancel{2!}}$$

$\Rightarrow$ 360 palabras.

b) $n=6$

$x=6$

$${}_6P_6 = \frac{6!}{0!} \Rightarrow 6\times5\times4\times3\times2\times1$$

$\Rightarrow$ 720 palabras

c)

2	5	4	3	2	1

$\downarrow$

0, A

$\Rightarrow 2\times5\times4\times3\times2\times1$

$\Rightarrow$ 240 palabras

113.- $n=18$

a) $x=3$

$${}_{18}C_3 = \frac{18!}{3!\,15!} \Rightarrow \frac{\cancel{18}^{3}\times17\times16\times\cancel{15!}^{1}}{\cancel{3\times2\times1}_{1}\times\cancel{15!}_{1}}$$

$\Rightarrow 816$

114.- 10 V

7 M

a) $\left.\begin{matrix}3V\\4M\end{matrix}\right\} 7 \Rightarrow {}_{10}C_3 * {}_7C_4$

$$\Rightarrow \frac{10!}{3!\,7!} * \frac{7!}{4!\,3!}$$

$$\Rightarrow \frac{10\times9\times\cancel{8}^{4}\times\cancel{7!}}{\cancel{3\times2}\times1\times\cancel{7!}} * \frac{7\times6\times5\times\cancel{4!}}{\cancel{4!}\,3\times2\times1}$$

$\Rightarrow$ 4200 comites

b)

$\left.\begin{matrix}3V\\4M\end{matrix}\right\} + \left.\begin{matrix}4V\\3M\end{matrix}\right\} \Rightarrow {}_{10}C_4 * {}_7C_3$

$$\frac{10!}{4!\,6!} * \frac{7!}{4!\,3!} \Rightarrow \frac{10\times9\times8\times7\times\cancel{6!}}{\cancel{6!}\,4\times3\times2\times1} * \frac{7\times6\times5\times\cancel{4!}}{\cancel{4!}\,3\times2\times1}$$

$\Rightarrow 7350$

$4200 + 7350$

$\Rightarrow \underline{11550}$

81

115.- $n = 21$
$x = 3$

$${}_{21}C_3 \Rightarrow \frac{21!}{3!\,18!}$$

$$\Rightarrow \frac{\cancel{21} \times \cancel{20} \times 19 \times \cancel{18!}}{\cancel{18!}\,\cancel{3} \times \cancel{2} \times 1}$$

$\Rightarrow 1330$ e) Correcto

116.- $(x+k)^5$

$$\cancel{\binom{5}{i}}(x)^{5-i}\cancel{(k)^{i}} = x^2$$

$$\cancel{x}^{5-i} = \cancel{x}^2$$

$$5-i = 2$$

$$-i = 2-5$$

$$\boxed{i=3}$$

$$\binom{5}{3} \cdot \cancel{x^2}(k)^3 = 80$$

$$\frac{5!}{3!\,2!}\cancel{x^2}k^3 = 80$$

$$\frac{5 \times 4 \times \cancel{3!}}{\cancel{3!}\,2!}k^3 = 80$$

$$10k^3 = 80$$

$$k^3 = 8$$

$\boxed{k=2}$ i) correcto

117.- a) $(1-2a)^3 = \cancel{\binom{3}{0}}(1)^3 - \cancel{\binom{3}{1}}(1)^2(2a) + \cancel{\binom{3}{2}}(1)(2a)^2 - \cancel{\binom{3}{3}}(2a)^3$

$$\Rightarrow 1 - 2a + 12a^2 - 8a^3$$

b) $\left(a - \frac{1}{a}\right)^5 = \cancel{\binom{5}{0}}a^5 - \cancel{\binom{5}{1}}a^4\left(\frac{1}{a}\right) + \cancel{\binom{5}{2}}a^3\left(\frac{1}{a}\right)^2 - \cancel{\binom{5}{3}}a^2\left(\frac{1}{a}\right)^3$

$$+ \cancel{\binom{5}{4}}a\left(\frac{1}{a}\right)^4 - \cancel{\binom{5}{5}}\left(\frac{1}{a}\right)^5$$

$$\Rightarrow a^5 - 5a^3 + 10a - 10\frac{1}{a} + 5\frac{1}{a^3} - \frac{1}{a^5}$$

$$\Rightarrow a^5 - 5a^3 + 10a - \frac{10}{a} + \frac{5}{a^3} - \frac{1}{a^5}$$

118.- $\left(\frac{1}{2}u - 2v\right)^{10}$ 7mo T

$i = 7-1$

$i = 6$

$$\binom{10}{6}\left(\frac{1}{2}u\right)^{10-6} \cdot \left(-2v\right)^{4} \Rightarrow \frac{10!}{6! \cdot 4!} \cdot \frac{u^4}{\cancel{2^4}_{1}} \cdot \cancel{2^6}^{2} v^6$$

$$\Rightarrow \frac{\cancel{10}\times\cancel{9}^{3}\times\cancel{8}\times 7\times \cancel{6!}}{\cancel{6!}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \Rightarrow 210 \cdot 2^2 u^4 v^6$$

$$\Rightarrow 840\, u^4 v^6$$

119.- Encontrar el Término medio en : $(2x + 3y^{-1})^6$

$a = 2x$ $b = 3y^{-1}$ $N = 6$ $R = 4$ $R-1 = 3$

Hay 7 términos

$$T.C. = \frac{7+1}{2} = \frac{8}{2} = 4.$$

$$\binom{N}{R-1}(a)^{N-(R-1)}(b)^{R-1} = \binom{6}{3}(2x)^{6-3}(3y^{-1})^3.$$

$$\frac{6\times 5\times 4\times \cancel{3!}}{\cancel{3!}\times \cancel{3}\times\cancel{2}\times 1}(2x)^3(2+y^{-3}) = 20(8x^3)(2+y^{-3}) =$$

$$\boxed{= 4320\, x^3 y^{-3}.}$$

120.- $\left(6x - \frac{1}{2x}\right)$ independiente de x $\Rightarrow x^0$

$$\cancel{\binom{10}{i}}(6x)^{10-i}\left(\frac{\cancel{1}}{\cancel{2x}}\right)^{i} = x^0$$

$$x^{10-i} \cdot x^{-i} = x^0 \Rightarrow \cancel{x}^{10-2i} = \cancel{x}^0$$

$10 - 2i = 0$

$2i = 10 \Rightarrow i = 5$ $T = 6^{10}$

$$\Rightarrow \frac{10!}{5!\,5!}$$

$$\Rightarrow \frac{\cancel{10}^{2}\times\cancel{9}^{3}\times\cancel{8}\times 7\times 6\times\cancel{5!}}{\cancel{5!}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times 2\times 1}$$

$$\Rightarrow 252$$

83

121.- $\left(x^2+\frac{3}{x^3}\right)^7$

$\cancel{\binom{7}{i}} \cdot (x^2)^{7-i} \cdot \left(\frac{3}{x^2}\right)^i = x^1 \Rightarrow (x^2)^{7-i} \cdot (x^{-2})^i = \cancel{x}^1$

$\Rightarrow x^{2(7-i)} \cdot x^{-2i} = x^1 \Rightarrow \cancel{x}^{14-2i-2i} = \cancel{x}^1$

$14-4i=1 \Rightarrow -4i = 1-14$

$i = \frac{13}{4}$

NO EXISTE
i debe ser un
número entero;
y no lo es

122.- $\left(x^2+\frac{1}{x}\right)^9$

Término $\Rightarrow x^0$

$\cancel{\binom{9}{i}} (x^2)^{9-i} \cdot \left(\frac{1}{x}\right)^i = x^0 \Rightarrow x^{2(9-i)} \cdot (x^{-1})^i = x^0$

$x^{18-2i} \cdot x^{-i} = x^0 \Rightarrow \cancel{x}^{18-3i} = \cancel{x}^0 \Rightarrow 18-3i=0$

$3i=18 \Rightarrow \boxed{i=6}$ 7mo Término

$\binom{9}{6} \Rightarrow \frac{9!}{3!\ 6!} \Rightarrow \frac{\cancel{9}^3 \times \cancel{8}^4 \times 7 \times \cancel{6!}^1}{\cancel{3}_1 \times \cancel{2}_1 \times 1 \times \cancel{6!}_1}$

$\Rightarrow 84$

123.- $(5+2x^2)^7$

$\cancel{\binom{7}{i}}\cancel{(5)}^{7-i}(\cancel{2}x^2)^i = x^{10}$

$\Rightarrow \cancel{x}^{2i} = \cancel{x}^{10} \Rightarrow 2i = 10$

$\boxed{i=5}$

$\binom{7}{5}(5)^2(2)^5x^{10} \Rightarrow \frac{7!}{5!\,2!} \times 25\times 32\, x^{10}$

$\Rightarrow \frac{7\times \overset{3}{\cancel{6}}\times \cancel{5!}}{\cancel{5!}\,\cancel{2}\times 1} \times 25\times 32\, x^{10} \Rightarrow 16800x^{10}$

124.- $\left(x^2 - \frac{y}{x}\right)^5$ Término que contiene x^3

$\cancel{\binom{5}{i}}(x^2)^{5-i}\cdot\cancel{\left(-\frac{y}{x}\right)}^i = x^3$

$\Rightarrow (x^2)^{5-i}\cdot(x^{-1})^i = x^3 \Rightarrow x^{10-2i}\cdot x^{-i} = x^3$

$\Rightarrow \cancel{x}^{10-3i} = \cancel{x}^3 \Rightarrow 10-3i = 3$

$-3i = 3-10$

$-3i = -7$

$i = 7/3$ No existe el término; i debe ser un número entero ≥ 0

e) correcto

125.- $S_n = 168$

$a_1 = 30$

$d = -2$

a) $S_n = \frac{n}{2}(2a_1+(n-1)d$

$168 = \frac{n}{2}(2(30)+(n-1)(-2))$

$336 = n(60+2-2n)$

$336 = 62n-2n^2$

$2n^2-62n-336=0$

$2(n^2-31n-168)=0 \Rightarrow n^2-31n-168$

$n = \frac{31\pm\sqrt{31^2-4(1)(-168)}}{2(1)} \Rightarrow \frac{31\pm 40,41}{2}$

$n = 35,7$

$a_n = a_1+(n-1)d$

$a_n = 30+(35,7-1)(-2)$

$a_n = -39,41$

126.- $S_{10} = 35$

$a_1 = 10$

$a_{10} = ¿?$

$n = 10$

$S_n = \frac{n}{2}(a_1+a_n)$

$35 = \frac{10}{2}(10+a_{10})$

$\frac{35}{5} = 10+a_{10}$

$7 = 10+a_{10}$

$a_{10} = 7-10$

$a_{10} = -3$

127 —

$n+2(n+1)=44$

$n+2n+2=44$

$3n=44-2$

$3n=42$

$n=14$

$\Rightarrow n+3$

$\Rightarrow 17$ c) correcto

128 — a) $S_n=\frac{n}{2}(a_1+a_n)$ VERDADERO

$S_n=\frac{100}{2}(1+100)$

$S_n=5050$

b) $\dfrac{\binom{20}{10}}{\binom{21}{10}} \Rightarrow \dfrac{\frac{20!}{\cancel{10!}\,10!}}{\frac{21!}{\cancel{10!}\,11!}} \Rightarrow \dfrac{\frac{\cancel{20!}}{\cancel{10!}}}{\frac{21\times \cancel{20!}}{11\times \cancel{10!}}} = \frac{11}{21}$

VERDADERO

c) $\binom{n}{0}=\binom{n}{n}$

VERDADERO

$\frac{\cancel{n!}}{0!\,n!}=\frac{\cancel{n!}}{n!\,\cancel{0!}}$

d) $n!=n(n-1)!$ VERDADERO

e) $2\binom{1000}{2} \Rightarrow \cancel{2}^{1}\times\frac{1000!}{998!\,\cancel{2!}_{1}} = \frac{1000\times 999\times \cancel{998!}}{\cancel{998!}} = 999000$; FALSO

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129.— $a_1=2$

$$a_1+a_1r+a_1r^2=86$$

$$2+2r+2r^2=86$$

$$2r^2+2r+2-86=0$$

$$2(r^2+r-42)=0$$

$$(r+7)(r-6)=0$$

$r=-7$; $\boxed{r=6}$. b) correcto

130.—

a) $f(n)= -1,1,-1,1,-1,1,\ldots$

P. geométrica , $r=-1$

b) $f(n)=\dfrac{1}{3^n}$

$f(n)=\dfrac{1}{3},\dfrac{1}{9},\dfrac{1}{27},\ldots$ P. geométrica

$r=\dfrac{\frac{1}{9}}{\frac{1}{3}} \Rightarrow r=\dfrac{1}{3}$

c) $f(n)=\dfrac{1}{n^2}$

$\Rightarrow 1,\dfrac{1}{4},\dfrac{1}{9},\dfrac{1}{16},\ldots$ La sucesión NO es ni geométrica o aritmética; la razón no es constante

$r_1=\dfrac{\frac{1}{4}}{1}$ $\quad r_2=\dfrac{\frac{1}{9}}{\frac{1}{4}}$

$\boxed{r_1=\frac{1}{4}}$ $\quad \boxed{r_2=\frac{4}{9}}$

d) $f(n)=\dfrac{1}{n!} \Rightarrow \dfrac{1}{1},\dfrac{1}{2},\dfrac{1}{6},\dfrac{1}{24},\ldots$ NO es geométrica o aritmética

131.- $\sqrt{2\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}}$

$\Rightarrow \left(2\left(2\left(2\left(2(2)^{1/2}\right)^{1/2}\right)^{1/2}\right)^{1/2}\right)^{1/2} \Rightarrow 2^{1/2}\cdot 2^{1/4}\cdot 2^{1/8}\cdot 2^{1/16}\cdot 2^{1/32}$

$\Rightarrow 2^{1/2+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}}$

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32} = \frac{16+8+4+2+1}{32}$

$\Rightarrow \frac{31}{32}$

$\Rightarrow 2^{\frac{31}{32}}$

a) correcto

132.-

a) $\frac{1}{1001}, \frac{1}{2001}, \frac{1}{3001}$

$\Rightarrow \frac{1}{1000n+1}$

b) $1, \frac{1}{9}, \frac{1}{25}, \frac{1}{49}$

$\Rightarrow \frac{1}{(2n-1)^2}$

133.- $a_1 = 3$
$r = 2$

$f(4) = a_1 r^3$
$\Rightarrow 3(2)^3$
$\Rightarrow 24$

$f(6) = a_1 r^5$
$\Rightarrow 3(2)^5$
$\Rightarrow 96$

$f(11) = a_1 r^{10}$
$= 3(2)^{10}$
$\Rightarrow 3072$

134 — $a_5 = 162$ $a_5 = a_1 r^4$ $a_8 = a_1 r^7$

$a_8 = 432$ $162 = a_1 r^4$ $432 = a_1 r^7$

$a_3 = ¿?$ $a_1 = \frac{162}{r^4}$ $a_1 = \frac{432}{r^7}$

$$\frac{162}{r^4} = \frac{432}{r^7}$$

$$\frac{r^7}{r^4} = \frac{432}{162}$$

$$r^3 = \frac{216}{81};\ r = \sqrt[3]{\frac{216}{81}} \Rightarrow \frac{\sqrt[3]{216}}{\sqrt[3]{81}}$$

$$r = \frac{6}{\sqrt[3]{27}\sqrt[3]{3}} \Rightarrow \frac{\cancel{6}^{2}}{\cancel{3}\sqrt[3]{3}} \Rightarrow \boxed{\frac{2}{\sqrt[3]{3}}}$$

$$a_1 = \frac{162}{\left(\frac{2}{\sqrt[3]{3}}\right)^4} \Rightarrow \frac{162}{\frac{16}{(\sqrt[3]{3})^3\sqrt[3]{3}}} \Rightarrow \frac{81 \times 3\sqrt[3]{3}}{4}$$

$$\boxed{a_1 = \frac{243}{4}\sqrt[3]{3}}$$

$$a_3 = a_1 r^2$$

$$a_3 = \frac{243}{4}\sqrt[3]{3} \times \left(\frac{2}{\sqrt[3]{3}}\right)^2$$

$$a_3 = \frac{243 \times \cancel{4}}{\cancel{4}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{9}}$$

$$a_3 = \frac{243}{1}\sqrt[3]{\frac{3}{9_3}}$$

$$a_3 = \frac{243}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$$

$$a_3 = \frac{243\sqrt[3]{9}}{3}$$

$$a_3 = 81\sqrt[3]{9}$$

135.— $S_3 = 9$

$S_6 = -63$

$S_{10} = ¿?$

$$9 = \frac{a_1(1-r^3)}{1-r}$$

$$9(1-r) = a_1(1-r^3)$$

$$9(1-r) = a_1(1-r^3)$$

$$9 = \frac{a_1(1-r^3)}{1-r}$$

$$-63 = \frac{a_1(1-r^6)}{1-r}$$

$$-63(1-r) = a_1(1-r^6)$$

$$-63(1-r) = a_1(1+r^3)(1-r^3)$$

$$-\frac{63}{1+r^3} = \frac{a_1(1-r^3)}{1-r}$$

$$\cancel{9}^{1} = -\frac{\cancel{63}^{7}}{1+r^3}$$

$$1+r^3 = -7$$

$$+r^3 = -7-1$$

$$r^3 = -8$$

$$\boxed{r = -2}$$

$$a_1 = \frac{9(1-r)}{1-r^3} \Rightarrow \frac{9(1-(-2))}{1-(-2)^3}$$

$$a_1 = \frac{27}{9}$$

$$\boxed{a_1 = 3}$$

$$S_{10} = \frac{a_1(1-r^{10})}{1-r}$$

$$S_{10} = \frac{3(1-(-2)^{10})}{1-(-2)}$$

$$S_{10} = \frac{\cancel{3}(1-1024)}{\cancel{3}}$$

$$S_{10} = -1023$$

136.- $n=10$ $\quad a_{10}=a_1 r^9$

$r=\frac{1}{2}$ $\quad a_1=\frac{a_{10}}{r^9} \Rightarrow \frac{10}{\left(\frac{1}{2}\right)^9}$

$a_{10}=10$

$a_1=¿?$ $\quad a_1=10*2^9$

$a_1=5120$

137.- $r=\frac{2}{5}$ $\quad a_{10}=a_1 r^{n-1}$

$a_{10}=150$ $\quad a_{10}=a_1 r^9$

$S_{10}=¿?$ $\quad a_1=\frac{a_{10}}{r^9} \Rightarrow \frac{150}{\left(\frac{2}{5}\right)^9} \Rightarrow \frac{150*5^9}{2^9}$

$$S_{10}=a_1\frac{(1-r^{10})}{1-r} \Rightarrow \frac{150*5^9}{2^9} * \frac{\left(1-\left(\frac{2}{5}\right)^{10}\right)}{1-\frac{2}{5}}$$

$$S_{10}=\frac{150*5^9}{2^9} * \frac{1-\frac{2^{10}}{5^{10}}}{\frac{5-2}{5}} \Rightarrow \frac{150*5^9}{2^9} * \frac{\frac{5^{10}-2^{10}}{5^{\cancel{10}\,9}}}{\frac{3}{\cancel{5}}}$$

$$S_{10}=\frac{150*5^9}{2^9} * \frac{\frac{5^{10}-2^{10}}{5^9}}{\frac{3}{1}} \Rightarrow \frac{\overset{\overset{25}{\cancel{50}}}{\cancel{150}}*\overset{1}{\cancel{5^9}}}{2^{\cancel{9}\,8}} \times \frac{5^{10}-2^{10}}{\underset{1}{\cancel{3}*\cancel{5^9}}}$$

$$S_{10}=\frac{25}{256}\left(5^{10}-2^{10}\right)$$

138.-

a) $0{,}675675675\ldots$

$$\Rightarrow \frac{675}{1000}+\frac{675}{1000000}+\frac{675}{1000000000}+\ldots$$

$$r=\frac{\frac{\cancel{675}}{\cancel{1000}000}}{\frac{\cancel{675}}{\cancel{1000}}}=\frac{1}{1000} \qquad N=\frac{a_1}{1-r}$$

$$N = \frac{\frac{675}{1000}}{1-\frac{1}{1000}} \Rightarrow \frac{\frac{675}{\cancel{1000}}}{\frac{1000-1}{\cancel{1000}}} \Rightarrow \frac{\cancel{675}^{75}}{\cancel{999}_{111}}$$

$$N = \frac{75}{111}$$

b) 3,4738247382...

$$\Rightarrow \underbrace{3 + \frac{47382}{100000} + \frac{47382}{10000000000}}+ \dots$$

$$r = \frac{1}{100000} \qquad N = \frac{\frac{47382}{100.000}}{1-\frac{1}{100.000}} \Rightarrow \frac{\frac{47382}{\cancel{100.000}}}{\frac{100.000-1}{\cancel{100.000}}}$$

$$N = \frac{\cancel{47382}^{15794}}{\cancel{99999}_{33.333}} \Rightarrow \frac{15794}{33.333} + 3$$

$$N = \frac{15794 + 99999}{33.333} = \frac{115793}{33.333}$$

c) 0,3754337543...

$$\Rightarrow \underbrace{\frac{37543}{100000} + \frac{37543}{10000000000}} + \dots$$

$$r = \frac{1}{100.000} \qquad N = \frac{\frac{37543}{100.000}}{1-\frac{1}{100000}} \Rightarrow \frac{\frac{37543}{\cancel{100.000}}}{\frac{100000-1}{\cancel{100.000}}}$$

$$N = \frac{37543}{99.999}$$

d) 12,213333213333...

$$\Rightarrow 12 + \frac{213333}{1000000} + \frac{213333}{10^{12}} + \dots$$

$$r = \frac{1}{1000000}$$

$$N = 12 + \frac{\frac{213333}{1000000}}{1 - \frac{1}{1000000}} \Rightarrow$$

$$N = 12 + \frac{\frac{\cancel{213333}}{\cancel{1000000}}}{\frac{\cancel{999999}}{\cancel{1000000}}} \quad \frac{71111}{333333}$$

$$N = 12 + \frac{71111}{333333} \Rightarrow \frac{3999996 + 71111}{333.333}$$

$$N = \frac{4071107}{333333}$$

139.- $a_1 + a_1 r = 15$ $\qquad 27 = \frac{a_1}{1-r}$

$27\left[a_1 + a_1\frac{(27-a_1)}{27}\right] = (15) * 27 \qquad 1 - r = \frac{a_1}{27}$

$27a_1 + a_1(27 - a_1) = 405 \qquad -r = \frac{a_1}{27} - 1 \Rightarrow \frac{a_1 - 27}{27}$

$27a_1 + 27a_1 - a_1^2 - 405 = 0$

$-a_1^2 + 54a_1 - 405 = 0 \qquad \boxed{r = \frac{27 - a_1}{27}}$

$a_1^2 - 54a_1 + 405 = 0$

$(a_1 - 45)(a_1 - 9) = 0 \qquad r = \frac{27-9}{27}$

$\cancel{a_1 = 45}$ $\qquad r = \frac{\cancel{18}\,2}{\cancel{27}\,3}$

$\boxed{a_1 = 9}$

$\boxed{r = \frac{2}{3}}$

140.- $a_1 = 100$ $\qquad S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$d = 20$ $\qquad 5800 = \frac{n}{2}(2(100) + (n-1)20)$

$n = ¿?$ $\qquad 11600 = n(200 + 20n - 20)$

$S_n = 5800$ $\qquad 1160\cancel{0} = 18\cancel{0}n + 2\cancel{0}n$

$2n + 18n - 1160 = 0$

$$2(n^2+9n-580)=0$$

$$(n+29)(n-20)=0$$

$$n=20$$

$$a_n=100+19(20)$$

$$a_n=480$$

141.— $[200+0{,}02(5000)]+200+0{,}02(4800)]+[200+0{,}02(4600)]+\ldots$

$$300+296+292+288+\ldots$$

a) $\frac{500\cancel{0}}{20\cancel{0}}=25$ pago

b) \$ 200

c) $S_{25}=\frac{n}{2}(2a_1+(n-1)d)$

$$\Rightarrow \frac{25}{2}(2(300)+24(-4))$$

$$S_{25}\Rightarrow \$\,6300$$

d) $I=6300-5000$

$$I=\$\,1300$$

142.— $a_1=1700$

$a_n=\$\,200$

$d=-150$

$n=¿?$

DEPRECIACION LINEAL

$$a_n=a_1+(n-1)d$$

$$200=1700+(n-1)(-150)$$

$$-150\cancel{0}=-15\cancel{0}n+15\cancel{0}$$

$$15n=150+15$$

$$n=11$$

c) $a_{20} = a_1 + 19d$

$a_{20} = 125 + 19(4)$

$a_{20} = \$\ 201$

143.- $S_n = 5490$

$d = 4$

$a_1 = 125$

a) $n = ¿?$

b) $a_n = ¿?$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$5490 = \frac{n}{2}(2(125) + (n-1)4)$

$10980 = n(250 + 4n - 4)$

$10980 = 246n + 4n^2$

$4n^2 + 246n - 10980 = 0$

$2n^2 + 123n - 5490 = 0$

$n = \frac{-123 \pm \sqrt{123^2 - 4(2)(-5490)}}{2(2)}$

$n = \frac{-123 \pm 243}{4} = \$\ 30$

$a_n = a_1 + 29(4)$

$a_n = 125 + 116$

$a_n = \$\ 241$

144.- Deuda = $\$\ 1800$

$\Rightarrow [150 + 0{,}01(1800)] + [150 + 0{,}01(1650)] + [150 + 0{,}01(1500)] + \ldots$

$168 + 166{,}5 + 165 + \ldots$

145.-

$a_1 = 1500$

$n = 9$

$a_9 = 420$

$d = ¿?$

$a_n = a_1 + (n-1)d$

$420 = 1500 + 8d$

$8d = 420 - 1500$

$d = \frac{-1080}{8}$

$d = -135$ depreciación: \$135

146.- $a_1 = \$2000$

$d = -160$

$n = ¿?$

$a_n = \$400$

Depreciación lineal

$a_n = a_1 + (n-1)d$

$400 = 2000 + (n-1)(-160)$

$400 - 2000 = -160n + 160$

$16\cancel{0}n = 160\cancel{0} + 16\cancel{0}$

$n = \frac{176}{16}$

$n = 11$

147.-

$a_8 = \$153$

$a_{15} = \$181$

a) $d = ¿?$

b) $a_1 = ¿?$

c) $a_{20} ¿?$

$a_n = a_1 + (n-1)d$

$(153 = a_1 + 7d)(-2)$ $181 = a_1 + 14d$

$-306 = -2a_1 - 14d$

$181 = a_1 + 14d$

$-125 = -a_1$

$a_1 = 125$

$153 = a_1 + 7d$

$7d = 153 - a_1$

$d = \frac{153 - 125}{7}$

$d = 4$

a) $a_1 = \$ 168$

b) $d = -1{,}5$

$a_n = ¿?$

$a_n = a_1 + (n-1)d$

$a_n = 168 + (11)(-1{,}5)$

$a_n = \$ 151{,}5$

$n = \frac{1800}{150} = 12$ pagos

c) $S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{12}{2}(168 + 151{,}5)$

$S_n = \$ 1917$

d) Intereses: $1917 - 1800$

$\$ 117$

148 – 9, 12, 15

$a_1 = 9$

$d = 3$

$S_n = 306$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$306 = \frac{n}{2}(2(9) + (n-1)3)$

$612 = n(18 + 3n - 3)$

$612 = 15n + 3n^2$

$3n^2 + 15n - 612 = 0$

$3(n^2 + 5n - 204) = 0$

$n^2 + 5n - 204 = 0$

$(n+17)(n-12) = 0$

$\cancel{n = -17}$

$n = 12$

149.- , -12, -7, -2, 3, 8, ...

$a_1 = -12$

$d = 5$

$n = ¿?$

$S_n = 105$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$105 = \frac{n}{2}(2(-12) + (n-1)5)$

$210 = n(-24 + 5n - 5)$

$210 = -29n + 5n^2$

$5n^2 - 29n - 210 = 0$

$(5n - 50)(5n + 21) = 0$

$5n - 50 = 0$ $\quad$ $\cancel{5n = -21}$

$5n = 50$

$n = 10$

150.- $n = 50$

$d = 3$

$a_1 = 8$

$S_{50} = ¿?$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$S_{50} = \frac{50}{2}(2(8) + 49(3))$

$S_{50} = 25(16 + 147)$

$S_{50} = 25(163)$

$\boxed{S_{50} = 4075}$

d) Correcto

151.- $a_1 = 105$

$a_n = 994$

$d = 7$

$a_n = a_1 + (n-1)d$

$994 = 105 + (n-1)7$

$994 - 105 = 7n - 7$

$7n = 889 + 7$

$n = 128$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_n = \frac{128}{2}(105 + 994) \Rightarrow \boxed{7.0336}$

e) Correcto

Solución
Ejercicios Propuestos

Capitulo TRES

MATEMATICAS

FUNCIONES DE UNA VARIABLE REAL

1.- FALSO; de ser así, deja de ser FUNCION

2.- $f(x) = \frac{2x+1}{x-3}$

Punto crítico

$x-3=0$

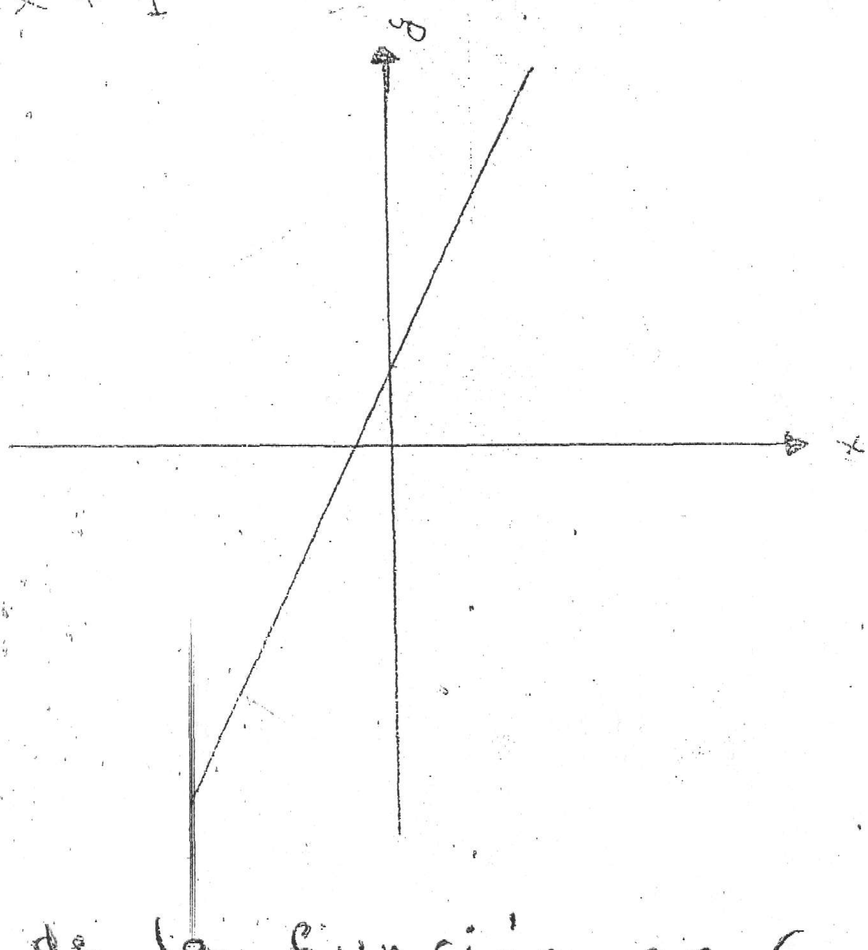
$\boxed{x=3}$

Máximo dominio: $(-\infty, 3) \cup (3, +\infty)$

a) CORRECTO

3) $f(x) = 2x + 1$

$(2, \infty)$



El rango de la función es $(-\infty, \infty)$

a) Verdadero

X Falso

4.-

a) $f(x) = \sqrt{\sqrt{x} - 1}$ dom: $[1, +\infty)$ VERDADERO

$\sqrt{x} - 1 \geq 0$

$(\sqrt{x})^2 \geq 1^2$

$\boxed{x \geq 1} \Rightarrow$ dom: $[1, +\infty)$

b) $f(x) = \dfrac{x^8 - x^3 + x - \sqrt{2}}{\sqrt{3} - 1}$ dom: $\mathbb{R}$ VERDADERO

NO HAY RESTRICCIONES
POR LO TANTO dom $f = \mathbb{R}$

c) $f(x) = \dfrac{1}{x-1}$; dom $f = \mathbb{R} - \{1\}$ VERDADERO

Punto crítico

$x - 1 = 0$

$\boxed{x = 1} \Rightarrow$ dom $f = \mathbb{R} - \{1\}$

d) $f(x) = \dfrac{\sqrt{x-1}}{x^2 - 4}$ FALSO

Punto crítico

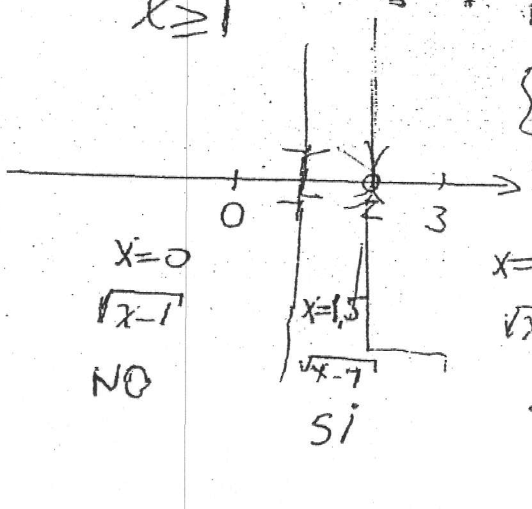
$x - 1 \geq 0$

$x \geq 1$

$x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{4}$

$\boxed{x = \pm 2}$



dom f: $[1, 2) \cup (2, +\infty)$

5. a) $g(x) = \dfrac{x}{x-1}$

Ponto crítico

$x - 1 = 0$

$\boxed{x = 1}$

$\text{dom } g = \mathbb{R} - \{1\}$

b) $h(x) = \dfrac{2x}{x+3}$

Ponto crítico

$x + 3 = 0$

$\boxed{x = -3}$

$\text{dom } h = \mathbb{R} - \{-3\}$

c) $f(x) = \sqrt{1 - x^2}$

$1 - x^2 \geq 0$

$-x^2 \geq -1$

$x^2 \leq 1$

$\boxed{x \leq \pm 1}$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$x = -2$	$x = 0$	$x = 2$
$\sqrt{1-x^2}$	$\sqrt{1-x^2}$	$\sqrt{1-x^2}$
NO	Si	NO

$\text{dom } f : [-1, 1]$

d) $r(x) = \sqrt{x^2 - 1}$

$x^2 - 1 \geq 0$

$x^2 \geq 1$

$\boxed{x \geq \pm 1}$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$x = -2$	$x = 0$	$x = 2$
$\sqrt{x^2-1}$	$\sqrt{x^2-1}$	$\sqrt{x^2-1}$
Si	No	Si

$\text{dom } r : (-\infty, -1] \cup [1, +\infty)$

e) $g(x) = \dfrac{2}{\sqrt{|x-2|-1}}$

$|x-2|-1 > 0$

$|x-2| > 1$

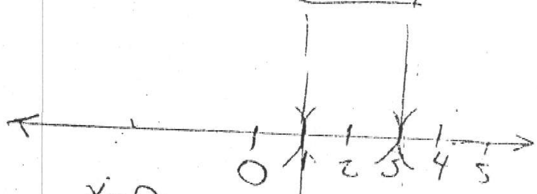
$x-2>1 \quad \vee \quad -x+2>1$

$\boxed{x>3}$

$-x > 1-2$

$-x > -1$

$\boxed{x<1}$



$x=0$ $\quad$ $x=2$ $\quad$ $x=4$

$\sqrt{|x-2|-1}$ $\quad$ $\sqrt{|x-2|-1}$ $\quad$ $\sqrt{|x-2|-1}$

si $\quad$ no $\quad$ si

dom : $(-\infty, 1) \cup (3, +\infty)$

f) $f(x) = \dfrac{x^2-1}{x^2+1}$

Punto crítico

$x^2+1=0$

$x^2 \neq -1$ NO HAY PUNTO

$x \neq \pm 1$ CRITICO

dom f : $x \in \mathbb{R}$

g) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x-2}$

$\Rightarrow \dfrac{x-2+x-1}{(x-1)(x-2)} \Rightarrow \dfrac{2x-3}{(x-1)(x-2)}$

Puntos críticos

$x-1=0 \quad x-2=0$

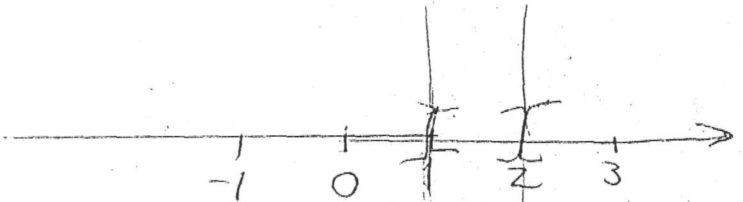
$\boxed{x=1} \quad \boxed{x=2}$

dom f : $\mathbb{R} - \{1, 2\}$

h) $h(x) = \sqrt{x-1} + \sqrt{x-2}$

$x-1 \geq 0 \qquad x-2 \geq 0$

$\boxed{x \geq 1} \qquad \boxed{x \geq 2}$



$x = 0$ | $x = 1{,}5$ | $x = 3$

$\sqrt{x-1}$ $\sqrt{x-2}$ | $\sqrt{x-2}$ | $\sqrt{x-1}$ $\sqrt{x-2}$

NO NO | NO | si si

dom h: $[2, +\infty)$

6.— $f(x) = x^2 - x$

Vértice

$x = -\dfrac{b}{2a} = \dfrac{-(-1)}{2(1)} = \dfrac{1}{2}$

$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)$

$y = -1/4$

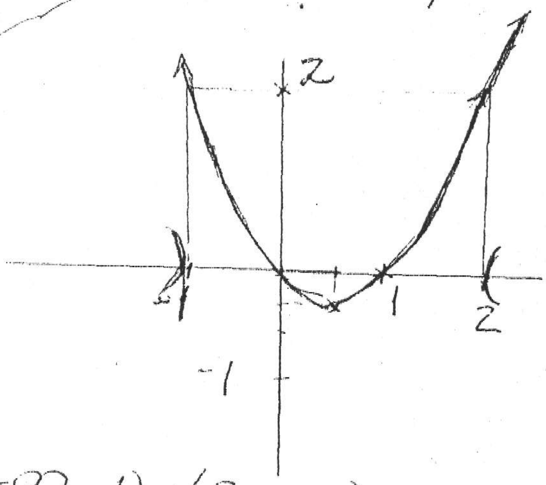
$V\left(\frac{1}{2}, -\frac{1}{4}\right)$

raíces

$x^2 - x = 0$

$x(x-1) = 0$

$\boxed{x=0}$ $\boxed{x=1}$



$x^2 - x > 2$

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$\boxed{x>2}$ $\boxed{x>-1}$

$(-\infty, -1) \cup (2, +\infty)$

7.— $f(x) = \dfrac{\sqrt{4 - x^2}}{x^2 + 6x - 7}$

$4 - x^2 \geq 0$

$-x^2 \geq -4$

$x^2 \leq 4$

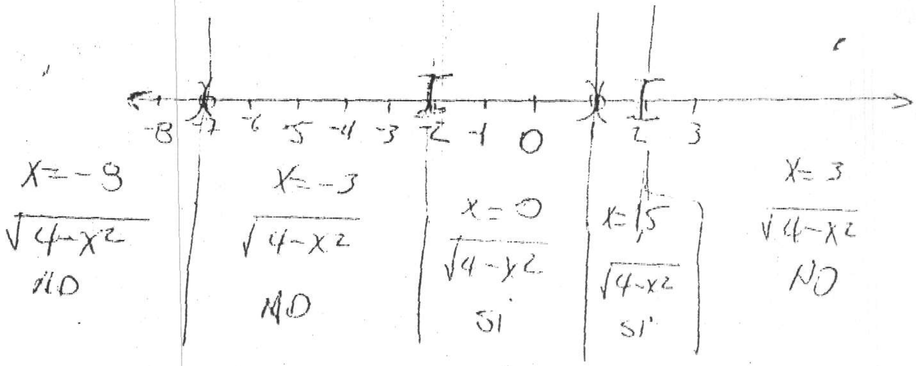
$\{x \leq \pm 2\}$

Pontos críticos

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$\{x = -7\}$ $\{x = 1\}$



dom: $[-2,1)\cup(1,2]$. c) correcto

8.— $h(x)=\sqrt{x-4+|3x-5|}$

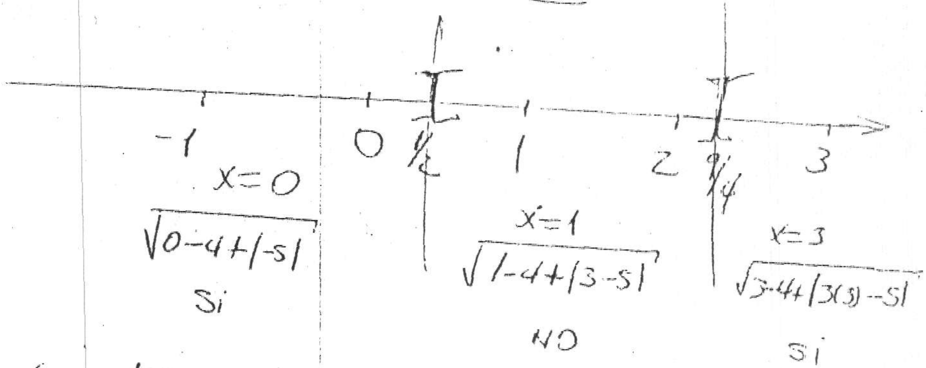
$x-4+|3x-5|\geq 0$

$x-4+3x-5\geq 0 \quad \vee \quad x-4-3x+5\geq 0$

$4x\geq 9 \qquad\qquad -2x\geq -1$

$\boxed{x\geq 9/4} \qquad\qquad 2x\leq 1$

$\boxed{x\leq 1/2}$

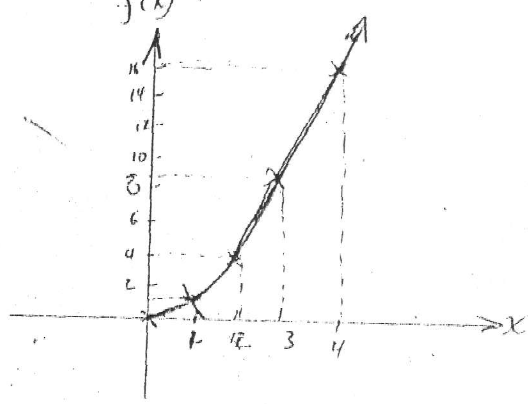


dom: $(-\infty,1/2]\cup[9/4,+\infty)$

dom $(1/4, 9/4)^c$ c) correcto

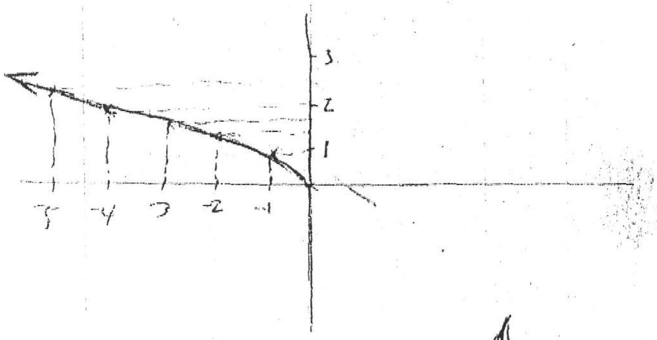
9.— a) $f(x)=x^2$; $x\geq 0$

x	$f(x)$
0	0
1	1
2	4
3	9
4	16
⋮	⋮



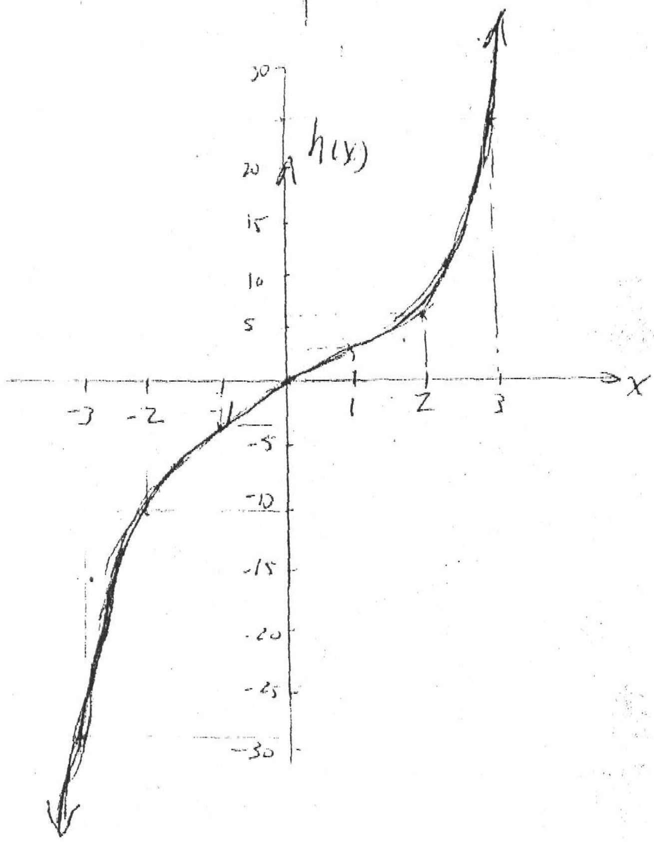
b) $g(x) = \sqrt{-x}$; $x \leq 0$

x	$g(x)$
0	0
-1	1
-2	$\sqrt{2}$
-3	$\sqrt{3}$
-4	2
-5	$\sqrt{5}$



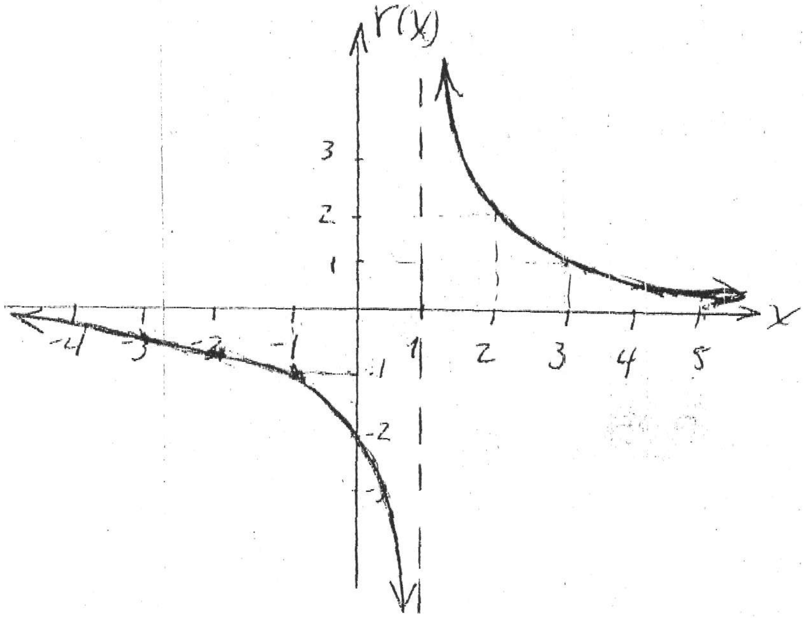
c) $h(x) = x^3 - 2$; $x \in \mathbb{R}$

x	$h(x)$
-3	-29
-2	-10
-1	-3
0	0
1	-1
2	6
3	25



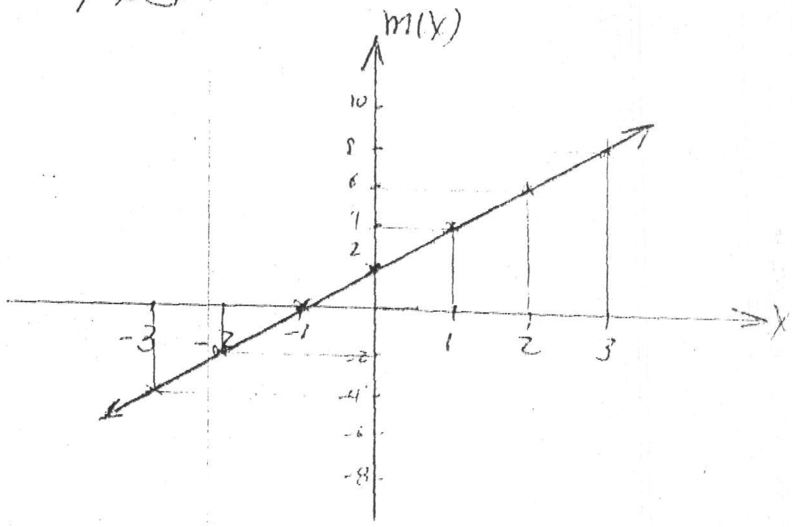
d) $r(x) = \dfrac{2}{x-1}$; $x \in \mathbb{R}, x \neq 1$

x	$r(x)$
-3	$-1/2$
-2	$-2/3$
-1	-1
0	-2
2	2
3	1
4	$2/3$
5	$1/2$



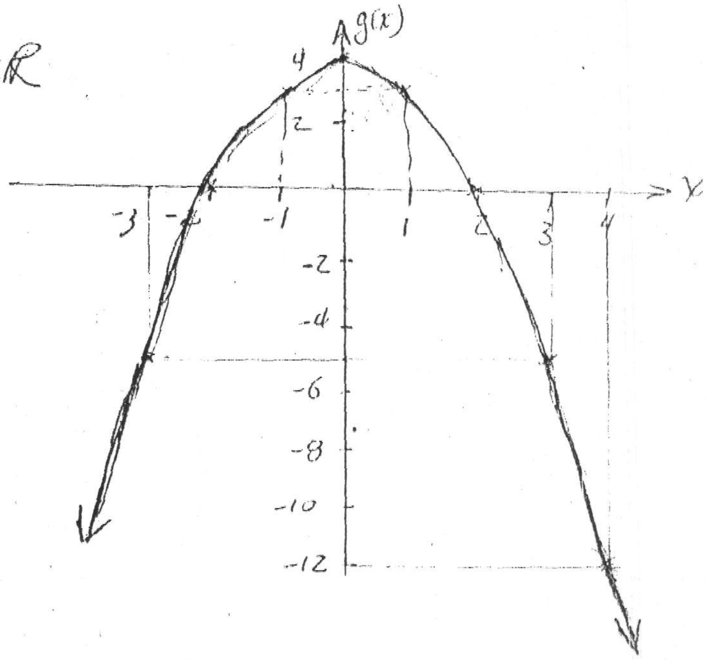
e) $m(x) = 2x+2 \quad ; x \in \mathbb{R}$

x	$m(x)$
-3	-4
-2	-2
-1	0
0	2
1	4
2	6
3	8



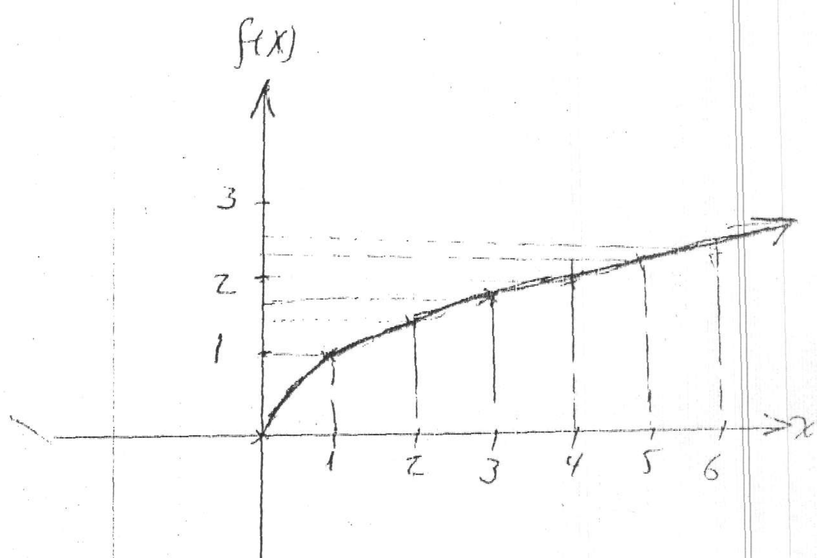
f) $g(x) = 4 - x^2 \quad ; x \in \mathbb{R}$

x	$g(x)$
-3	-5
-2	0
-1	3
0	4
1	3
2	0
3	-5
4	-12



g) $f(x) = \sqrt{x} \quad ; x \geq 0$

x	$f(x)$
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$
4	2
5	$\sqrt{5}$
6	$\sqrt{6}$



10.- Si al trazar una recta vertical, intercepta más de 1 punto, la relación NO ES FUNCION.

a) Si es función

b) Si es función

c) Si es función

d) Si es función

e) Si es función

f) NO es función

g) NO es función.

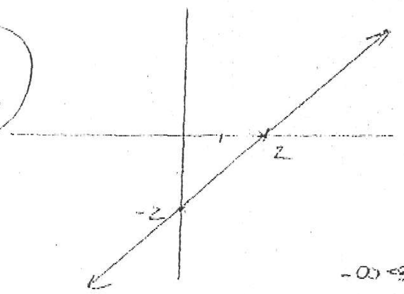
11.- b) Falso, si eso ocurre deja de ser función.

12.- $f(x) = |X-2| + 1$

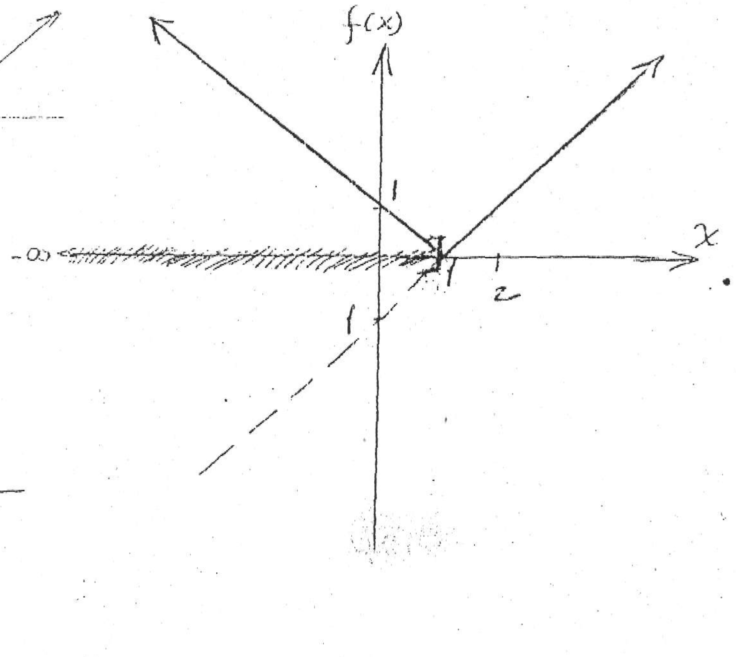
$y = x - 2$

$x = 0$
$y = -2$

$y = 0$
$x = 2$

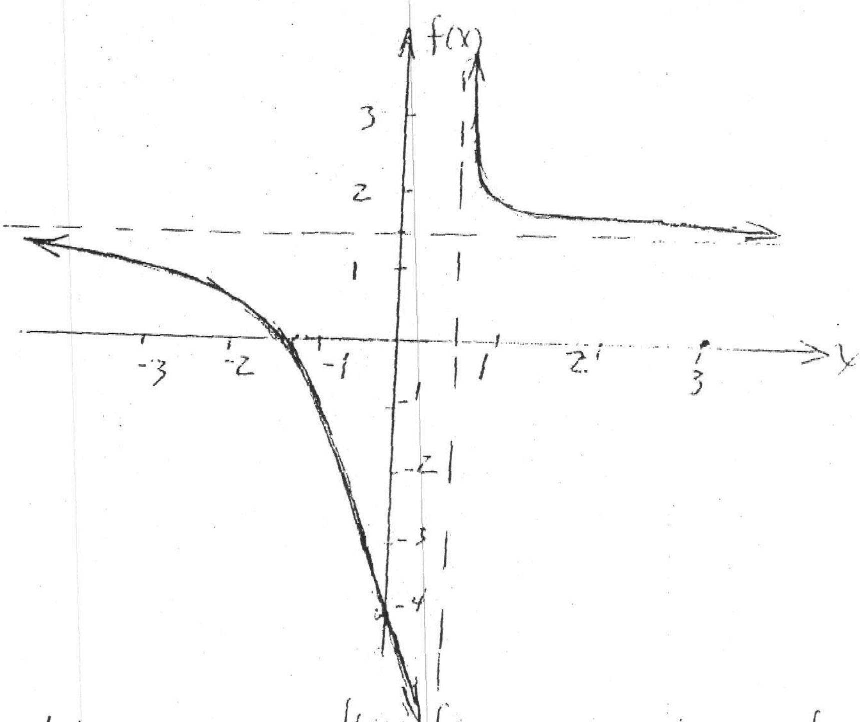


$f(x) = |X-2| + 1$



a) correcto

13.- $f(x) = \dfrac{3x+4}{2x-1}$



asintota vertical

$2x-1=0$

$2x=1$

$\boxed{x=1/2}$

asintota horiz

$\dfrac{3\cancel{x}}{2\cancel{x}}$

$\boxed{y=3/2}$

raices

$3x+4=0$

$3x=-4$

$\boxed{x=-4/3}$

Interseccion en y

$x=0$

$\boxed{f(0)=-4}$

14.- $g(x) = \dfrac{f(x)+f(-x)}{2}$; $h(x) = \dfrac{f(x)-f(-x)}{2}$

a) $h(-x) = \dfrac{f(-x)-f(x)}{2} \neq g(x)$; FALSO

b) Si es impar $h(x) = -h(-x)$

$\Rightarrow -\dfrac{f(-x)-f(x)}{2}$

$\Rightarrow \dfrac{f(x)-f(-x)}{2}$; CORRECTO

c) $f(a) = g(-a) - h(-a)$

$\Rightarrow \dfrac{f(-a)+f(a)}{2} - \dfrac{f(-a)-f(a)}{2}$

$\Rightarrow \dfrac{\cancel{f(-a)}+f(a)-\cancel{f(-a)}+f(a)}{2} \Rightarrow \dfrac{\cancel{2}f(a)}{\cancel{2}}$; CORRECTO

$\Rightarrow f(a)$.

d) Si $g(x)$ es par, entonces $g(-x) = g(x)$

$$g(-x) = \frac{f(-x)+f(x)}{2}$$

$$\Rightarrow \frac{f(x)+f(-x)}{2}$$; CORRECTO

e) $-g$ es par $\Rightarrow -g(x) = -g(-x)$

$g(x) = g(-x)$; CORRECTO

15.— a) $f(x) = \operatorname{sen} x + x^3$; $\operatorname{seno}(x)$ no es par.

¿impar?

$f(x) = -f(-x)$

$\Rightarrow -(\operatorname{sen}(-x) + (-x)^3) \Rightarrow -(-\operatorname{sen} x - x^3)$

$\Rightarrow \operatorname{sen} x + x^3$; $f(x)$ es impar.

b) $g(x) = |x| + 1$

¿par?

$|x| + 1 = |-x| + 1$

$\Rightarrow |x| + 1$; $g(x)$ es par

c) $h(x) = |-x| - x^2$

$\Rightarrow |x| - x^2$

¿par?

$|x| - x^2 = |-x| - (-x)^2$

$\Rightarrow |x| - x^2$; $h(x)$ es par.

d) $f(x) = |2-x| - |x+2|$.

¿par?

$|+2-x| - |x+2| = |-2+x| - |-x-2|$

$= |x-2| - |x+2|$; $f(x)$ es par

e) $f(1-x) = x+2$

NO ES PAR; porque es lineal

¿impar?

$f(1-x) = -f(x-1)$

$= -(x+2)$

$\Rightarrow x-2$; NO ES IMPAR

f) $h(x) = x^2 - |x|$

¿par?

$x^2 - |x| = (-x)^2 - |-x|$

$\Rightarrow x^2 - |x|$; $h(x)$ es par

16.— $g(x) = x^2 - x + 1$

Vértice

$x = \frac{+1}{2(1)} = \boxed{1/2}$

$y = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 1$

$y = \frac{1}{4} - \frac{1}{2} + 1$

$y = 3/4$

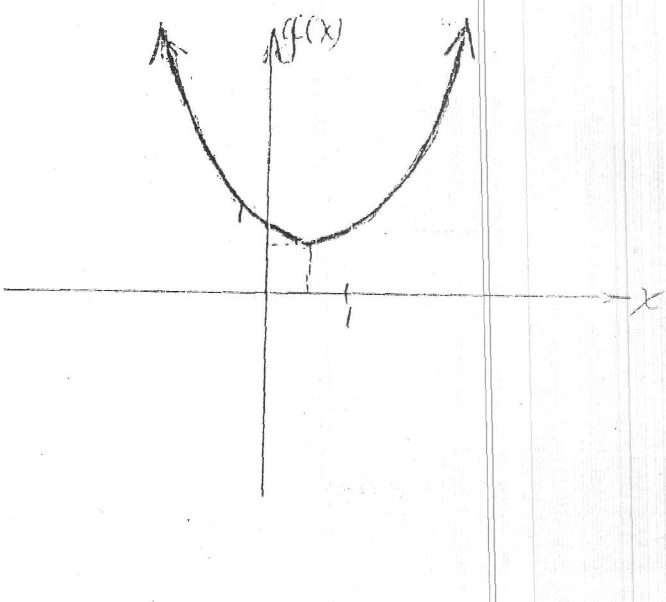
$V\left(\frac{1}{2}, 3/4\right)$

Raíces

$x^2 - x + 1 = 0$

$x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$

NO HAY raíces



17.- $K \Rightarrow$ pendiente o tangente del ángulo

$K > 0$; $\tan\theta$ (entre 0 y 90°) ; debe ser creciente

$K < 0$; $\tan\theta$ (entre 90 y 180) ; debe ser decreciente

18.- f es función impar y estrictamente decreciente $f(x) = g(x)$.

$$\frac{2g(4) + 3f(4)}{-f(-4) + 4g(-4)} = \frac{5f(4)}{3f(-4)} = -\frac{5}{3}$$

$f(x) = -f(-x)$ $\qquad f(4)/f(-4) = -1$

a) 5/3 $\qquad$ b) -5/2 $\qquad$ ~~c)~~ -5/3 $\qquad$ d) 1 $\qquad$ e) -1

19.- $f(x) = \dfrac{3x^3+2x^2+5}{x^2-4}$

Asíntotas Verticales

$x^2-4=0$

$(x+2)(x-2)=0$

$x=2$ / $x=-2$

a) Correcto

20.-

$g(x) = \dfrac{12x-3}{9x^2-4}$

Asíntotas Verticales

$9x^2-4=0$

$(3x-2)(3x+2)=0$

$3x-2=0 \qquad 3x+2=0$

$3x=2 \qquad 3x=-2$

$x=2/3 \qquad x=-2/3$

Asíntotas Horizontales

$$\frac{12x}{9x^2} \Rightarrow \frac{4}{3x}$$

$\Rightarrow y=0$

b) FALSO

21.- $f(x) = \dfrac{4x^2 - x}{x^2 - 1}$

Asíntotas V.

$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$\boxed{x = -1}$ $\boxed{x = 1}$

Asíntotas H.

$\dfrac{4x^2}{x^2} \Rightarrow y = 4$

raices

$4x^2 - x = 0$

$x(4x-1) = 0$

$x = 0$ $4x - 1 = 0$

$4x = 1$

$x = 1/4$

a) correcto

b) falso

c) correcto

d) correcto

e) correcto

22.- $h(x) = \dfrac{2x}{x^2 + x - 2}$

Asíntotas V.

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$\boxed{x = -2}$ $\boxed{x = 1}$

Asíntotas H.

$\dfrac{2x}{x^2} \Rightarrow \dfrac{2}{x}$

$y = 0$

raices

$2x = 0$

$\boxed{x = 0}$

a) correcto

23.- $g(x) = \dfrac{1}{x^2 + 1}$

Asíntotas V.

$x^2 + 1 = 0$

$\boxed{x^2 = -1}$

NO TIENE ASINTOTAS

Asíntotas H.

$\dfrac{1}{x^2}$

$\Rightarrow \boxed{y = 0}$

raices

NO HAY

C) FALSO

24.- a) $f(x) = \dfrac{x^2-1}{x^2+7x-8}$

Asíntotas V

$x^2+7x-8=0$

$(x+8)(x-1)=0$

$\boxed{x=-8}$ $\boxed{x=1}$

Asíntotas H

$\dfrac{x^2}{x^2} \Rightarrow y=1$

raices

$x^2-1=0$

$x^2=1$

$\boxed{x=\pm 1}$

Intersección con y

$x=0$

$\boxed{f(x)=\frac{1}{8}}$

b) $g(x) = \dfrac{2}{x^2+1}$

Asíntotas V

$x^2+1=0$

$\boxed{x^2=-1}$

NO EXISTEN ASINTOTAS

Asíntotas H

$\boxed{y=0}$

raices

NO EXISTEN

Intersección con y

$g(x)=2$

c) $h(x) = \dfrac{x^2-3x+2}{x^2+1}$

Asíntotas V

$x^2+1=0$

$\boxed{x^2=-1}$

NO EXISTEN

Asíntotas H

$\dfrac{x^2}{x^2} \Rightarrow \boxed{y=1}$

raices

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$\boxed{x=2}$ $\boxed{x=1}$

Intersección con y

$\boxed{h(x)=2}$

d) $r(x) = \dfrac{2x^2}{-x^2+9}$

Asíntotas V

$-x^2+9=0$

$x^2-9=0$

$(x-3)(x+3)=0$

$x=3$ $x=-3$

Asíntotas H

$\dfrac{2x^2}{-x^2}$

$y=-2$

raices

$2x^2=0$

$x=0$

Intersección con Y

$r(x)=0$

e) $f(x) = \dfrac{x}{\sqrt{1-x^2}}$

Asíntotas V

$\sqrt{1-x^2}=0$

$1-x^2=0$

$-x^2=-1$

$x^2=1$

$x=\pm 1$

Asíntotas H

No Tiene

raices

$x=0$

Intersección con Y

$r(x)=0$

f) $g(x) = \dfrac{x^3}{x^2-4}$

Asíntotas V

$x^2-4=0$

$(x-2)(x+2)=0$

$x=2$ $x=-2$

Asíntotas Oblicua

$y=x$

raices

$x=0$

25.- $\dfrac{a}{x^2+bx+c}$

asíntotas verticales

$x=-3 \wedge x=1$

$x+3=0 \quad x-1=0$

$(x+3)(x-1)$ $y(2)$

$\Rightarrow x^2-x+3x-3$

$\Rightarrow x^2+2x-3$ $\Rightarrow \dfrac{2}{x^2+2x-3}$

26.- I) $f(x)=\dfrac{1}{2(2x-1)(2x+1)}$

<u>Asíntotas V</u>

$2x-1=0 \quad 2x+1=0$

$2x=1 \quad 2x=-1$

$x=1/2 \quad x=-1/2$

<u>Asíntota H</u>

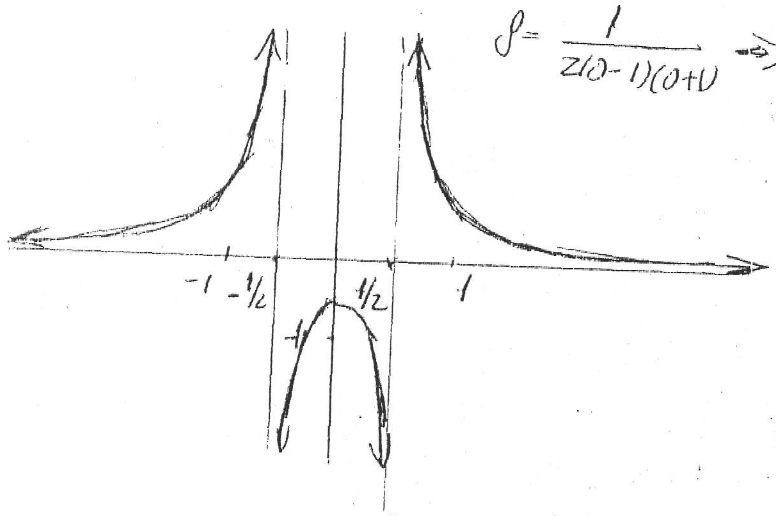
$y=0$

<u>raíces</u>

NO EXISTE

<u>Intersecciones con y</u>

$y=\dfrac{1}{2(0-1)(0+1)} \Rightarrow -\dfrac{1}{2}$



x	f(x)
-0,4	-
-0,3	-
-0,2	-
-0,1	-
0	-
0,1	-
0,2	-
0,3	-

II) $g(x) = \dfrac{7x^2}{4(2x-1)(2x+1)}$

<u>Asíntotas V.</u>

$2x-1=0 \quad 2x+1=0$

$2x=1 \quad 2x=-1$

$\boxed{x=1/2} \quad \boxed{x=-1/2}$

<u>Asíntotas H</u>

$y = \dfrac{7\cancel{x^2}}{16\cancel{x^2}} \approx 0{,}5$

$\boxed{y \approx 0{,}5}$

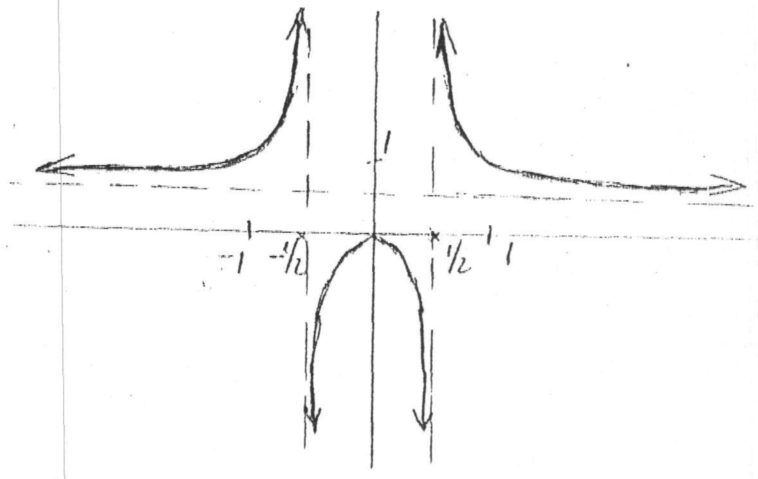
<u>raíces</u>

$7x^2=0$

$\boxed{x=0}$

<u>Intersecciones con y</u>

$\boxed{y=0}$



x	f(x)
-0,4	-
-0,2	-
-0,1	-
0	+
0,1	-
0,2	-

III) $h(x) = \dfrac{7x^3}{4(2x+1)(2x-1)}$

<u>Asíntotas V</u>

$2x+1=0 \quad 2x-1=0$

$2x=-1 \quad 2x=1$

$x=-1/2 \quad x=1/2$

<u>Asíntota oblicua</u>

$\dfrac{7x^3}{16x^2-4} = \dfrac{7}{16}x + \dfrac{7x}{4(16x^2-4)}$

$\boxed{y = \frac{7}{16}x}$

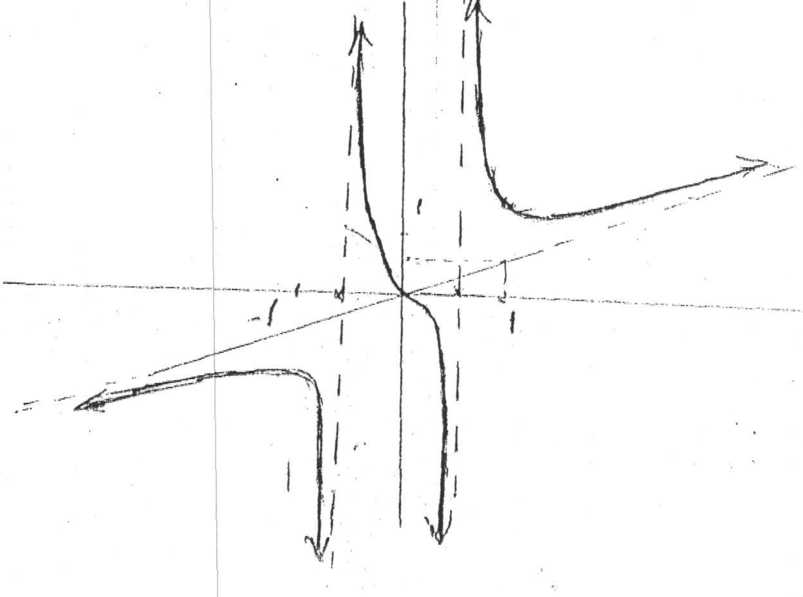
<u>raíces</u>

$7x^3=0$

$\boxed{x=0}$

<u>Intersecciones con eje y</u>

$y=0$



x	f(x)
-0,4	+
-0,2	+
-0,1	+
0	+
0,1	-
0,2	-
0,4	-

IV) $r(x) = \dfrac{7x^4}{4(2x+1)(2x-1)}$

Asíntota V.

$2x+1=0$ $\quad$ $2x-1=0$

$2x=-1$ $\quad$ $2x=1$

$x=-1/2$ $\quad$ $x=1/2$

Asíntota Oblicua

$y = \frac{7}{16}x^2$

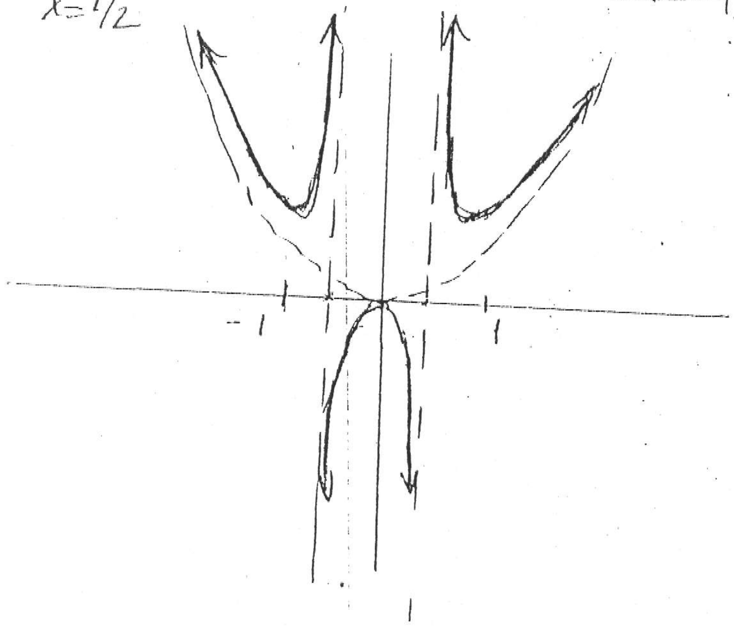
raíces

$7x^4=0$

$(x=0)$

Intersecciones con Y

$\boxed{y=0}$



V) $m(x) = \dfrac{7x}{(2x-1)(2x+1)}$

Asíntotas V.

$2x-1=0$ $\quad$ $2x+1=0$

$2x=1$ $\quad$ $2x=-1$

$(x=1/2)$ $\quad$ $(x=-1/2)$

Asíntota H

$y=0$

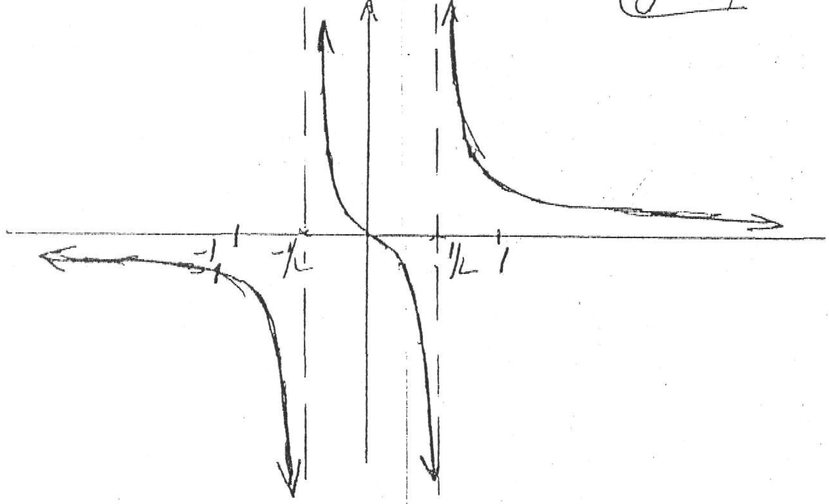
raíces

$7x=0$

$\boxed{x=0}$

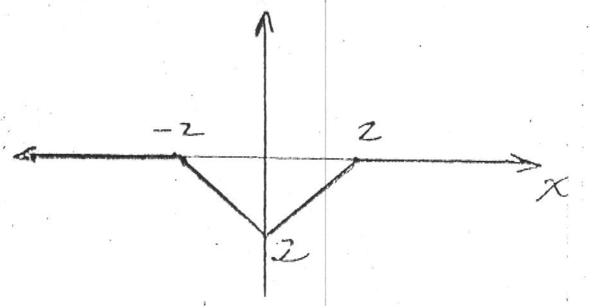
Intersección con eje Y

$(y=0)$

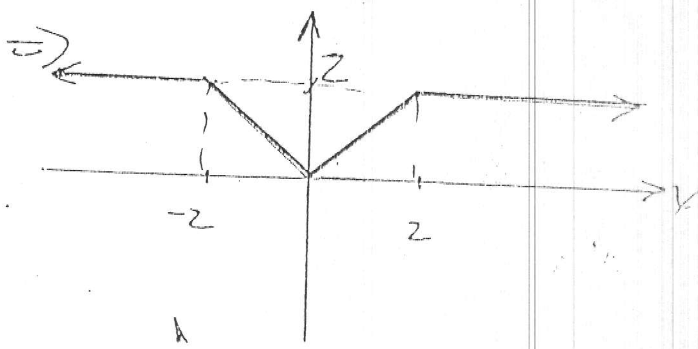


x	m(x)
-0,4	+
-0,2	+
-0,1	+
0	•
0,1	−
0,2	−
0,4	−

27.- $y = -f(x)$



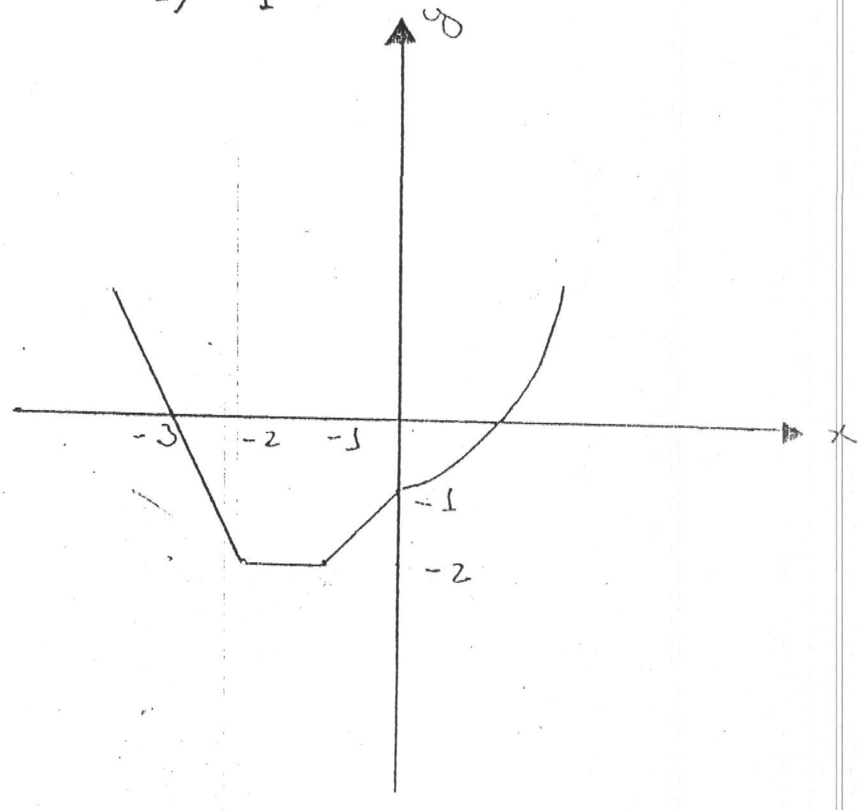
$y = -f(x) + 2$



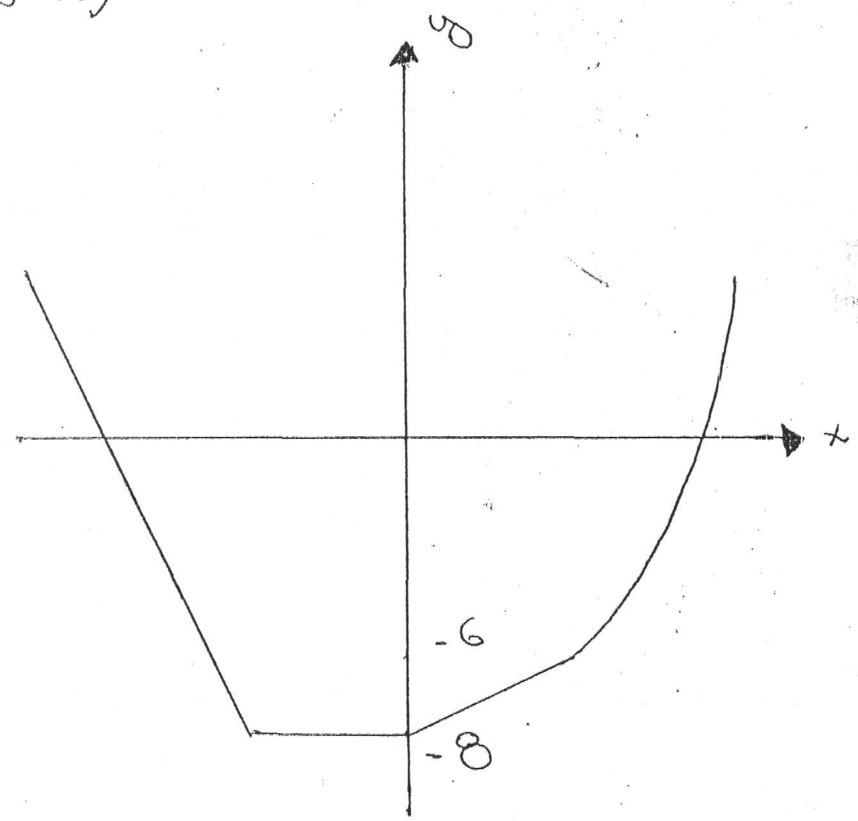
28.- $g(x) = f(|x|)$

a) correcto

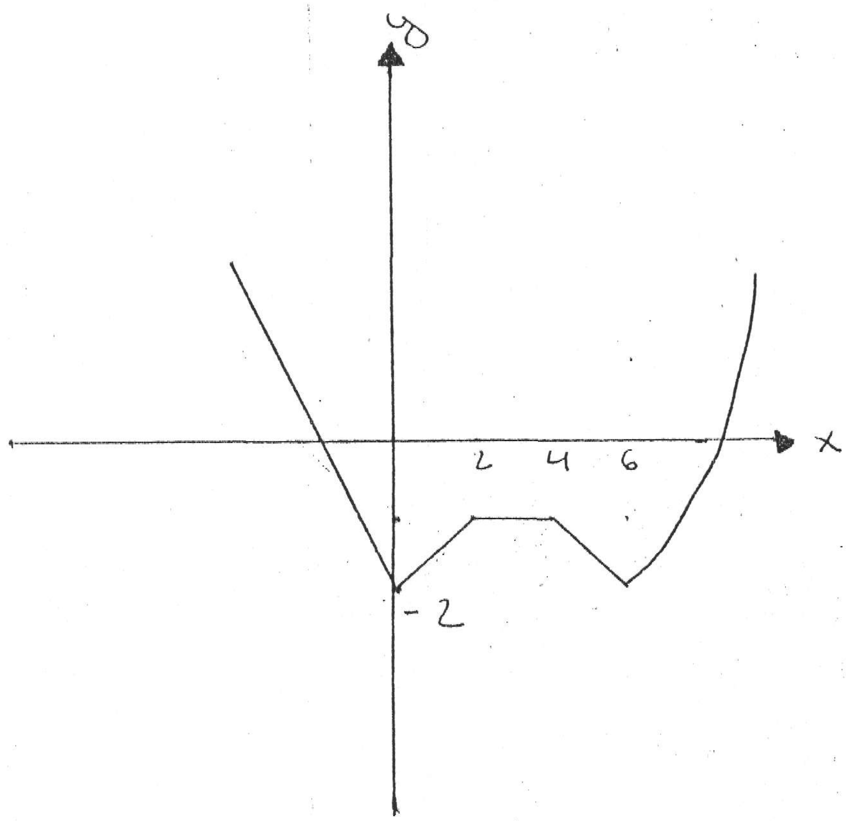
29.- a) $y = f(x+1) - 1$



) $y = -2f(3-x)$



) $y = |f(2x-4)| - 2$



30.-

$$h(x) = \begin{cases} 4+2x & ; -2 \leq x \leq 0 \\ 4-2x & ; \quad 0 < x \leq 2 \\ 0 & ; \quad |x| > 2 \end{cases}$$

$h(-3) = 0$

$h(0) = 4$

$h(5) = 0$

$h\left(-\frac{5\pi}{2}\right) = 0$

$h(1) = 2$

$h(-1) = 2$

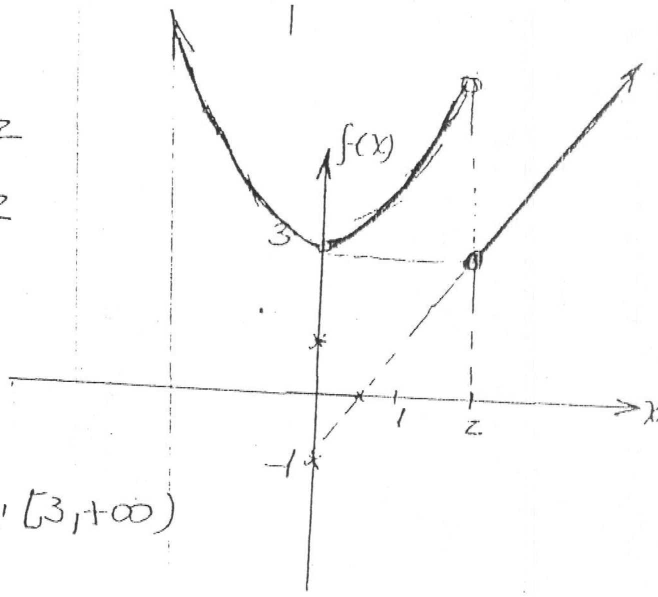
$h(\pi) = 0$

$h(-e) = 0$

$$\Rightarrow \frac{0+4-0+0}{2+2-0+0} \Rightarrow \frac{4}{4} \Rightarrow 1$$ a) correcto

31.-

$$f(x) = \begin{cases} 2x-1 & ; x \geq 2 \\ x^2+3 & ; x < 2 \end{cases}$$



$y = 2x-1$

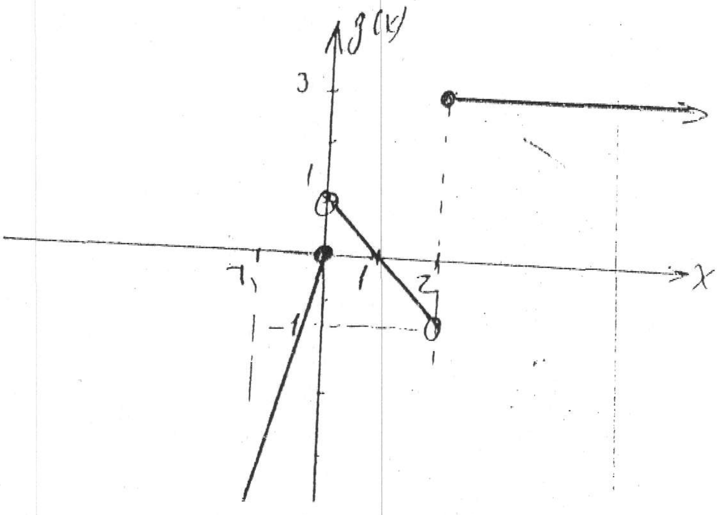
$x=0$, $y=-1$ $\quad$ $y=0$, $x=1/2$

rango: $[3, +\infty)$

$$g(x) = \begin{cases} 3 & ; x \geq 2 \\ 1-x & ; 0 < x < 2 \\ 4x & ; x \leq 0 \end{cases}$$

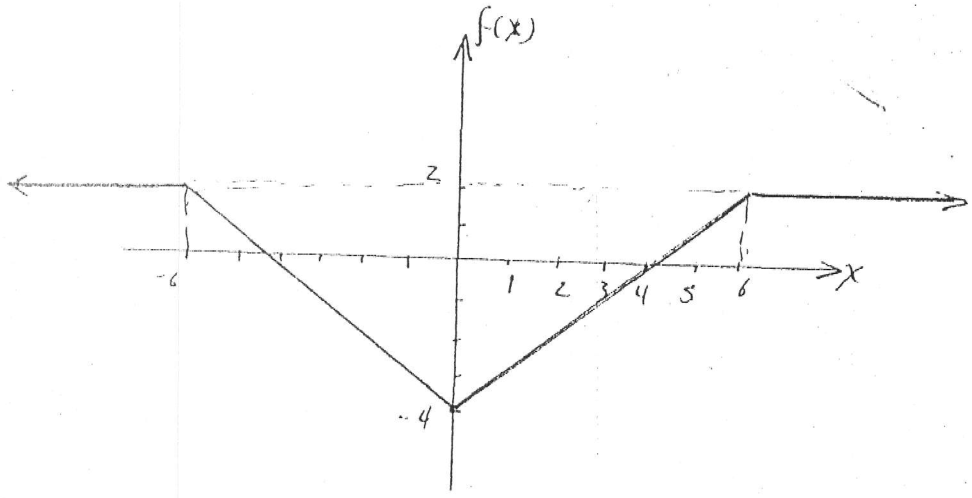
$y = 1-x$

$x=0$, $y=1$ $\quad$ $y=0$, $x=1$



rango: $(-\infty, 1) \cup \{3\}$

32.- $f(x) = \begin{cases} |x| - 4 & ; |x| \leq 6 \\ 2 & ; |x| > 6 \end{cases}$



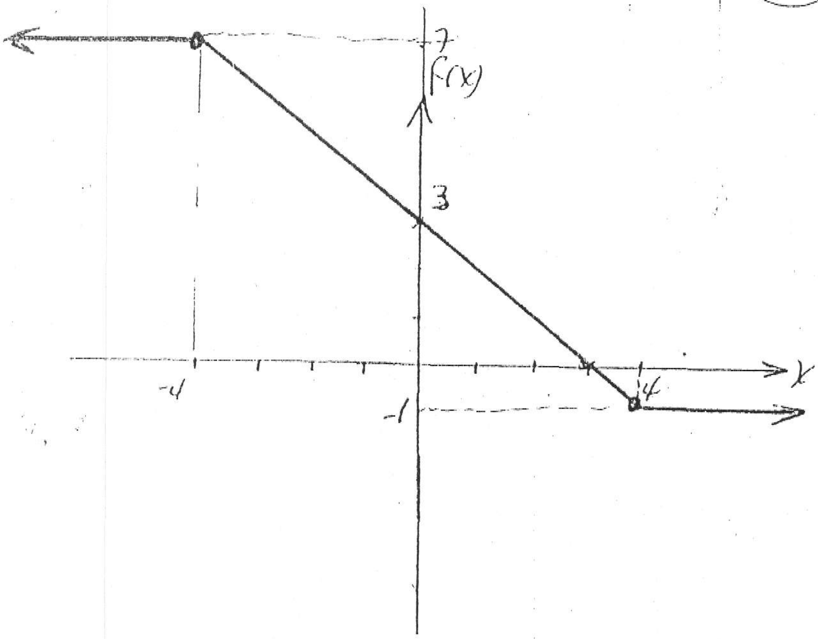
a) correcto
b) correcto
c) Falso
d) correcto
e) correcto

33.- $f(x) = \begin{cases} 7 & ; x < -4 \\ 3 - x & ; -4 \leq x \leq 4 \\ -1 & ; x > 4 \end{cases}$

$y = 3 - x$

$x = 0$
$y = 3$

$y = 0$
$x = 3$



a) FALSO
b) FALSO
c) FALSO
d) FALSO
e) CORRECTO

34.-

1^{o} $f(x) = -x$

2^{o} $f(x) = 2 - x$ (sube 2 unidades)

3^{o} $f(x) = 2 - (x+2)$ (se mueve a la izq)

4^{o} $f(x) = -2 + (x+2)$ (se voltea)

$\Rightarrow f(x) = x.$

$f(0) = 0$ b) FALSO

35.-

$C(r) = nr + s$

① $\boxed{12r + s = 2925}$ ② $\boxed{20r + s = 4525}$

① (-1) + ②

$-12r - s = -2925$

$20r + s = 4525$

$8r = 1600$

$\boxed{r = \$200}$

$12r + s = 2925$

$s = 2925 - 12r$

$\boxed{s = \$525}$

36.-

$D(n) = 1420 + 100n$

$n = 10$

$D(10) = 1420 + 100(10)$

$\boxed{D_{10} = 2420 \text{ médicos}}$

$1420 + 100n \geq 2000$

$100n \geq 2000 - 1420$

$n \geq \frac{580}{100}$

$n \geq 5{,}8$ años $\rightarrow$ 5 años y 9 meses

$1994 + 5 + 9$ mes. $\Rightarrow$ $\boxed{1999 \text{ y } 9 \text{ meses}}$

37.-

C.F = $ 2500

CT = $ 3300

X = 200

P.V.P = $ 5,25

$C(x) = CF + CV \cdot x$

$33\cancel{00} = 25\cancel{00} + C.V \cdot 2\cancel{00}$

$33 - 25 = 2CV$

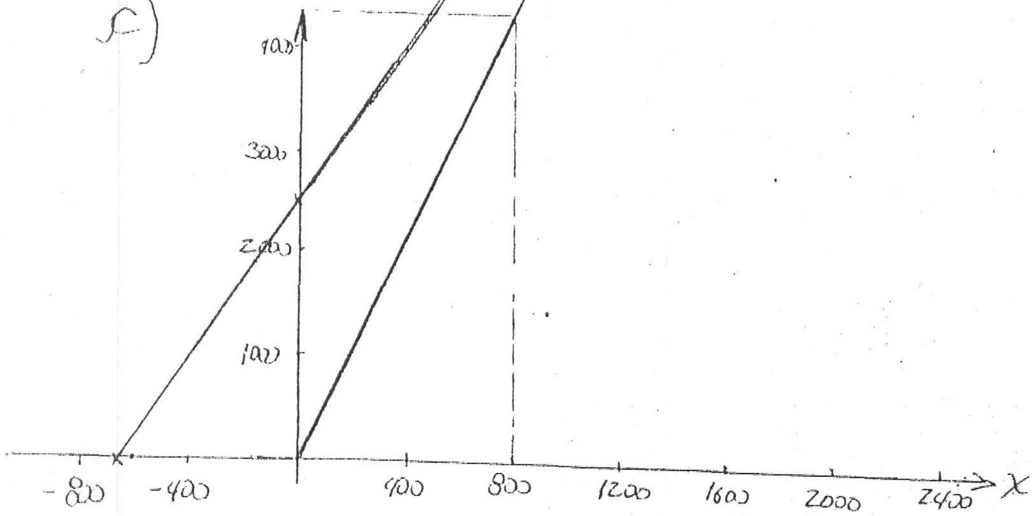
$8 = 2CV$

$CV = 4$

a) $C(x) = 2500 + 4X$

b) $I(x) = 5,25X$

c)



$y = 2500 + 4X$

$X = 0$, $y = 2500$

$y = 0$, $X = -625$

$y = 5,25X$

$X = 800$, $y = 4200$

d)

$200 = 5,25X - (2500 + 4X)$

$200 = 5,25X - 2500 - 4X$

$2700 = 1,25X$

$X = 2160$

38.-

$y_c = 2,8X + 600$

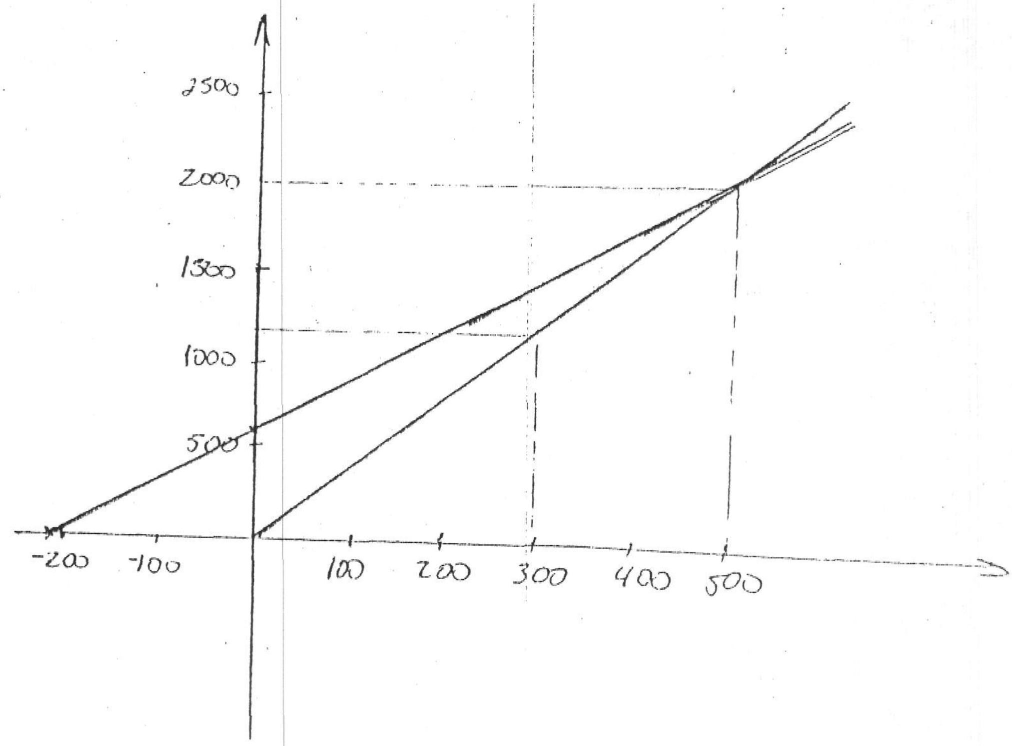
$I(x) = 4X$

a) $4X = 2,8X + 600$

$1,2X = 600$

$X = 500$ unidades

b)



$y = 2{,}8x + 600$

$x = 0$
$y = 600$

$y = 0$
$x = -214{,}2$

$y = 4x$

$x = 300$
$y = 1200$

c) Sin pérdida

$I(x) = y_c$

$P \cdot X = 2{,}8x + 600$

$45\cancel{0}P = 2{,}8(45\cancel{0}) + 60\cancel{0}$

$45P = 126 + 60$

$P = \frac{186}{45}$

$P = \$4{,}13$

39.- $\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho}$

$V = \frac{5000}{0{,}859}$

$V = 5820 \; m^3$

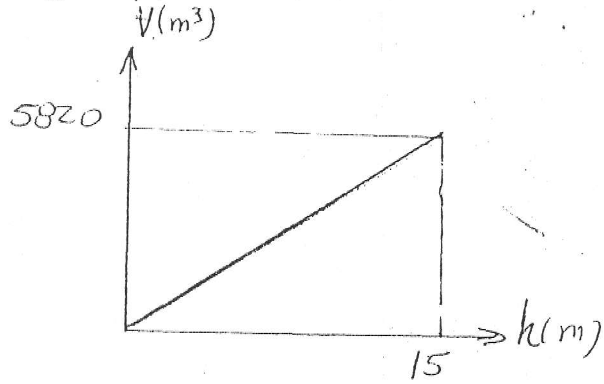
$V = A\,h$

$h = \frac{V}{A} \Rightarrow \frac{5820{,}72}{388}$

$h = 15m$

b)

$V = 388h$

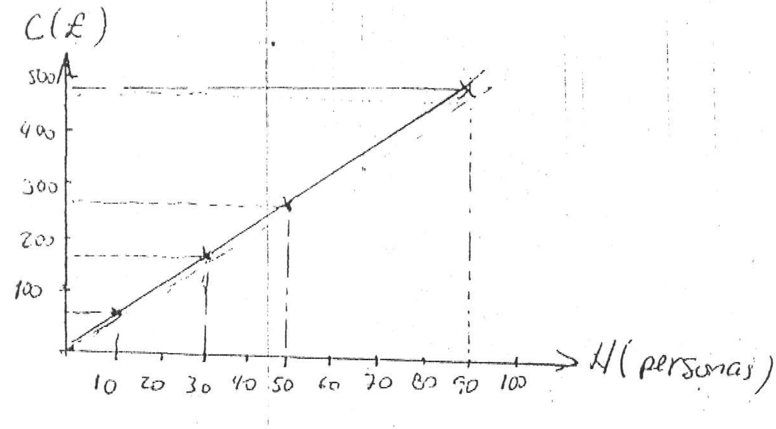


dom f : (0, 15) m.

40.- a.1)

N	10	30	50	90
C	70	170	270	470

a.2)

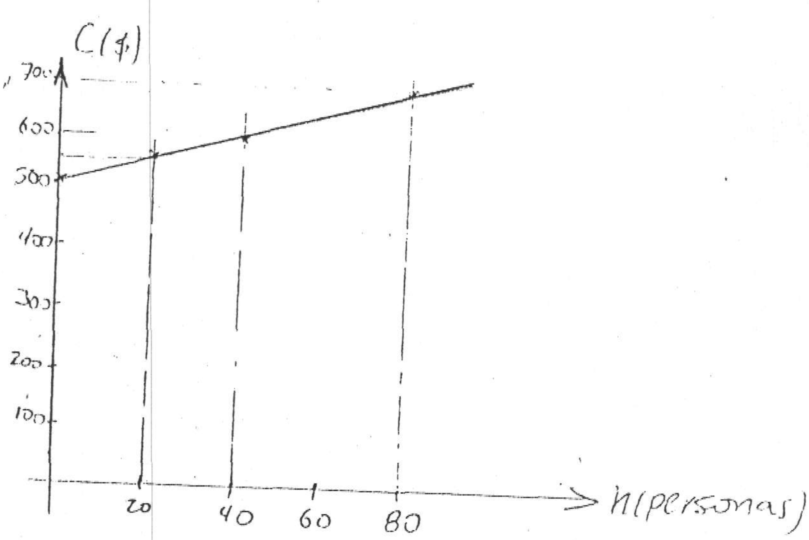


a.3) $C(n) = 5n + 20$

b.1)

N	0	20	40	80
C	500	550	600	700

b.2)



c.1) es una extrapolación; se usará ecuación:

$$C = \frac{5N}{2} + 500$$

$$C = \frac{5}{2}(208) + 500$$

$$C = \$\ 1020$$

c.2)

$$C = 2{,}5n + 500$$

$$1060 = 2{,}5n + 500$$

$$2{,}5n = 1060 - 500$$

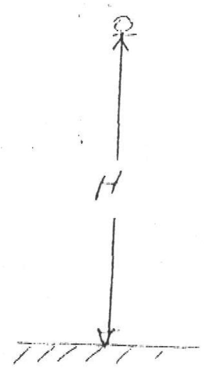
$$n = \frac{560}{2{,}5} \Rightarrow n = 224$$

41.-

$$H = 25 - 16t^2$$

$$H = 25 - 16(1{,}25)^2$$

$$\underline{H = 0}$$



a) correcto

12.-

Producto de raices: $\frac{c}{a}$

Suma de raices: $-\frac{b}{a}$

Producto r. = Suma r.

$$\frac{c}{\cancel{a}} = -\frac{b}{\cancel{a}}$$

$b = -c$ b) correcto

13.- $g(x) = x^2 - bx + 1$

a) Dom g: $x \in \mathbb{R}$; verdadero

b) $\Delta = b^2 - 4(1)(1)$

$\Delta = b^2 - 4 \geq 0$

$b^2 \geq 4$; para que haya solución real.

$b^2 < 4$; no hay soluciones reales (no hay raices) $g(x) \neq 0$

Verdadero

c) rango g: $\left[1 - \frac{b^2}{4}, +\infty\right)$

$$y = \frac{-\Delta}{4a} \Rightarrow \frac{-b^2 + 4ac}{4a} \Rightarrow \frac{-b^2 + 4}{4} \Rightarrow -\frac{b^2}{4} + 1 \Rightarrow \boxed{1 - \frac{b^2}{4}}$$

FALSO

d) Verdadero

e) Verdadero; $b \geq 2 \vee b \leq -2$

44.- $f(x) = |2x^2 - 3x + 1| - 2$

$f_1(x) = 2x^2 - 3x + 1$

Vértice

$x = -\frac{b}{2a} \Rightarrow -\frac{-3}{2(2)}$

$x = \frac{3}{4}$

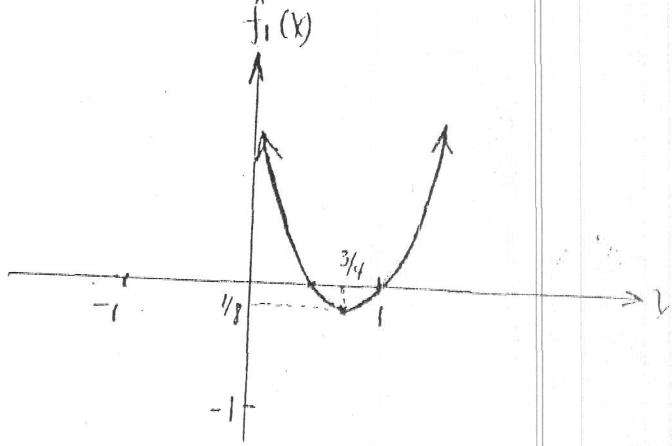
$y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$

$\Rightarrow \frac{\cancel{18}^{9}}{\cancel{16}_{8}} - \frac{9}{4} + 1$

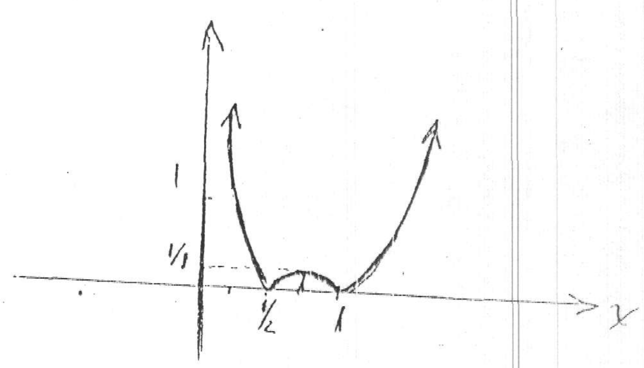
$\Rightarrow \frac{9}{8} - \frac{9}{4} + 1 \Rightarrow \frac{9-18+8}{8}$

$\Rightarrow -\frac{1}{8}$

$V\left(\frac{3}{4}, -\frac{1}{8}\right)$



$f_2(x) = |2x^2 - 3x + 1|$



raíces

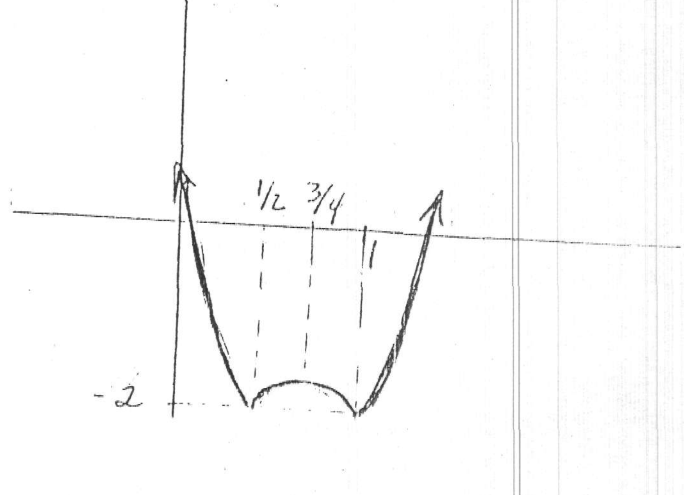
$2x^2 - 3x + 1 = 0$

$(2x-2)(2x-1) = 0$

$2x - 2 = 0 \qquad 2x - 1 = 0$

$x = 1 \qquad x = 1/2$

$f_3(x) = |2x^2 - 3x + 1| - 2$



a) FALSO

b) VERDADERO

c) FALSO

d) FALSO

e) FALSO

45.- $f(x) = x^2 + x$

Vértice

$x = -\frac{1}{2(1)} \Rightarrow -\frac{1}{2}$

$y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)$

$y = \frac{1}{4} - \frac{1}{2} \Rightarrow -\frac{1}{4}$

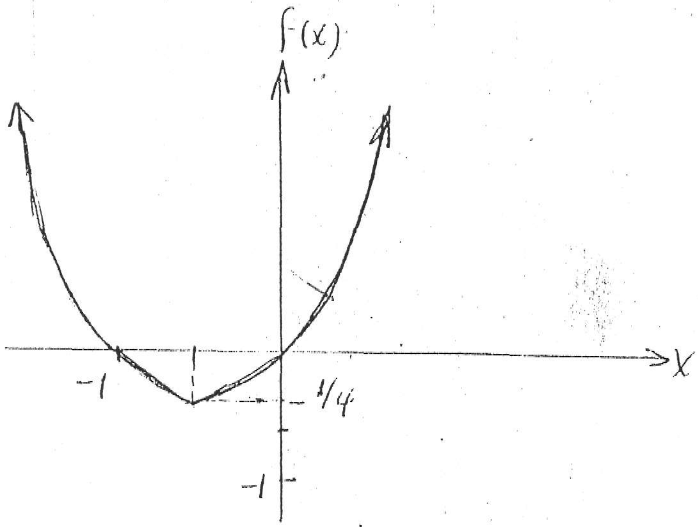
$V\left(-\frac{1}{2}, -\frac{1}{4}\right)$

raíces

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0 \; ; \; x = -1$



a) FALSO

b) FALSO

c) FALSO; $[-1/4, +\infty)$

d) CORRECTO

e) FALSO

46.- $V(-1, 2) \quad ; \quad A(1, 0)$

$x = -\frac{b}{2a} \Rightarrow -1 = -\frac{b}{2a}$

$\Rightarrow \boxed{b = +2a}$

$y = -\frac{b^2 - 4ac}{4a} \Rightarrow 2 = -\frac{(+2a)^2 - 4ac}{4a}$

$\Rightarrow -2 = \frac{\cancel{4}a^{\cancel{2}} - \cancel{4a}c}{\cancel{4a}} \Rightarrow -2 = a - c$

$\Rightarrow \boxed{c = a + 2}$

$y = ax^2 + bx + c$

$(1,0) \quad ax^2 + (2a)x + a + 2 = 0$

$a + 2a + a + 2 = 0$

$4a = -2 \Rightarrow \boxed{a = -1/2}$

$a = -1/2$

$b = -1$

$c = 3/2$

$y = -\frac{1}{2}x^2 - x + \frac{3}{2}$

$y = -\frac{1}{2}(x^2 + 2x - 3)$

$y = -\frac{1}{2}(x^2 + 2x + 4 - 4 - 3)$

$y = -\frac{1}{2}(x+2)^2 + 7/2$

$y = \frac{-1}{2}(x+2)^2 + 7/2$

$h = -2$

$k = +7/2$

$a = -\frac{1}{2}$

47.- $V(1,8)$ $A(3,0)$

$y = ax^2 + bx + c$

$x = -\frac{b}{2a} \Rightarrow 1 = -\frac{b}{2a}$

$\boxed{b = -2a}$

$y = -\frac{b^2 - 4ac}{4a} \Rightarrow 8 = -\frac{(-2a)^2 - 4ac}{4a}$

$-8 = \frac{4a^{\cancel{2}} - 4ac}{\cancel{4a}} \Rightarrow -8 = a - c$

$\boxed{c = a+8}$

$y = ax^2 - 2ax + a + 8$

$0 = a(3)^2 - 2a(3) + a + 8$

$0 = 9a - 6a + a + 8$

$-4a = 8$

$\boxed{a = -2}$ $\boxed{b = 4}$ $\boxed{c = 6}$

$\boxed{y = -2x^2 + 4x + 6}$

$y = -2(x^2 - 2x + 3)$

$y = -2(x-2)(x-1)$

48.- $V(3,1)$ $P(5,9)$

$y = ax^2 + bx + c$

$x = \frac{-b}{2a} \Rightarrow 3 = \frac{-b}{2a}$

$b = -6a$

$y = -\frac{b^2 - 4ac}{4a}$

$-1 = \frac{(-6a)^2 - 4ac}{4a}$

$-1 = \frac{\overset{9}{\cancel{36}}a^{\cancel{2}} - \cancel{4a}c}{\cancel{4a}}$

$-1 = 9a - c \Rightarrow \boxed{c = 9a+1}$

$\Longrightarrow y = ax^2 - 6ax + 9a + 1$

$P(5,9)$

$9 = a(5)^2 - 6a(5) + 9a + 1$

$25a - 30a + 9a = 8$

$4a = 8$

$\boxed{a = 2}$ $\boxed{b = -12}$ $\boxed{c = 19}$

$y = 2x^2 - 12x + 19$

$y = (2x^2 - 12x + 18) + 1$

$y = 2(x^2 - 6x + 9) + 1$

$$y = 2(x-3)^2 + 1$$

$a = 2$

$h = 3$

$k = 1$

49.- $f(x) = 30x - 5x^2$

a) raíces : $30x - 5x^2 = 0$

$5x(6 - x) = 0$

$\boxed{x = 0}$ $\quad x - 6 = 0$

$\boxed{x = 6}$

b) c = vértice

$$x = \frac{-30}{2(-5)} = 3$$

$y = 30(3) - 5(3)^2$ $\quad C(3, 45)$

$y = 90 - 45$

$y = 45$

c) $x = 3 \Rightarrow x - 3 = 0$

50.- $V(2,3)$ $\quad A(0,5)$

$$x = -\frac{b}{2a}$$

$$2 = -\frac{b}{2a} \Rightarrow \boxed{b = -4a}$$

$$y = -\frac{b^2 - 4ac}{4a}$$

$$3 = -\frac{(-4a)^2 - 4ac}{4a}$$

$$-3=\frac{\cancel{16}^{4}a^{\cancel{2}}-\cancel{4a}c}{\cancel{4a}} \Rightarrow -3=4a-c$$

$$\boxed{c=4a+3}$$

$$y=ax^2-4ax+4a+3$$

$A(0,5)$ $5=0-0+4a+3$

$$4a=2$$

$$\boxed{a=1/2} \quad \boxed{b=-2} \quad \boxed{c=5}$$

$$y=\frac{1}{2}x^2-2x+5$$

$$\Rightarrow y=\frac{1}{2}x^2-2x+2+3$$

$$\Rightarrow y=\frac{1}{2}(x^2-4x+4)+3$$

$$\Rightarrow y=\frac{1}{2}(x-2)^2+3$$

$h=2$
$a=1/2$

51.- $f(x)=x^2+bx+c$ $A(2,0)$ $B(3,0)$

$0=2^2+b(2)+c$ $\quad$ $0=3^2+b(3)+c$

(1) $\boxed{2b+c=-4}$ $\quad$ (2) $\boxed{3b+c=-9}$

$\Downarrow$

$(1)*(-1)+(2)$ $\quad$ $c=-9-3b$

$c=-9+15$

$\boxed{c=6}$

$$\begin{array}{r} -2b-c=4 \\ 3b+c=-9 \\ \hline \boxed{b=-5} \end{array}$$

52.- $f(x) = 2x^2 + x + K$

$f(x) \Rightarrow$ no se intercepta con el eje x; entonces no hay raíces; por lo tanto $b^2 - 4ac < 0$

$1^2 - 4(2)K < 0$

$-8K < -1$

$8K > 1$

$\{K > 1/8\}$ c) correcto

53.- Pventa = \$100

$y_c = x^2 + 20x + 700$

a) $I(v) = 100x$

b) $U(x) = 100x - x^2 - 20x - 700$

c) $U(x) = -x^2 + 80x - 700$

vértice

$x = \frac{-80}{2(-1)} =$ 40 unidades

$U(x) = -(40)^2 + 80(40) - 700$

$U(x) = \$900$

54.- Demanda

$P^2 + x^2 = 169$

$P_1 = \sqrt{169 - x^2}$

Oferta

$P_2 = x + 7$

$$P_1 = P_2$$

$$\left(\sqrt{169 - x^2}\right)^2 = (x+7)^2$$

$$169 - x^2 = x^2 + 14x + 49$$

$$x^2 + x^2 + 14x + 49 - 169 = 0$$

$$2x^2 + 14x - 120 = 0$$

$$x^2 + 7x - 60 = 0$$

$$(x+12)(x-7) = 0$$

$$\boxed{x = 7}$$

$$P = x + 7$$

$$P = \$14$$

55.- a)

Longitud(m)	Ancho(m)	Área(m²)
1	11	11
2	10	20
3	9	27
4	8	32

b) $x \to$ largo

$24 - x \to$ ancho

$$A(x) = x(12 - x)$$

$2x + 2a = 24$

$x + a = 12$

$a = 12 - x$

c) $A(x) = x(12 - x)$

$\Rightarrow 12x - x^2$

$x = \frac{-12}{2(-1)} = 6\,m$

56.- a) $h(t) = -5t^2 + 30t$

$t = \frac{-b}{2a} = \frac{-30}{2(-5)}$

$t = 3s$

b) $h = -5(3)^2 + 30(3)$

$h = 45m$

57.- CV = $15.

número : 100.000 − 4000x

P.V.P = x

a) $U = x \cdot (100.000 - 4000x) - 15(100.000 - 4000x)$

b) $U(x) = 100.000x - 4000x^2 - 1'500.000 + 60.000x$

$U(x) = -4000x^2 + 160.000x - 1'500.000$

$x = \frac{-160.\cancel{000}}{2(-4\cancel{000})}$

$x = 20$

c) 100.000 − 4000(20) ⇒ 20.000 libros

58.-

$f(x)$: x^2+3 for $x<2$; $2x-1$ for $x\geq 2$ (marks 0, 1, 2)

$g(x)$: $4x$ for $x\leq 0$; $1-x$ for $0<x<2$; 3 for $x\geq 2$ (marks 0, 1, 2)

$f-g$: x^2-4x+3 ; x^2+x+2 ; $2x-4$ (marks 0, 2)

$\frac{f}{g}$: $\frac{x^2+3}{4x}$; $\frac{x^2+3}{1-x}$; $\frac{2x-1}{3}$ (marks 0, 2)

$f+g$: x^2+4x+3 ; x^2-x+4 ; $2x+2$ (marks 0, 2)

fg: $4x(x^2+3)$; $(x^2+3)(1-x)$; $3(2x-1)$ (marks 0, 2)

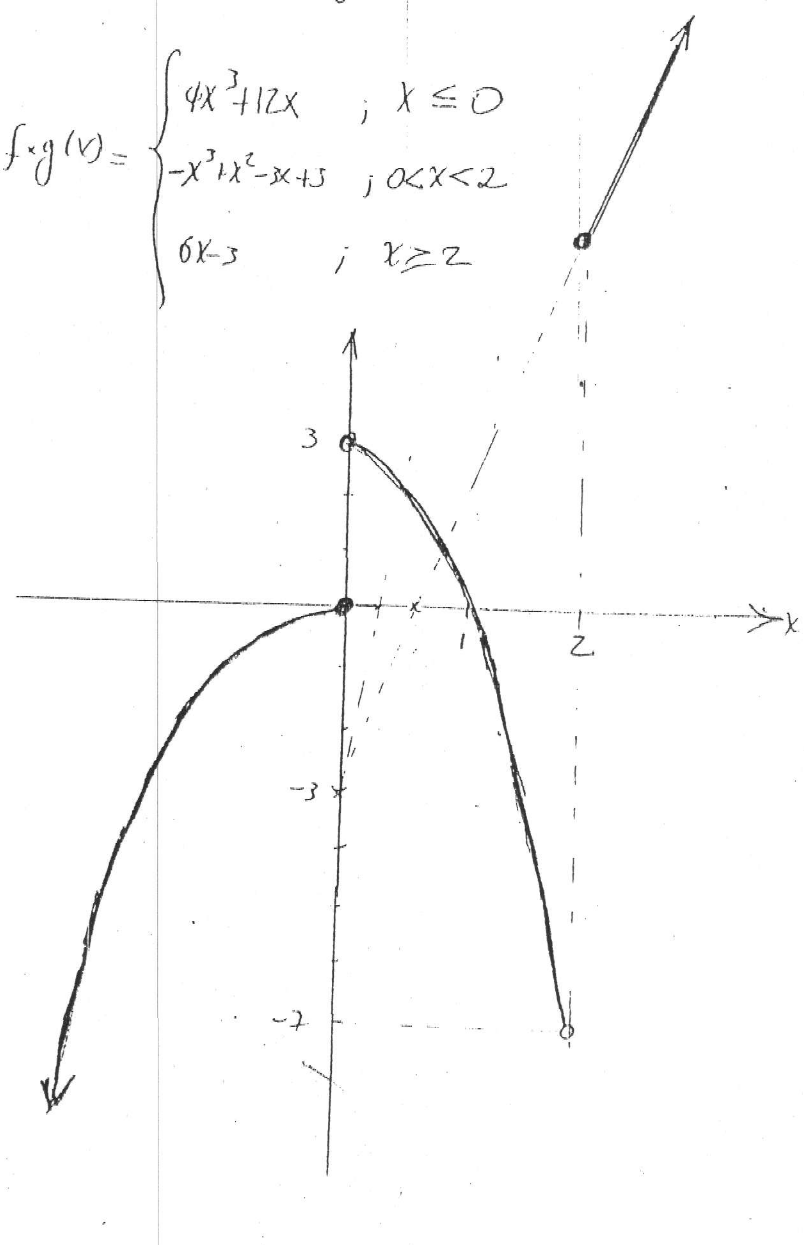
$$\begin{array}{r} x^2+3 \\ 1-x \\ \hline x^2+3-x^3-3x \end{array}$$

$$f\cdot g(x)=\begin{cases} 4x^3+12x & ; x\leq 0 \\ -x^3+x^2-3x+3 & ; 0<x<2 \\ 6x-3 & ; x\geq 2 \end{cases}$$

$y=6x-3$

$x=0$, $y=-3$

$y=0$, $x=1/2$

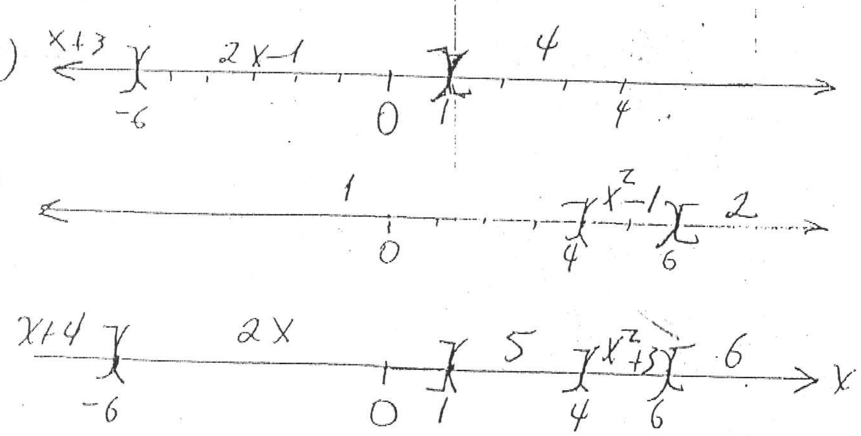


$4x^3+12x=0$

$4x(x^2+3)=0$

$\boxed{x=0}$

59.- $f(x)$

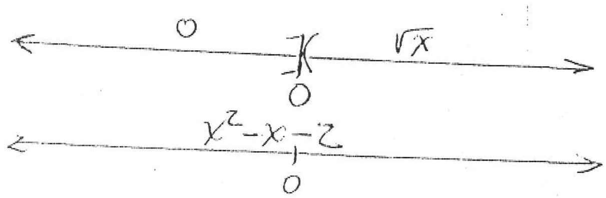


$g(x)$

$$(f+g)(x)\begin{cases} x+4 & ; x\leq -6 \\ 2x & ; -6<x\leq 1 \\ 5 & ; 1<x\leq 4 \\ x^2+3 & ; 4<x<6 \\ 6 & ; x\geq 6 \end{cases}$$

C) correcto

60.- $f(x)$



$g(x)$

a) $\begin{cases} x^2-x-2 & ; x\leq 0 \\ x^2-x+\sqrt{x}-2 & ; x>0 \end{cases}$

b) $\begin{cases} x^2-x-2 & ; x\leq 0 \\ x^2-x-\sqrt{x}-2 & ; x>0 \end{cases}$

d) $\begin{cases} 0 & ; x\leq 0 \\ \dfrac{\sqrt{x}}{x^2-x-2} & ; x>0 \end{cases}$

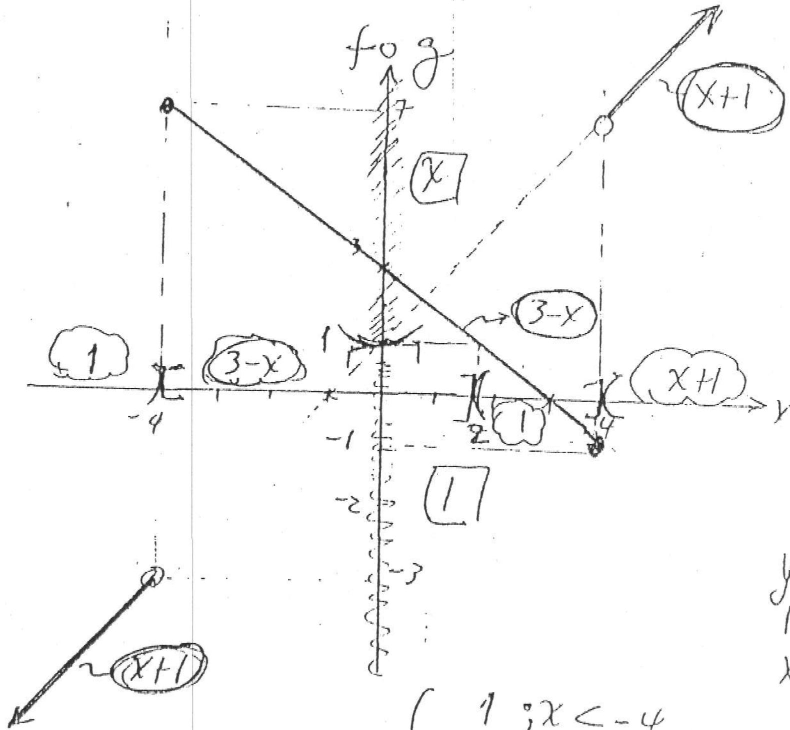
e) $\begin{cases} 0 & ; x\leq 0 \\ \dfrac{\sqrt{x}}{x^2-x-2} & ; x>0 \end{cases}$

e) $\begin{cases} \text{NO DEFINIDO} & ; x\leq 0 \\ \dfrac{x^2-x-2}{\sqrt{x}} & ; x>0 \end{cases}$

f) $\begin{cases} 0 & ; x\leq 0 \\ \sqrt{x}(x^2-x-2) & ; x>0 \end{cases}$

61.- $f(x)=\begin{cases} x & ;x>1 \\ 1 & ;x\le 1\end{cases}$

$g(x)=\begin{cases} 3-x & ;|x|\le 4 \\ x+1 & ;|x|>4\end{cases}$



$y=3-x$

$x=0$, $y=3$ $\quad$ $y=0$, $x=3$

$y=x+1$

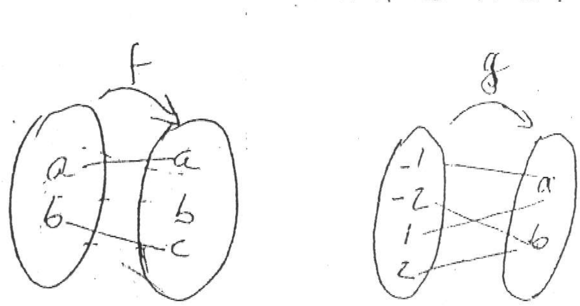
$x=0$, $y=1$ $\quad$ $y=0$, $x=-1$

$y=3-x$
$1=3-x$
$x=2$

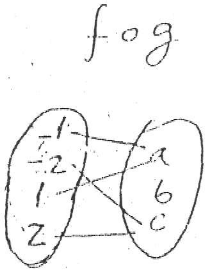
$fog\begin{cases} 1 & ;x<-4 \\ 3-x & ;-4\le x\le 2 \\ 1 & ;2\le x\le 4 \\ x+1 & ;x>4\end{cases}$

e) correcto

62.- FALSO, NO NECESARIAMENTE.

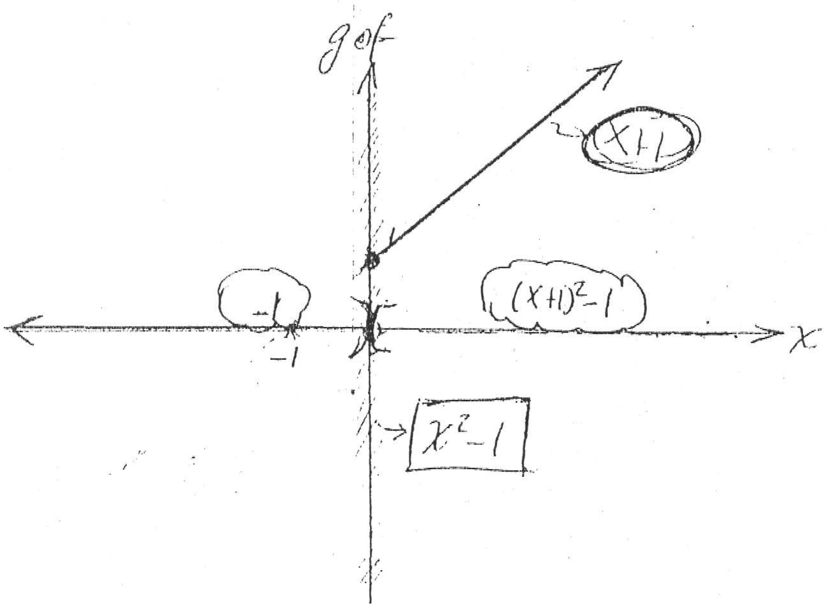


fog



63.-

$$f(x) = \begin{cases} 0 & ; x < 0 \\ x+1 & ; x \geq 0 \end{cases} \qquad g(x) = x^2 - 1 \ ; x \in \mathbb{R}$$



$y = x+1$

$x=0$
$y=1$

$y=0$
$x=-1$

$$gof = \begin{cases} -1 & ; \ x < 0 \\ x^2+2x & ; \ x \geq 0 \end{cases}$$

64.-

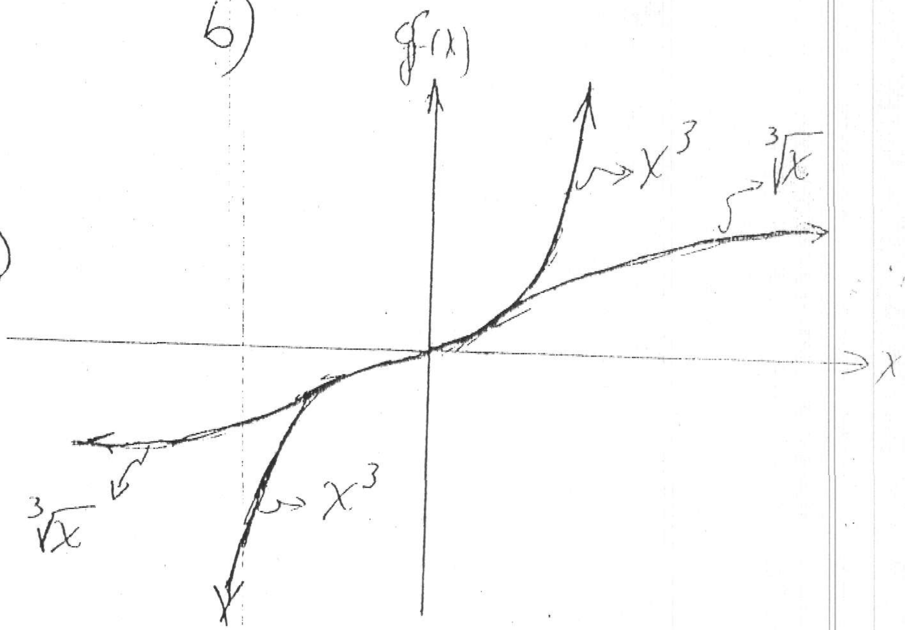
$$g(x) = x^3$$

a) $\sqrt[3]{g(x)} = \sqrt[3]{x^3}$

$x = \sqrt[3]{g(x)}$

$g(x)^{-1} = \sqrt[3]{x}$

b)



c) $g(x)$: $x^3 = \sqrt[3]{x}$

$$(x^3)^3 = (\sqrt[3]{x})^3$$

$\Rightarrow x^9 = x$

$\Rightarrow x^9 - x = 0$

$\Rightarrow x(x^8 - 1) = 0 \Rightarrow x(x^4+1)(x^4-1) = 0 \Rightarrow x(x^4+1)(x^2+1)(x^2-1) = 0$

$\Rightarrow x(x^4+1)(x^2+1)(x+1)(x-1) = 0$ $\quad A g(x) = \{-1, 0, 1\}$

$\boxed{x=0}$ $\quad x-1=0$ $\quad x+1=0$

$\boxed{x=1}$ $\quad \boxed{x=-1}$

65.- $f(x) = \sqrt{x}$ $\quad g(x) = x^2$ $\quad h(x) = x-1$

$f \circ g \circ h$

$g \circ h = (x-1)^2 \Rightarrow f \circ (g \circ h) = \sqrt{(x-1)^2}$

$f \circ (g \circ f) = x - 1$

56.- $\int og(x) = x^2+2x+6$

$f(0)=9$

$f(0)= ax^2+6x+9$

$f(x)= x^2+6x+9$

$f(x)= x^2+4x+9$

a)

$x-k$

$fog = x^2+4x+9$

$fog = (x-k)^2+4(x-k)+9$

$fog = x^2-2kx+k^2+4x-4k+9$

$fog = x^2+(-2k+4)x+(k^2-4k+9) = x^2+2x+6$

$\cancel{x}(-2k+4) = 2\cancel{x}$

$-2k+4=2$

$2k=-2$

$\boxed{k=1}$ $\quad k \in N$

b) $f(x)=x^2+4x+9 \qquad k=1 \; ; \; k \in Z$

57.- $f(x) = \begin{cases} 2(x-3) \; ; & x \leq 3 \\ (x-2)^2 \; ; & x > 3 \end{cases}$

$g(x) = 1-2x \quad ; x \leq 0$

Determinar la regla de correspondencia de $f \circ g$

$(f \circ g)(x) = f(g(x))$

$f(g(x)) = \begin{cases} 2(g(x)-3) \; ; & g(x) \leq 3 \\ (g(x)-2)^2 \; ; & g(x) > 3 \; * \end{cases}$

No se puede efectuar ya que $g(x) \; ; x \leq 0$

$f(g(x)) = 2(1-2x-3) \quad ; x \leq 0$

$f(g(x)) = 2(-2-2x) \quad ; x \leq 0$

$f(g(x)) = -4-4x \quad ; x \leq 0$

68.- $f(x) = -[\![x]\!]$

a) verdadero

69.- $f(x) = x^3$

$$y = x^3$$
$$\sqrt[3]{y} = \sqrt[3]{x^3}$$
$$x = \sqrt[3]{y}$$
$$f^{-1}(x) = \sqrt[3]{x}$$

$$f(x) = f^{-1}(x)$$
$$x^3 = \sqrt[3]{x}$$
$$(x^3)^3 = (\sqrt[3]{x})^3$$
$$x^9 = x$$

$$x^9 - x = 0 \Rightarrow x(x^8-1)=0 \Rightarrow x(x^4+1)(x^4-1)=0 \Rightarrow x(x^4+1)(x^2+1)(x^2-1)=0$$

$$\Rightarrow x(x^4+1)(x^2+1)(x+1)(x-1)=0 \quad \boxed{x=0} \quad \boxed{x=-1} \quad \boxed{x=1}$$

$Ap(x): \{-1, 0, 1\}$

Suma: $-1+0+1 \Rightarrow \underline{\underline{0}}$ b) correcto

70.-

Opción A: $A(x) = 6+3x$

a) Costo fijo: \$6.

b) $A(x) = 6+3(2,4)$

$A(x) = S/. 13,2$

c) $B(x)\begin{cases} \$4 & ; 0{,}5 \le x < 1 \\ \$6 & ; 1 \le x < 1{,}5 \\ \$8 & ; 1{,}5 \le x < 2 \\ \$10 & ; 2 \le x \le 2{,}5 \\ \$12 & ; 2{,}5 \le x < 3 \\ \$14 & ; 3 \le x < 3{,}5 \end{cases}$

d) $B(x) = 2 * ([\![2x]\!] + 1)$

e) $B(1{,}6) = 2 * ([\![2(1{,}6)]\!] + 1)$

$\Rightarrow 2 * ([\![3{,}2]\!] + 1)$

$\Rightarrow 2 * 4$

$\Rightarrow \$8$

f) $A(x) = 6 + 3x$

$22{,}5 = 6 + 3x$

$\dfrac{22{,}5 - 6}{3} = x \Rightarrow x = 5{,}5\,kg$

g) $B(5{,}5) = 2 * ([\![2(5{,}5)]\!] + 1)$

$\Rightarrow 2 * ([\![11]\!] + 1)$

$\Rightarrow \$24$

h) $6 + 3x = 2([\![2x]\!] + 1)$

$6 + 3x = 2[\![2x]\!] + 2$

$2[\![2x]\!] - 3x = 4$

$x = 1$ NO

$x = 2$ NO

$x = 3$ NO

$x = 4$ SI

$x = 4$

$A(x) = 6 + 3x$

$A(x) = \$18$

71.- $h(x) = |x-2| - |x| + 2$

$|x-2| - |x| + 2$

$x-2-|x|+2=0 \quad \wedge \quad -x+2-|x|+2=0$

$\cancel{x-2-x+2=0} \wedge x-2+x+2=0 \wedge \quad -x+2-x+2=0 \wedge -x+2+x+2=0$

$0 \wedge 2x=0 \wedge \quad -2x+4=0 \quad 4=0$ NO.

$x=0 \qquad -2x=-4$

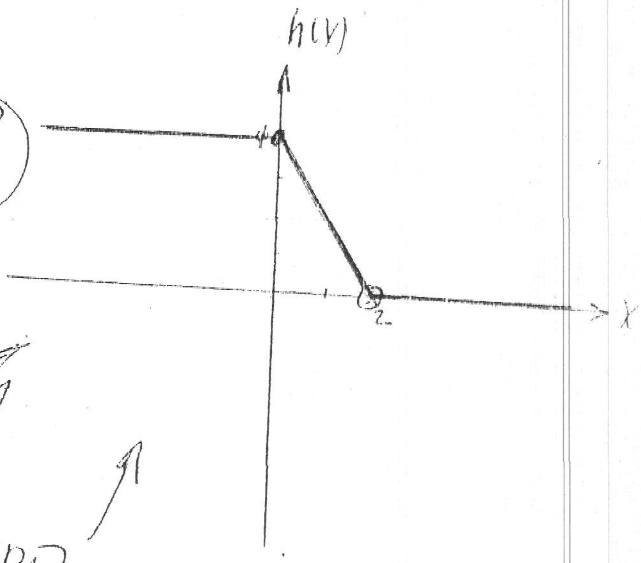
$x=2$

	$x<0$	0	$0<x<2$	2	$x>2$
$\lvert x-2 \rvert$	$-x+2$		$-x+2$		$x-2$
$-\lvert x \rvert$	$+x$		$-x$		$-x$
2	2		2		2
$h(x)$	4		$-2x+4$		0

$y=-2x+4$

$x=0$
$y=4$

$y=0$
$x=2$



a) FALSO

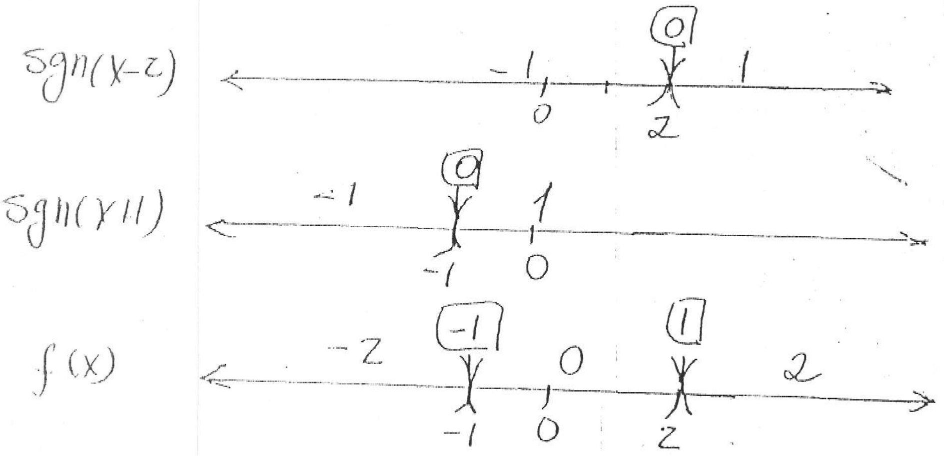
b) FALSO

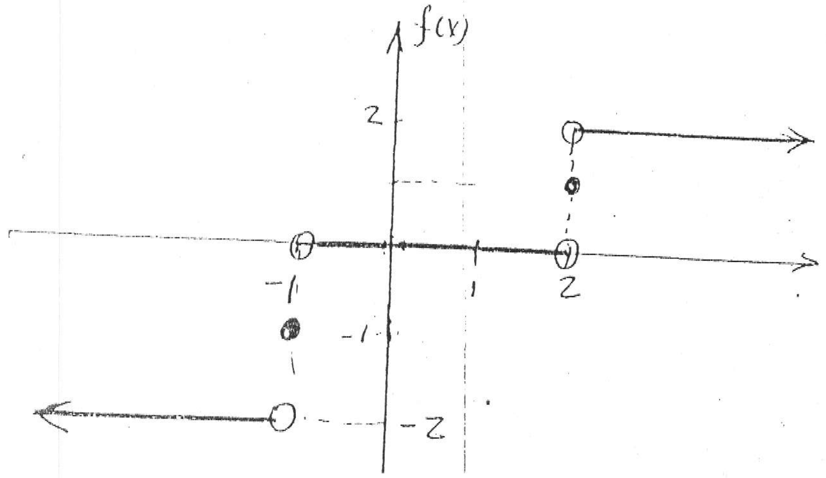
c) VERDADERO

d) FALSO

e) FALSO

72.- $f(x) = \operatorname{sgn}(x-2) + \operatorname{sgn}(x+1)$





a) Verdadero

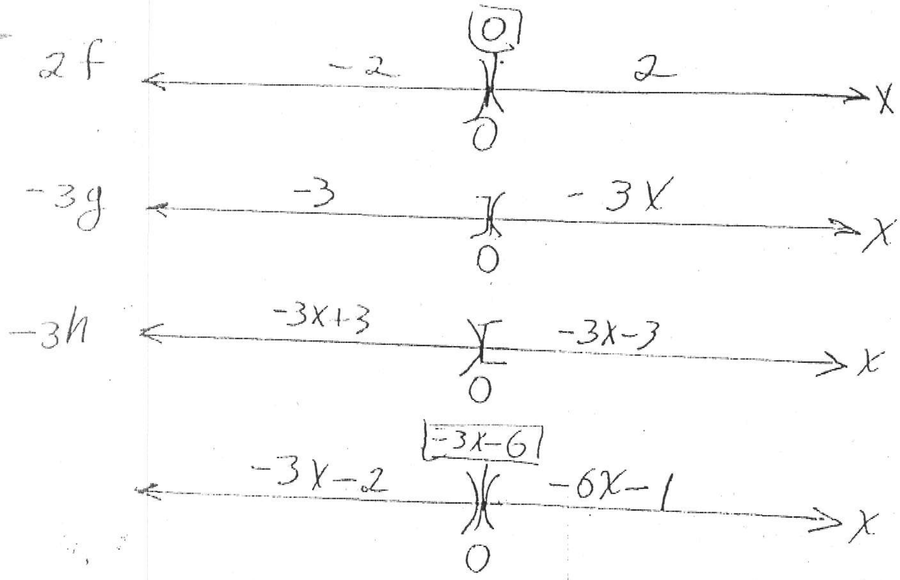
b) Verdadero

c) Verdadero

d) Falso

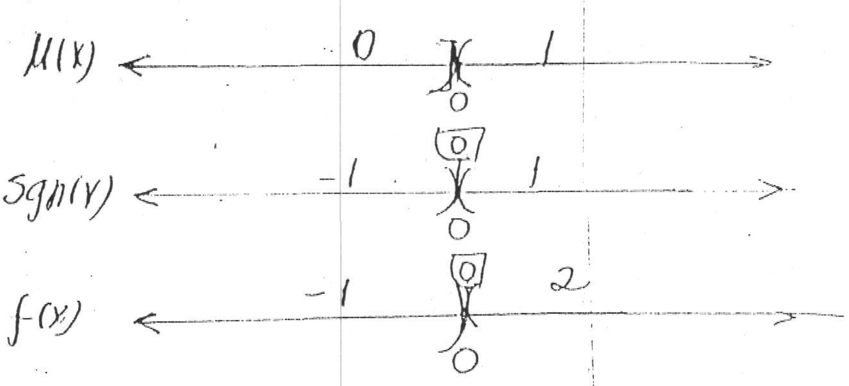
e) Verdadero

73.-



$$(2f-3g-3h)(x)\begin{cases} -3x-2 & ; x<0 \\ -3x-6 & ; x=0 \\ -6x-1 & ; x>0 \end{cases}$$

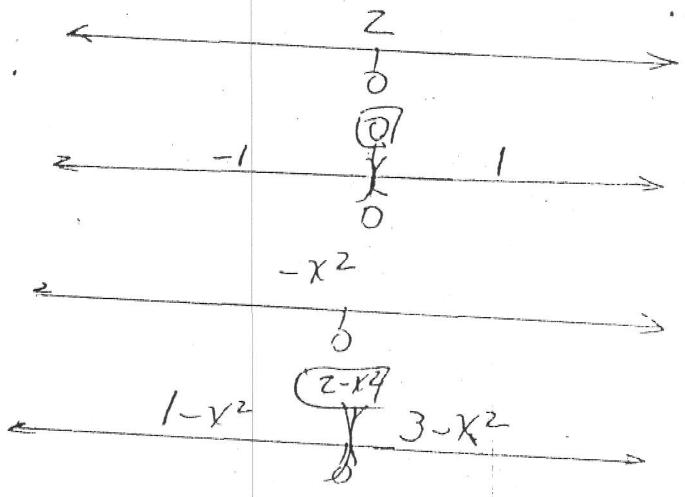
74.-

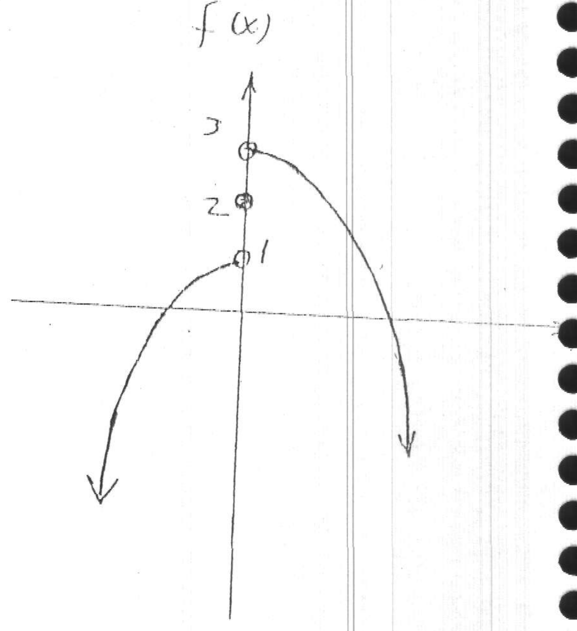


$$f(x)=\begin{cases} 2 & ; x>0 \\ 0 & ; x=0 \\ -1 & ; x<0 \end{cases}$$

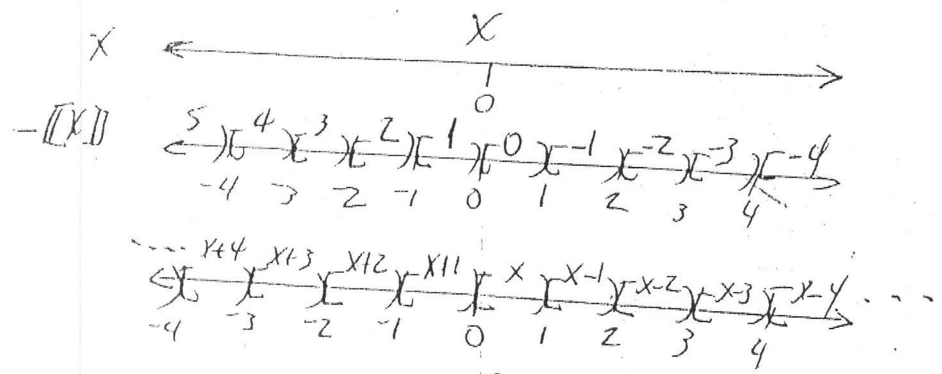
d) correcto

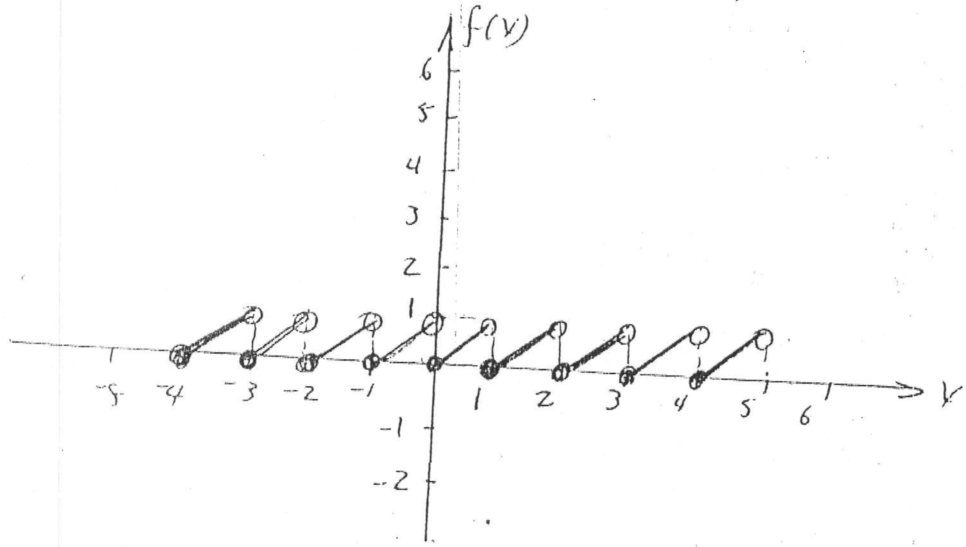
75.- a)



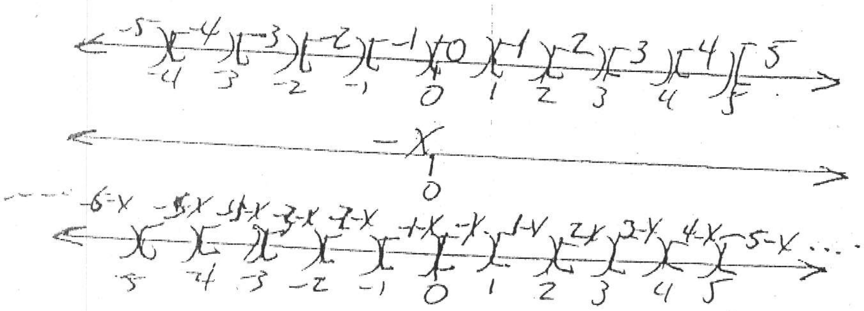


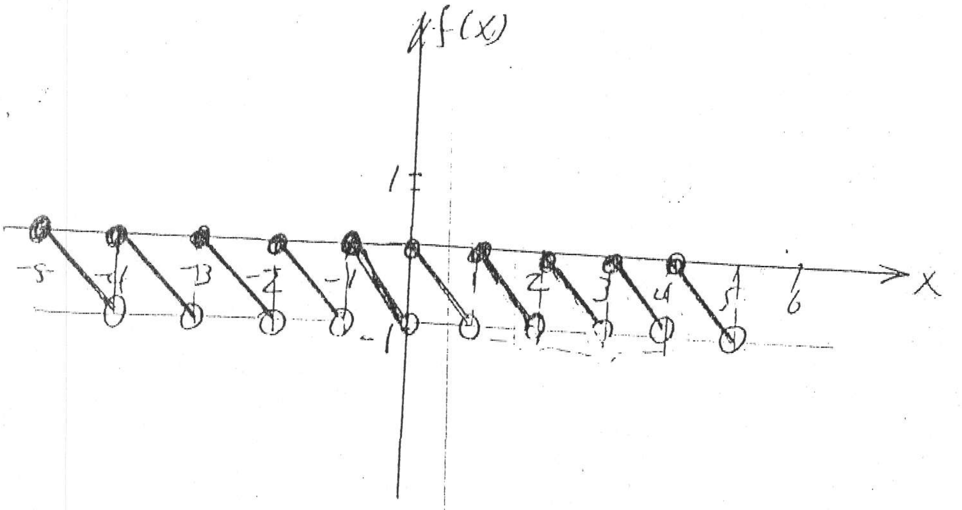
b) $f(x) = x - [\![x]\!]$





c) $f(x) = \begin{cases} x - [\![x]\!] \; ; \; x \geq 0 \\ [\![x]\!] - x \; ; \; x < 0 \end{cases}$



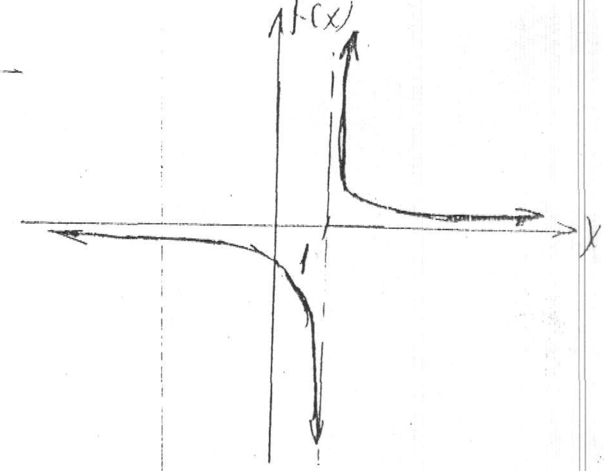


d) $f(x)=\frac{1}{x-1}$

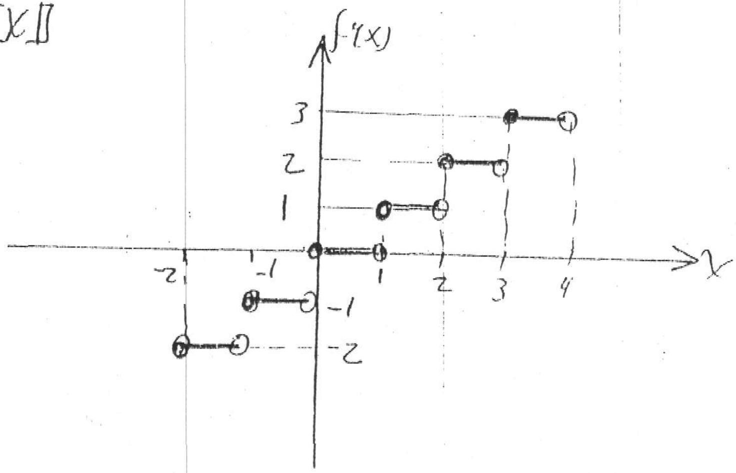
asíntota

$x-1=0$

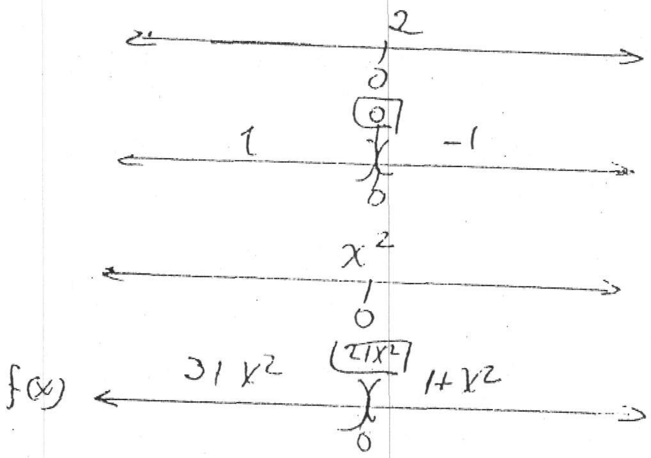
$x=1$

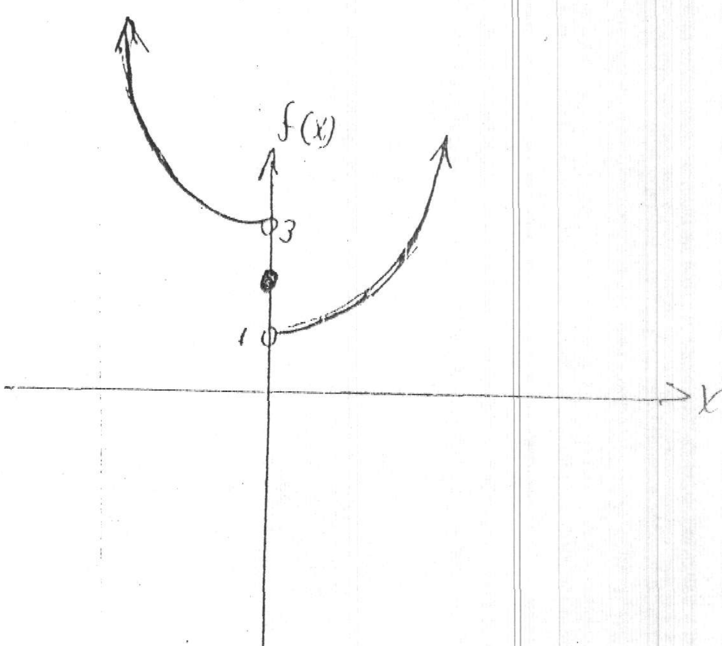


e) $f(x)\ [\![x]\!]$



f) $f(x)=[2-\text{sgn}(x)]+x^2$





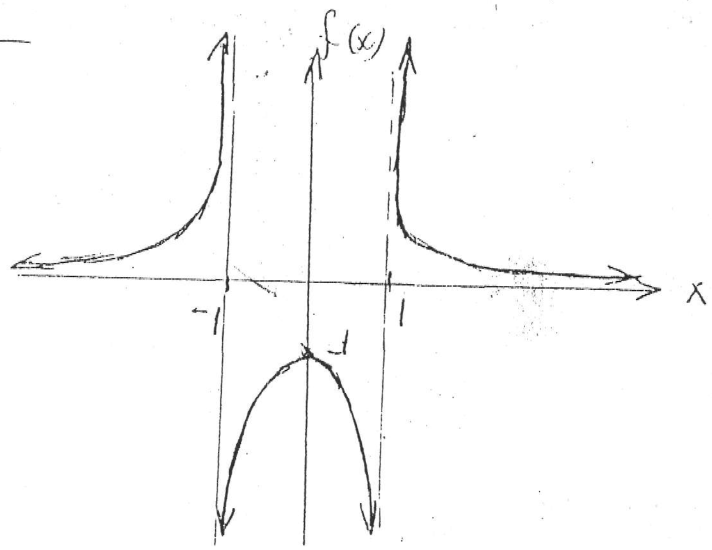
g) $f(x)=\frac{1}{x^2-1}$

asíntota

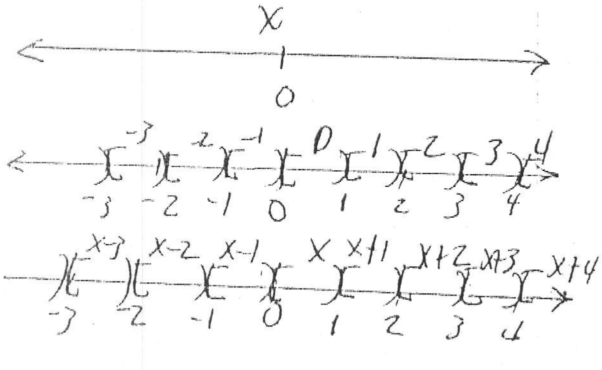
$x^2-1=0$

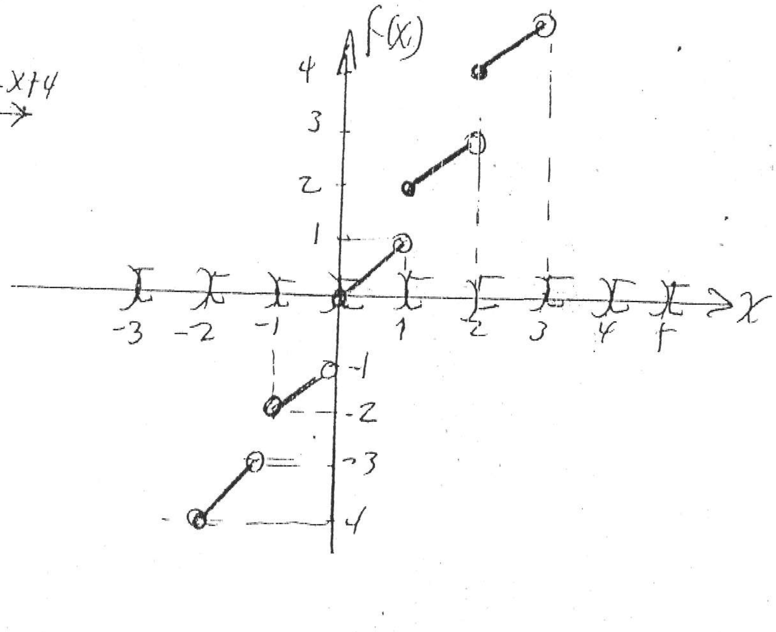
$x^2=1$

$x=\pm 1$



h) $f(x)=x+[\![x]\!]$





a) verdadero

b) Falso; tomando en cuenta el ejercicio anterior, la inversa también deberá estar en el 1er cuadrante

78.— $f^{-1}(a) = 2$

$f(f^{-1}(a)) = f(2)$

$a = f(2)$ a) verdadero

79.— $x^2 - 4x - 3$ $x \in (-\infty, 2)$

rango: $(-7, +\infty)$

$y = x^2 - 4x - 3$

$y = x^2 - 4x + 4 - 4 - 3$

$y = (x-2)^2 - 7$

$\sqrt{y-7} = \sqrt{(x-2)^2}$

$x - 2 = \sqrt{y-7}$

$x = \sqrt{y-7} + 2 \Rightarrow f^{-1}(x) = \sqrt{x-7} + 2$; $x \geq -7$ c) correcto

80.— a) $f(x) = \frac{8}{x}$

$y = \frac{8}{x}$

$x = \frac{8}{y} \Rightarrow f^{-1}(x) = \frac{8}{x}$, NO es ni par ni impar

b) $g(x) = x^2$ $f^{-1}(x) = \frac{8}{x}$

$f^{-1} \circ g(x) = \frac{8}{x^2} \Rightarrow$ es función par.

c) $(f^{-1}(x) \circ g) = x$

$$\frac{8}{x^2} = x$$

$$\sqrt{x^3} = \sqrt{8}$$

$$x = 2$$

81.- a) $g(x) = \frac{6-x}{2}$

$$y = \frac{6-x}{2}$$

$$2y = 6-x$$

$$x = 6-2y$$

$$f^{-1}(x) = 6-2x$$

b) $f(x) = 4(x-1)$ $\quad g(x) = \frac{6-x}{2}$

$$y = 4x-4$$

$$y-4 = 4x$$

$$x = \frac{y-4}{4}$$

$f^{-1} \circ g$

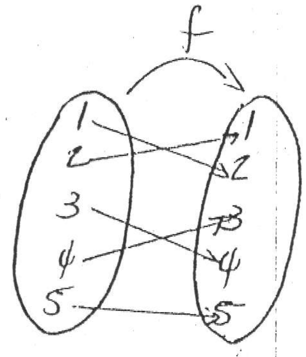
$$f^{-1}(x) = \frac{x-4}{4} \Rightarrow \frac{\left(\frac{6-x}{2}\right)-4}{4}$$

$$\Rightarrow \frac{\frac{6-x-8}{2}}{4} \Rightarrow \frac{-x-2}{8}$$

$$\frac{-x-2}{8} = 4 \Rightarrow -x-2 = 32$$

$$-x = 34 \Rightarrow x = -34$$

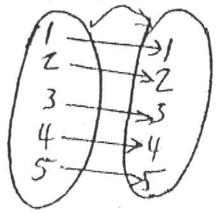
82.—



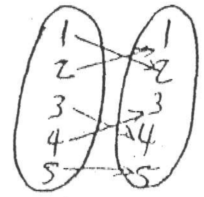
a) correcto

b)

fof



(fof)of



correcto

c) correcto

d) correcto

e) correcto

83.—

a) Son funciones lineales, tienen que ser inyectivas,

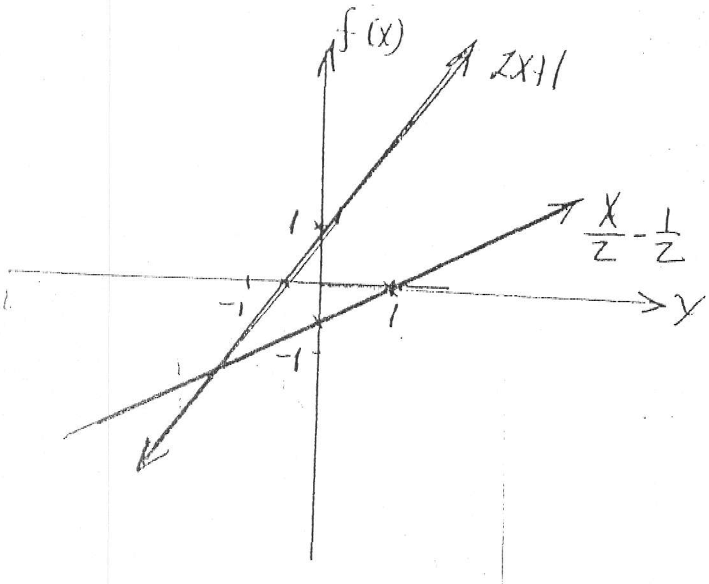
$(x_1 = x_2) \Rightarrow f(x_1) = f(x_2)$

$f(x) = 2x+1$	$g(x) = 3x-4$
$y = 2x+1$	$y = 3x-4$
$y-1 = 2x$	$y+4 = 3x$
$x = \frac{y-1}{2}$	$x = \frac{y+4}{3}$

Solo a un valor de y, corresponde x

b) $f(x)^{-1} = \frac{x-1}{2}$

c)



$y = 2x+1$

$x=0$, $y=1$

$y=0$, $x=-\frac{1}{2}$

$y = \frac{x}{2} - \frac{1}{2}$

$x=0$, $y=-\frac{1}{2}$

$y=0$, $x=1$

d) $(g \circ f)(-2)$

$f(-2) = -3$

$g(-3) = -13$ //

e) $(f \circ g)(x) = 2(3x-4)+1$

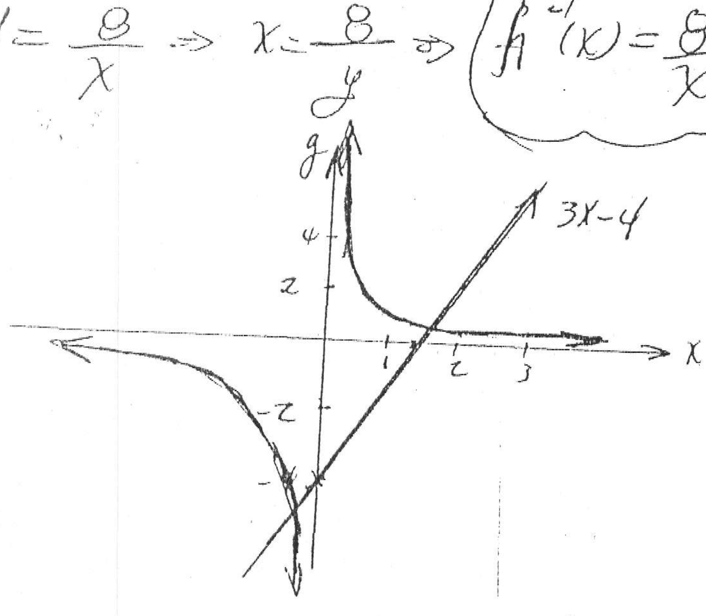
$\Rightarrow 6x-8+1$

$\Rightarrow 6x-7$

f) $h(x) = \frac{8}{x}$

$y = \frac{8}{x} \Rightarrow x = \frac{8}{y} \Rightarrow h^{-1}(x) = \frac{8}{x}$

g)



$g = 3x-4$

$x=0$, $g=-4$

$g=0$, $x=4/3$

h) $h^{-1}_{(x)} = \frac{8}{x}$; $r(x) = x^2$

$(h^{-1} o r)(x) = \frac{8}{x^2}$

i) $(h^{-1} o r)(x) = \frac{1}{2}$

$\frac{8}{x^2} = \frac{1}{2}$

$16 = x^2$

$\boxed{x = \pm 4}$

84.- $f(x) = \sqrt{x-5} - 5$ dom: $[5, +\infty)$

rango: $[-5 +\infty)$

$f^{-1}(x) \Rightarrow$ dom: $[-5, +\infty)$

85.- $f(x) = \frac{1}{ax^2 - 1}$ $A(2, \frac{1}{3})$

a) $\frac{1}{3} = \frac{1}{a(2)^2 - 1} \Rightarrow 4a - 1 = 3 \Rightarrow 4a = 4$

$a = 1$

b) $y = \frac{1}{x^2 - 1} \Rightarrow x^2 - 1 = \frac{1}{y} \Rightarrow x^2 = \frac{1}{y} + 1$

$x = \sqrt{\frac{1+y}{y}} \Rightarrow f^{-1}(x) = \sqrt{\frac{1+x}{x}}$

c) $g(x) = \frac{1}{f(x)}$ x^2-1

d) $f \circ g(x) \Rightarrow \frac{1}{(x^2-1)^2-1} \Rightarrow \frac{1}{x^4-2x^2+1-1} \Rightarrow \frac{1}{x^4-2x^2}$

– Una raiz (en una ecuación) corresponde a un factor en un polinomio

a) correcto

– b) FALSO, No hay relación alguna entre a, b y el residuo.

– $\frac{P(x)}{x-z} = D(x) + \frac{k}{x-z}$

$\frac{P(x)}{\cancel{x-z}} = \frac{D(x)(x-z)+k}{\cancel{x-z}}$

$P(x) = \underbrace{Dx(x-z)}_{\downarrow} + k$

grado 4

también debe ser grado 4.

a) Verdadero

89.- $P(x) = x^3 + x^2 - (k+7)x + \frac{21}{8}$

$(x - \frac{1}{2}) \Rightarrow$ factor $\Rightarrow x = \frac{1}{2}$ $P(x) = 0$

$\Rightarrow \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - (k+7)\left(\frac{1}{2}\right) + \frac{21}{8} = 0$

$8 \times \left(\frac{1}{8} + \frac{1}{4} - \frac{k+7}{2} + \frac{21}{8}\right) = 0 \times 8$

$1 + 2 - 4(k+7) + 21 = 0$

$-4(k+7) = -24$

$k+7 = 6$

$k = -1$ a) correcto

90.- $x^3 + ax^2 + b \quad | \quad x^2 - x - 2$

$-x^3 + x^2 + 2x \quad | \quad x + (a+1)$

$// \ (a+1)x^2 + 2x + b$

$-(a+1)x^2 + (a+1)x + 2(a+1)$

$// \ (a+3)x + (2a+b+2)$

Para que sea divisible $P(x)/q(x)$ ¡residuo $\underline{\underline{0}}$

$a+3=0$

$a=-3$

$2a+b+2=0$

$b=-2a-2$

$b=-2(-3)-2$

$b=4$

$a+b=1$

a) correcto.

91 $q(x) = k^2x^3 - 4kx + 4 \quad ; x=1$

$k^2(1)^3 - 4k(1) + 4 = 1$

$k^2 - 4k + 3 = 0$

$(k-3)(k-1) = 0$

$k=3 \quad k=1$

$k_1 + k_2 = 4$

a) correcto

92.- $P(x): x^3+mx^2-x-2$; $x=2$; residuo: 4

$(2)^3+\cdot(2)^2-(2)-2=4$

$8+4m-4=4$

$4m=8-8$

$m=0$ b) correcto

93.- $P(x)=x^4-ax^2-5x+b$; $x=2$; $P(1)+10=0$

$(2)^4-(2)^2 a-5(2)+b=0$

$16-4a-10+b=0$

$\boxed{-4a+b=-6}$

$(1)^4-a(1)^2-5(1)+b+10=0$

$1-a-5+b+10=0$

$\boxed{-a+b=-6}$

Es un sistema de ecuaciones inconsistente, ESTA MAL PLANTEADA LAS CONDICIONES

94.- $P(x): (a+1)x^5+(b-2)x^4-31x^3-39x^2+76x-20$

$x=1$ residuo: 0

$(a+1)+(b-2)-31-39+76-20=0$

$\boxed{a+b=+15}$

$(a+1)(-3)^5+(b-2)(-3)^4-31(-3)^3-39(-3)^2+76(-3)-20=400$

$-243(a+1)+81(b-2)+837-351-228-20=400$

$-243(a+1)+81(b-2)=162 \Rightarrow 3(a+1)+b-2=2$

$3a+3+b-2=2$

$\boxed{3a+b=1}$ $\boxed{a+b=15}\,|(-1)$

$-a-b=-15$
$3a+b=1$
$2a=-14$
$\boxed{a=-7}$

$b=15-a$
$b=22$

$a+b=15$ e) correcto

95.- $q(x)=x^2+ax+b$; $x=1$; residuo: -3
; $x=2$; residuo: -7

$(1)^2+a(1)+b=-3$ $(2)^2+a(2)+b=-7$

$\boxed{a+b=-4}\,|(-1)$ $\boxed{2a+b=-11}$

$-a-b=4$
$2a+b=-11$
$\boxed{a=-7}$

$b=-11-2a$
$b=-11-2(-7)$
$\boxed{b=3}$

$a\times b=-21$ e) correcto

96.- $P(x): x^3-x^2-14x+24$

$\Rightarrow (x^2+x-12)(x-2)$

$\Rightarrow (x+4)(x-3)(x-2)$

$x_1=-4$ $x_2=3$ $x_3=2$

97.- $P(x): ax^4+bx^3+cx^2+dx+e$

$P(x): x^4+bx^3+cx^2+dx+e$

$P(1): 1+b+c+d+e=0$

① $\{\boxed{b+c+d+e=-1}$

$P(0)= x^4+bx^3+cx^2+dx+e=-2$

$0+0+0+0+e=-2$

$e=-2$

$$\begin{cases} b+c+d=1 \\ 2b+2c+d=0 \\ 4b-2c=6 \end{cases}$$

$$(-2),(-2)\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & -2 & 1 & 0 \\ 2 & -1 & 0 & 3 \end{array}\right)$$

$$\Rightarrow \begin{array}{c} \\ (-3) \\ (4) \end{array}\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -1 & -2 \\ 0 & -3 & -2 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -4 & -1 & -2 \\ 0 & 0 & -5 & 10 \end{array}\right)$$

$-5d=10$

$d=-2$

$$\begin{array}{l|l} x^4+bx^3+cx^2+dx+e & x^2+2x+2 \\ -x^4-2x^3-2x^2 & x^2+(b-2)x \\ \hline // (b-2)x^3+(c-2)x^2+dx & (-2b+c+2) \\ -(b-2)x^3-2(b-2)x^2-2(b-2)x & \\ \hline // (c-2-2b+4)x^2+(-2b+4+d)x & \\ (c-2b+2)x^2+(-2b+d+4)x+e & \\ -(-2b+c+2)x^2-2(-2b+c+2)x-2(-2b+c+2) & \\ \hline // (-2b+d+4+4b-2c-4)x+(4b-2c & \\ (2b-2c+d)x+(4b-2c+e-4) & \end{array}$$

residuo: 0

② $\{\boxed{2b+2c+d=0}$

③ $\boxed{4b-2c+e=4}$

$-3c-2d=1$

$-3c=1+2d$

$-3c=1+(-4)$

$-3c=-3$

$c=1$

$b+c+d=1$

$b=1-c-d$

$b=\cancel{1}-\cancel{1}+2$

$b=2$

$P(x): x^4+2x^3+x^2-2x-2$

98.- $g(x) = x^3 - 8x^2 + 9x + K$

$\left.\begin{array}{l} x_1 = a \\ x_2 = 2a \end{array}\right\}$ Son raíces

$(a)^3 - 8(a)^2 + 9(a) + K = 0$

$a^3 - 8a^2 + 9a + K = 0$

$(2a)^3 + 9(2a) - 8(2a)^2 + K = 0$

$8a^3 - 32a^2 + 18a + K = 0$

$a^3 - 8a^2 + 9a = -K \qquad 8a^3 - 32a^2 + 18a = -K$

$a^3 - 8a^2 + 9a = 8a^3 - 32a^2 + 18a$

$7a^3 - 24a^2 + 9a = 0$

$a(7a^2 - 24a + 9) = 0$

$a = 0 \wedge a = \dfrac{24 \pm \sqrt{24^2 - 4(7)(9)}}{2(7)}$

$a = \dfrac{24 \pm 18}{14} = \begin{cases} 3 \\ \dfrac{3}{7} \end{cases}$

$K = -a^3 + 8a^2 - 9a$

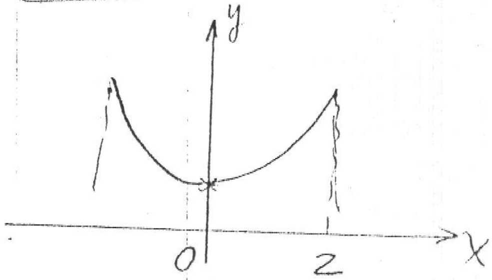
$K = -(3)^3 + 8(3)^2 - 9(3) \wedge K = (-\frac{3}{7})^3 + 8(\frac{3}{7})^2 - 9(\frac{3}{7})$

$K = 18$

$K = -\dfrac{846}{343}$

Exponenciales

99.- $y = 2^{x} + 2^{-x}$



$x = 0$

$y = 2^{0} + 2^{0}$

$y = 2$ $P(0,2)$ b) correcto

100.- $y = 2^{2} + 2^{-2}$

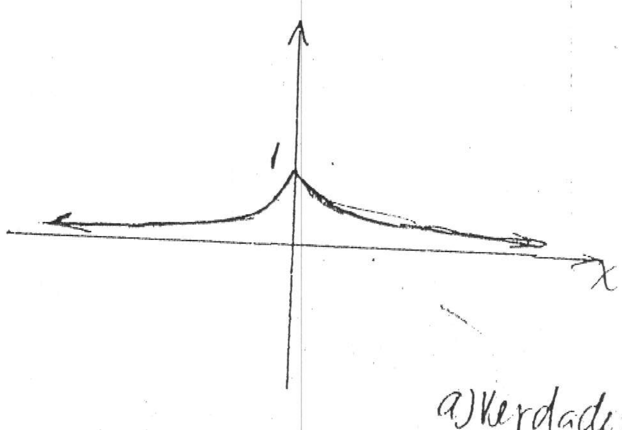
$y = 4 + \frac{1}{4} \Rightarrow \frac{16+1}{4} = \frac{17}{4}$

rango: $\left[2, \frac{17}{4}\right]$ d) correcto

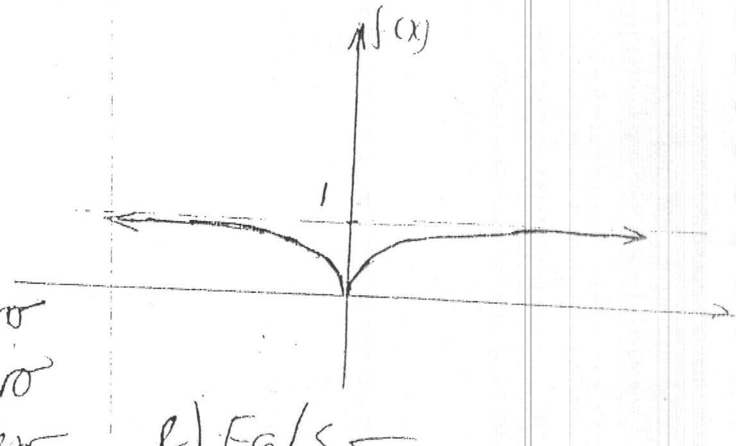
101.- $f(x) = -\frac{1}{2^{|x|}} + 1$

$\Rightarrow f(x) = -\left[\frac{1}{2}\right]^{|x|} + 1$

$f(x) = \left(\frac{1}{2}\right)^{|x|}$



$f(x) = -\left(\frac{1}{2}\right)^{|x|} + 1$



a) Verdadero
b) Verdadero
c) Verdadero
d) Verdadero

f) Falso.

102.- $4 \times 4^{1/3} \times 4^{1/9} \times 4^{1/27} \ldots$

$\Rightarrow 4^{1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots}$

$S = 1+\frac{1}{3}+\frac{1}{9}+\ldots$

$\boxed{r=\frac{1}{3}}$

$S = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{3}}$

$S = \frac{1}{\frac{2}{3}} \Rightarrow \frac{3}{2}$

$\Rightarrow 4^{3/2} \Rightarrow (\sqrt{4})^3 \Rightarrow \underline{\underline{8}}$ c) correcto

103.-

a) $P(x)$: $3^{x+1} + 3^x + 3^{x-1} = 39$

$\Rightarrow 3 \cdot 3^x + 3^x + \frac{3^x}{3} = 39$

$\Rightarrow 3^x(\underbrace{3+1}_{4}+\frac{1}{3}) = 39 \Rightarrow 3^x\left(\frac{12+1}{3}\right) = 39$

$\Rightarrow 3^x = \frac{\cancel{39}^{3} \times 3}{\cancel{13}_{1}} \Rightarrow \cancel{3}^x = \cancel{3}^2$

$x = 2$

b) $q(x)$: $2^{x+1} + 4^x = 80$

$\Rightarrow 2 \cdot 2^x + (2^x)^2 - 80 = 0$

$a = 2^x$

$2a + a^2 - 80 = 0 \Rightarrow a^2 + 2a - 80 = 0$

$(a+10)(a-8) = 0$

$\cancel{a = -10}$ $a = 8$ $\quad 2^x = 8 \Rightarrow \cancel{2}^x = \cancel{2}^3$

$x = 3$

c) $r(x): 6(3^{2x}) - 13(6^{x}) + 6(2^{2x}) = 0$

$$\Rightarrow 6(3^x)^2 - 13(2\times3)^x + 6(2^x)^2 = 0$$

$$a = 3^x \quad ; b = 2^x$$

$$6a^2 - 13ab + 6b^2 = 0$$

$$(6a - 9b)(6a - 4b) = 0$$

$6a - 9b = 0$ $\qquad$ $6a - 4b = 0$

$6a = 9b$ $\qquad$ $6a = 4b$

$\boxed{a = \frac{3}{2}b}$ $\qquad$ $\boxed{a = \frac{2}{3}b}$

$\frac{a}{b} = \frac{3}{2}$ $\qquad$ $\frac{a}{b} = \frac{2}{3}$

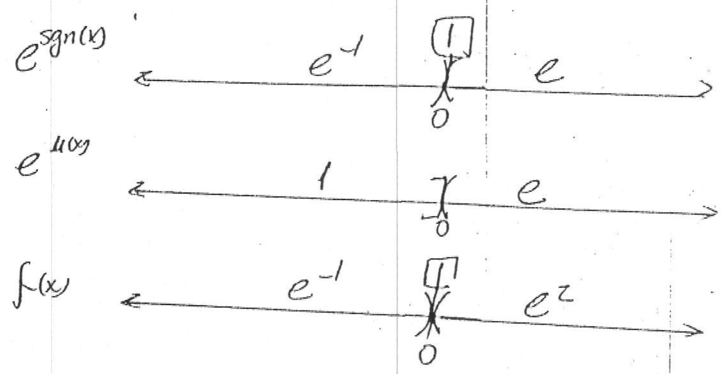
$\Rightarrow \frac{3^x}{2^x} = \frac{3}{2}$ $\qquad$ $\Rightarrow \frac{3^x}{2^x} = \frac{2}{3}$

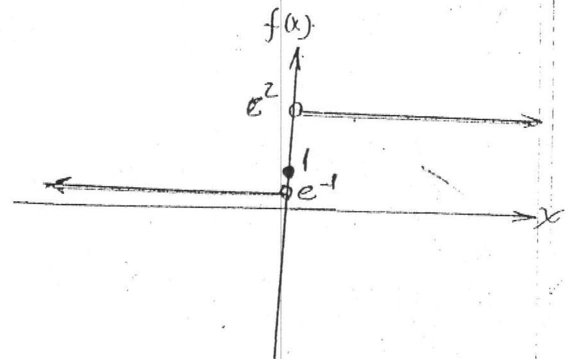
$\Rightarrow \cancel{\left(\frac{3}{2}\right)}^x = \cancel{\frac{3}{2}}$ $\qquad$ $\Rightarrow \cancel{\left(\frac{3}{2}\right)}^x = \cancel{\left(\frac{3}{2}\right)}^{-1}$

$x = 1$ $\qquad$ $x = -1$

104.- $f(x) = e^{sgn(x) + \mu(x)}$

$$f(x) = e^{sgn(x)} \cdot e^{\mu(x)}$$





a) FALSO; es creciente

b) FALSO

c) FALSO

d) VERDADERO

e) FALSO; rango $f: \{\frac{1}{e}, e^2\}$

105.-

a) $P(x): 4^x + 2^{x+1} = 8$

$(2^x)^2 + 2^x . 2 - 8 = 0$

$a = 2^x$

$a^2 + 2a - 8 = 0$

$(a+4)(a-2) = 0$

$a+4=0 \qquad a-2=0$

$\cancel{a=-4} \qquad a=2 \qquad 2^x = 2$

$\log_2 2^x = \log_2 2$

$x . \cancel{\log_2 2} = \cancel{\log_2} 2$

$x = \underline{\underline{1}}$

b) $h(x): 2^x + (0{,}5)^{2x-3} - 5(0{,}5)^{x-1} = -1$

$\Rightarrow 2^x + \left(\frac{1}{2}\right)^{2x}\left(\frac{1}{2}\right)^{-3} - 5\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{-1} = -1$

$\Rightarrow 2^x + \left[\left(\frac{1}{2}\right)^x\right]^2 \times 2^3 - 5\left(\frac{1}{2}\right)^x \times 2 = -1$

$\Rightarrow 2^x + \left(\frac{1}{2^x}\right)^2 \times 8 - 10 \times \frac{1}{2^x} + 1 = 0$

$\Rightarrow \qquad a = 2^x$

$\Rightarrow a + \frac{8}{a^2} - \frac{10}{a} + 1 = 0 \Rightarrow \frac{a^3 + 8 - 10a + a^2}{a^2} = 0$

$\Rightarrow a^3 + a^2 - 10a + 8 = 0 \Rightarrow (a^2 + 2a - 8)(a-1) = 0$

$\Rightarrow (a+4)(a-2)(a-1) = 0$

$a=4 \; ; \; a=2 \; ; \; a=1$

$\cancel{2^x=-4}$ $\quad$ $\cancel{2}^x=\cancel{2}^1$ $\quad$ $2^x=1$

$x=1$ $\quad$ $\cancel{2}^x=\cancel{2}^0$

$x=0$

c) $g(x): 16^x-6(4^x)=-8$

$\Rightarrow (4^x)^2-6(4^x)+8=0$

$a=4^x$

$\Rightarrow a^2-6a+8=0$

$\Rightarrow (a-4)(a-2)=0$

$a=4 \; ; \; a=2$

$4^x=2^1$ $\quad$ $\cancel{4}^x=\cancel{4}^1$

$\Rightarrow \cancel{2}^{2x}=\cancel{2}^1$ $\quad$ $x=1$

$2x=1$

$x=1/2$

d) $r(x)= 9^x+3^{x+1}-4=0$

$(3^x)^2+3^x\cdot 3-4=0$

$a=3^x$ $\quad \Rightarrow \cancel{3^x=-4} \; ; \; 3^x=1$

$a^2+3a-4=0$ $\quad \Rightarrow \cancel{3}^x=\cancel{3}^0$

$(a+4)(a-1)=0$ $\quad x=0$

$a=-4 \; ; \; a=1$

$g(x)$: $3^x + 9^x = 6642$

$\Rightarrow 3^x + (3^x)^2 - 6642 = 0$

$a = 3^x$

$a + a^2 - 6642 = 0$

$a^2 + a - 6642 = 0$

$(a+82)(a-81) = 0$

$a = -82 \quad a = 81$

$\cancel{3^x = 82} \quad 3^x = 81$

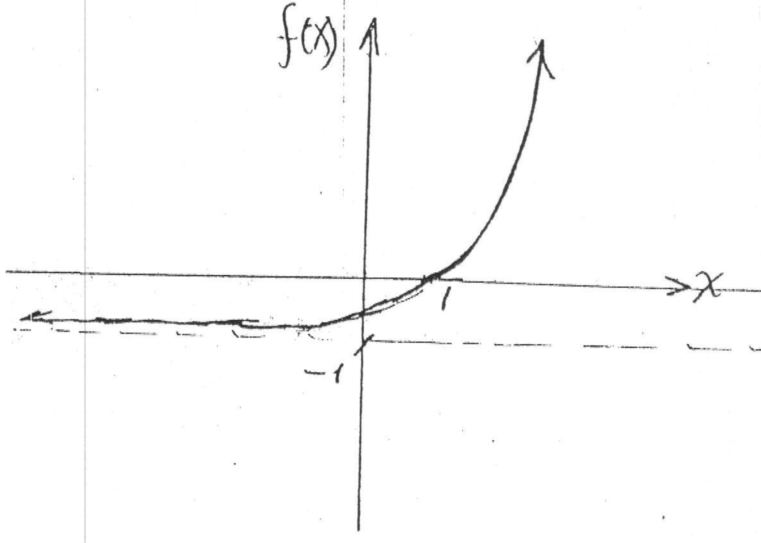
$\cancel{3}^x = \cancel{3}^4$

$x = 4$

6642	2
3321	3
1107	3
369	3
123	3
41	41
0	

106.-

$F(x) = e^{x-1} - 1$



a) FALSO

b) FALSO

c) VERDADERO

d) FALSO

e) FALSO

107.-

a) $\sqrt[3]{2^{\frac{3x-1}{x-1}}} < 8^{\frac{x-3}{3x-7}}$

$\cancel{2}^{\frac{1}{3}\cdot\left(\frac{3x-1}{x-1}\right)} < \cancel{2}^{3\cdot\frac{x-3}{3x-7}}$

$\frac{3x-1}{3x-3} < \frac{3x-9}{3x-7}$

$\frac{3x-1}{3x-3} - \frac{3x-9}{3x-7} < 0$

$\frac{(3x-1)(3x-7)-(3x-9)(3x-3)}{(3x-3)(3x-7)} < 0$

$9x^2-21x-3x+7-(9x^2-9x-27x+27)<0$

$\cancel{9x^2}-24x+7-\cancel{9x^2}+36x-27<0$

$12x-20<0$

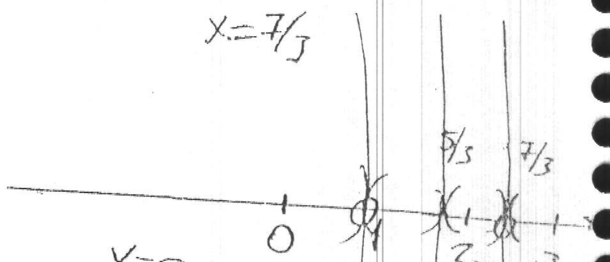
$\boxed{x<\frac{5}{3}}$

Pto críticos

$3x-3=0$ $\quad$ $3x-7=0$

$\boxed{x=1}$ $\quad$ $3x=7$

$x=7/3$



$x=0$

$\frac{1}{3}<\frac{9}{7}$

Si

$x=1,5$

$\frac{3,5}{0,5}<\frac{-4,5}{-2,5}$

NO

$x=2$

$\frac{5}{3}<$ ...

Si

Sol: $(-\infty,1)\cup\left(\frac{5}{3},7/3\right)$

b) $(0{,}04)^{5x-x^2} < 625$

$\Rightarrow \left(\frac{\cancel{4}\,1}{\cancel{100}\,25}\right)^{5x-x^2} < 5^4$

$\Rightarrow \left(\frac{1}{25}\right)^{5x-x^2} < 5^4 \Rightarrow \left(5^{-2}\right)^{5x-x^2} < 5^4$

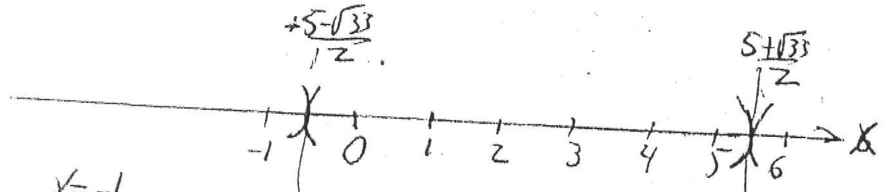
$\Rightarrow \cancel{5}^{-10x+2x^2} < \cancel{5}^{4} \Rightarrow 2x^2-10x<4$

$\Rightarrow 2(x^2-5x-2)<0$

$$x=\frac{5\pm\sqrt{25-4(1)(-2)}}{2(1)}$$

$\Rightarrow x^2-5x-2<0$

$$x=\frac{5\pm\sqrt{33}}{2}$$



$x=-1$

$(0{,}04)^{-6}<625$

$\Rightarrow \left(\frac{1}{25}\right)^{-6}<625$

$\Rightarrow 25^6<25^2$

NO

$x=0$

$(0{,}04)^0<625$

$1<625$

si

$x=6$

$(0{,}04)^{-6}<625$

$\left(\frac{1}{25}\right)^{-6}<25^2$

$25^6<25^2$

NO

Sol: $\left(\frac{5-\sqrt{33}}{2}, \frac{5+\sqrt{33}}{2}\right)$

c) $2^{x+2} - 2^{x+3} - 2^{x+4} \geq \cancel{5^{x+1}} - \cancel{5^{x+1}}$

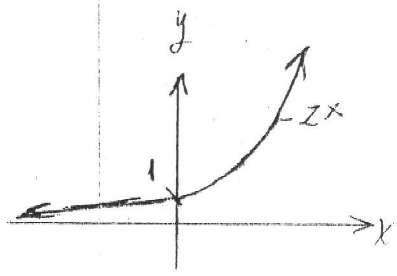
$\Rightarrow 2^x \cdot 2^2 - 2^x \cdot 2^3 - 2^x \cdot 2^4 \geq 0$

$\Rightarrow 2^2(2^x - 2\cdot 2^x - 2^2 \cdot 2^x) \geq 0$

$2^x - 2\cdot 2^x - 4\cdot 2^x \geq 0$

$5 \cdot 2^x \geq 0$

$2^x \geq 0$



Sol: $x \in \mathbb{R}$

d.) $\dfrac{1}{(0.5)^x - 1} - \dfrac{1}{1-(0.5)^{x+1}} \geq 0$

$\Rightarrow \dfrac{1}{\left(\frac{1}{2}\right)^x - 1} - \dfrac{1}{1 - \left(\frac{1}{2}\right)^{x+1}} \geq 0 \Rightarrow \dfrac{1}{\frac{1}{2^x} - 1} - \dfrac{1}{1 - \frac{1}{2^{x+1}}} \geq 0$

$\Rightarrow \dfrac{1}{\frac{1-2^x}{2^x}} - \dfrac{1}{\frac{2^{x+1}-1}{2^{x+1}}} \geq 0 \Rightarrow \dfrac{2^x}{1-2^x} - \dfrac{2^{x+1}}{2^{x+1}-1} \geq 0$

$\Rightarrow \dfrac{2^x(2^{x+1}-1) - 2^{x+1}(1-2^x)}{(1-2^x)(2^{x+1}-1)} \geq 0 \Rightarrow 2^x(2^x \cdot 2 - 1) - 2^x \cdot 2(1-2^x) \geq 0$

$\Rightarrow 2^x[(2\cdot 2^x - 1) - 2(1-2^x)] \geq 0 \quad \Rightarrow 2^x[2\cdot 2^x - 1 - 2 + 2\cdot 2^x] \geq 0$

$\Rightarrow 2^x[4\cdot 2^x - 3] \geq 0 \Rightarrow 2^x \geq 0$

$x \in \mathbb{R}$

$4 \cdot 2^x - 3 \geq 0$

$4 \cdot 2^x \geq 3$

$2^x \geq \frac{3}{4}$

$\log_2 2^x \geq \log_2(3/4)$

$x \geq \log_2(3/4)$

l) $0<8^x+18^x-2(27)^x$

$\Rightarrow 2^{3x}+(9\times2)^x-2(3^3)^x>0$

$\Rightarrow 2^{3x}+3^{2x}\times2^x-2\times3^{3x}>0$

$a=2^x \quad ; \quad b=3^x$

$\Rightarrow a^3+ab^2-2b^3>0$

$\Rightarrow (a^3-ab^2)+(2ab^2-2b^3)>0 \Rightarrow a(a^2-b^2)+2b^2(a-b)>0$

$\Rightarrow a(a+b)(a-b)+2b^2(a-b)>0 \Rightarrow (a-b)(a(a+b)+2b^2)>0$

$a-b>0 \qquad a(a+b)+2b^2>0$

$a>b$

$2^x>3^x$

$\frac{2^x}{3^x}>1 \Rightarrow \left(\frac{2}{3}\right)^x>1 \Rightarrow \cancel{\left(\frac{2}{3}\right)}^x>\cancel{\left(\frac{2}{3}\right)}^0$

$x>0$

108.-

a) $f\circ g=2^{\frac{x}{x-2}}$

b) $g(x)=\frac{x}{x-2}$

$\Rightarrow y=\frac{x}{x-2}$

$\Rightarrow y(x-2)=x$

$\Rightarrow yx-2y=x \Rightarrow$

$xy-x=2y$

$x(y-1)=2y$

$x=\frac{2y}{y-1}$

$f^{-1}(x)=\frac{2x}{x-1}$

dom; $x\in\mathbb{R}-\{1\}$

c) $g^{-1} \circ g$ $\quad g^{-1}(x) = \dfrac{2x}{x-1}$ $\quad g(x) = \dfrac{x}{x-2}$

$$g^{-1} \circ g = \frac{2\left(\frac{x}{x-2}\right)}{\frac{x}{x-2} - 1} \Rightarrow \frac{\frac{2x}{\cancel{x-2}}}{\frac{x-(x-2)}{\cancel{x-2}}} \Rightarrow \frac{\cancel{2}x}{\cancel{x-x+2}}$$

$g^{-1} \circ g = x$ dom $x \in \mathbb{R}$

109.- $\log(x) \cdot \log(x) \cdot \log(x) = 3\log(x)$

$\Rightarrow (\log(x))^3 \neq 3\log(x)$ b) Falso

110.- $p(x)$: $\ln(\log_x 2) = -1$

$e^{-1} = \log_x 2 \Rightarrow \log_x 2 = e^{-1}$

$\Rightarrow x^{e^{-1}} = 2 \Rightarrow x^{\frac{1}{e}} = 2 \Rightarrow (x^{\frac{1}{e}})^e = (2)^e$

$x = 2^e$ a) Verdadero

111 $\quad a < b \quad c \in \mathbb{R}^+ - \{1\}$

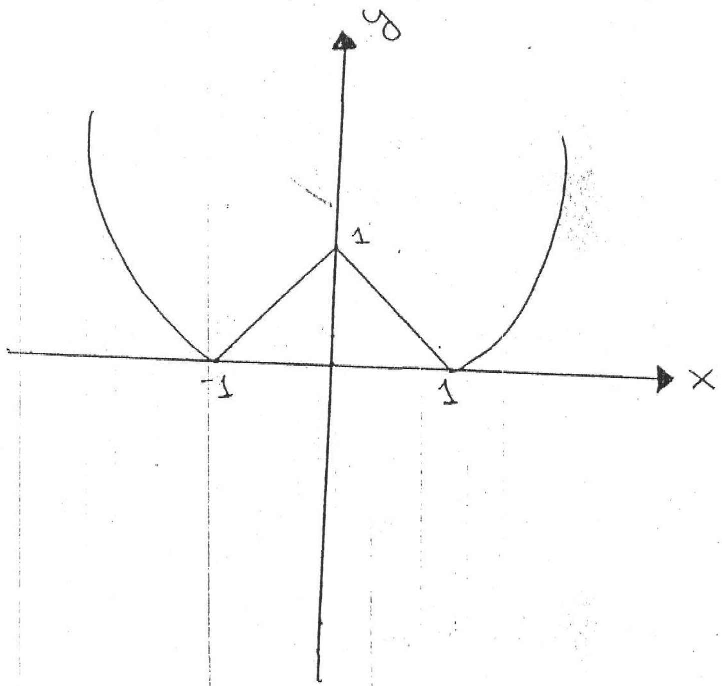
$\log_c a < \log_c b$

$\log_2 1 < \log_2 2 \quad a = 1 \quad b = c = 2$

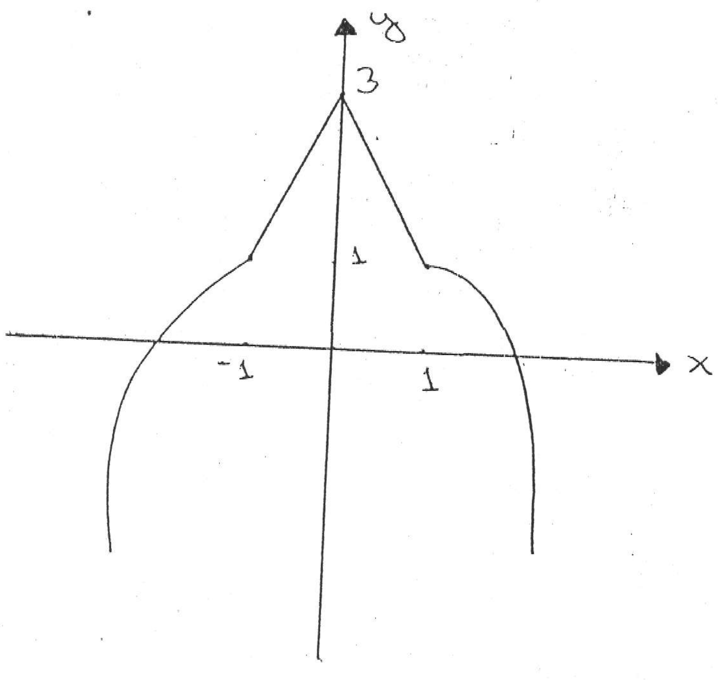
~~a)~~ Verdadero $\quad 0 < 1$

b) Falso

d) $y = |f(-|x|)|$



e) $y = 1 - 2f(|x|)$



$-2 > -5$ b) Falso

112.- $\rightarrow$ $(1,0)$ $(a,1)$

$\log_a 1 = 0$ $\log_a a = 1$

a) verdadero

113.- $\log 2 = a$ $\qquad \log 3 = b$

$$\log(75) = \log\left(\frac{300}{4}\right)$$

$$\Rightarrow \log 300 - \log 4$$

$$\Rightarrow \log(3 \times 100) - \log 2^2$$

$$\Rightarrow \log 3 + \log 100 - 2 \cdot \log 2$$

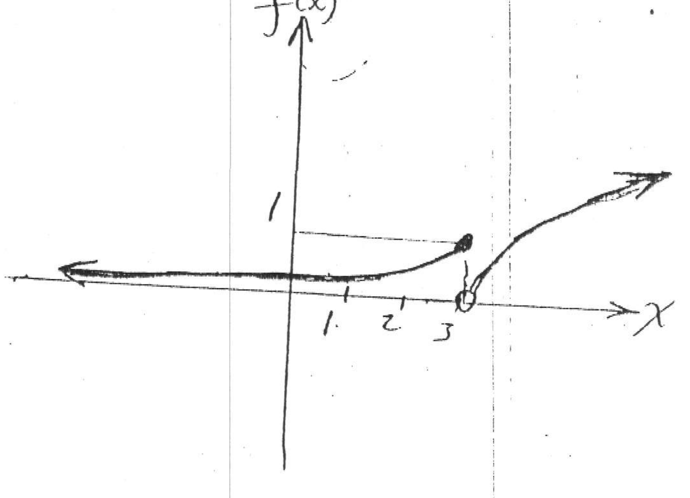
$$\Rightarrow \log 3 + \log 10^2 - 2\log 2$$

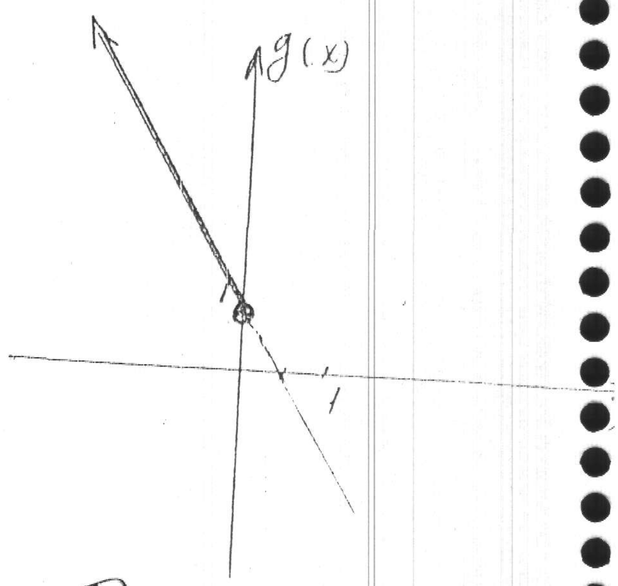
$$\Rightarrow b + 2 - 2a$$

e) correcto

114.- $f(x) = \begin{cases} 2^{x-3} & ; x \leq 3 \\ \log_3(x-2) & ; x \in (3, +\infty) \end{cases}$ $\qquad g(x) = 1 - 2x \; ; \; x \leq 0$

b





$y = \log_3(x-2)$

asintota

$x - 2 = 0$

$x = 2$

raiz

$x - 2 = 1$

$x = 3$

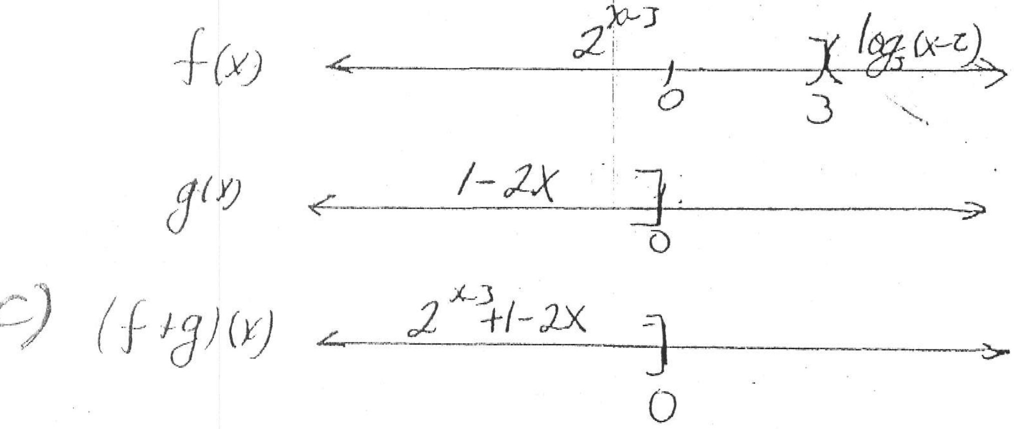
$y = 1 - 2x$

$x = 0$ $\quad y = 0$

$y = 1$ $\quad x = 1/2$

a) rango de f : $(0, +\infty)$

rango g : $[1, +\infty)$



$(f+g)(x) = 2^{x-3}+1-2x \quad ; x \leq 0$

15.- a) $p(x): \log(x-1) = \log\sqrt{5+x} + \log\sqrt{5-x}$

$\log(x-1) = \log[\sqrt{5+x}][\sqrt{5-x}]$

$\log(x-1) = \log\sqrt{25-x^2}$

$(x-1) = \sqrt{25-x^2}$

$(x-1)^2 = (\cancel{\sqrt{25-x^2}})^{\cancel{2}}$

$x^2-2x+1 = 25-x^2$

$x^2-2x+1+x^2-25=0$

$2x^2-2x-24=0$

$2(x^2-x-12)=0$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x-4=0$

$x=4$

$x+3=0$

$x=-3$ *

77

Pero $x=-3$ no satisface a $\log(x-1)$

$x=4$ es solución ya que:

$$\log(4-1) = \log\sqrt{5+4} + \log\sqrt{5-4}$$
$$\log 3 = \log\sqrt{9} + \log\sqrt{1}$$
$$\log 3 = \log 3 + \log 1 \qquad \log 1 \equiv 0$$
$$\log 3 = \log 3$$

$$A\,p(x) = \{4\}$$

b) $q(x)$: $\log(x^2-4) - \log(x+2) = 3\log(x-2)$

$$\log\frac{x^2-4}{x+2} = \log(x-2)^3$$

$$\frac{x^2-4}{x+2} = (x-2)^3 \qquad \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = (x-2)^3$$

$$\frac{(x-2)^{\cancel{3}2}}{\cancel{x-2}} = 1 \qquad (x-2)^2 = 1$$

$$x^2-4x+4 = 1 \qquad x^2-4x+4-1=0$$
$$x^2-4x+3=0 \qquad (x-3)(x-1)=0$$
$$x-3=0 \qquad x-1=0$$
$$x=3 \qquad x=1 \ *$$

Pero $x=1$ no satisface a $\log(x-2)$ ni a $\log(x^2-4)$

$x=3$ es solución ya que:

$$\log(3^2-4) - \log(3+2) = 3\log(3-2)$$
$$\log(9-4) - \log(3+2) = 3\log(3-2)$$
$$\log 5 - \log 5 = 3\log 1 \qquad \log 1 = 0$$
$$0 = 3(0) \qquad 0=0$$

$$A\,q(x) = \{3\}$$

c) $p(x): e^x - e^{-x} = 1$

$e^x(e^x - e^{-x}) = e^x(1)$

$e^{2x} - 1 = e^x$ $\qquad e^{2x} - e^x - 1 = 0$

* Hagamos $e^x = y$

$y^2 - y - 1 = 0$ $\qquad a = 1 \quad b = -1 \quad c = -1$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{1 \pm \sqrt{1+4}}{2} \qquad y = \frac{1 \pm \sqrt{5}}{2}$$

$$y_1 = \frac{1+\sqrt{5}}{2} \qquad y_2 = \frac{1-\sqrt{5}}{2}$$

* Luego reemplazamos $y = e^x$

$e^x = \dfrac{1+\sqrt{5}}{2}$ $\qquad \ln(e)^x = \ln\left(\dfrac{1+\sqrt{5}}{2}\right)$

$x \ln e = \ln\left(\dfrac{1+\sqrt{5}}{2}\right)$ $\qquad x_1 = \ln\left(\dfrac{1+\sqrt{5}}{2}\right)$

$e^x = \dfrac{1-\sqrt{5}}{2}$ $\qquad \ln(e)^x = \ln\left(\dfrac{1-\sqrt{5}}{2}\right)$

$x \ln e = \ln\left(\dfrac{1-\sqrt{5}}{2}\right)$ $\qquad x_2 = \ln\left(\dfrac{1-\sqrt{5}}{2}\right)$

x_2 no satisface ya que $\dfrac{1-\sqrt{5}}{2} < 0$.

d) $r(x): 5^{x} - 5^{-x} = 2$

$Ap(x) = \left\{ \ln\left(\frac{1+\sqrt{5}}{2}\right) \right\}$

$5^{x}(5^{x} - 5^{-x}) = 2(5^{x})$

$5^{2x} - 1 = 2(5^{x})$ $\qquad (5)^{2x} - 2(5)^{x} - 1 = 0$

* Hagamos $y = 5^{x}$

$y^2 - 2y - 1 = 0 \qquad a = 1 \qquad b = -2 \qquad c = -1$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$y = \frac{2 \pm \sqrt{4+4}}{2} \qquad y = \frac{2 \pm \sqrt{8}}{2}$$

$$y = \frac{2 \pm 2\sqrt{2}}{2} \qquad y = \frac{\cancel{2}(1 \pm \sqrt{2})}{\cancel{2}}$$

$y = 1 \pm \sqrt{2}$

$y_1 = 1 + \sqrt{2} \qquad y_2 = 1 - \sqrt{2}$

* Reemplazamos en $y = 5^{x}$

$5^{x} = 1 + \sqrt{2} \qquad \log_5 (5)^{x} = \log_5(1 + \sqrt{2})$

$x \log_5 5 = \log_5(1 + \sqrt{2})$

$x_1 = \log_5(1 + \sqrt{2})$

$^{x} = 1 - \sqrt{2}$ $\qquad$ $\log_5 (5)^x = \log_5 (1 - \sqrt{2})$

$\log_5 (5) = \log_5 (1 - \sqrt{2})$

$_2 = \log_5 (1 - \sqrt{2})$

no es solución ya que $1 - \sqrt{2} < 0$, y
argumento no puede ser negativo.

$A\, r(x) = \{ \log_5 (1 + \sqrt{2}) \}$

) $h(x)$: $5^{1+2x} + 6^{1+x} = 30 + 150^x$

$(5)^{2x} + 6(6)^x = 30 + (5^2 \cdot 6)^x$

$(5)^{2x} + 6(6)^x = 30 + (5)^{2x}(6)^x$

$(5)^{2x} - (5)^{2x}(6)^x = 30 - 6(6)^x$

$)^{2x} [5 - 6^x] = 6 [5 - 6^x]$

$5^{2x} = 6 \qquad 25^x = 6$

$og_{25} (25)^x = \log_{25} (6)$

$\log_{25} (25) = \log_{25} (6)$

$= \log_{25} (6)$

$A\, h(x) = \{ \log_{25} (6) \}$

116.- a) $10^{x^2+3x} = 200$

$\log(10)^{x^2+3x} = \log(200)$

$(x^2+3x)\log 10 = \log(100 \cdot 2)$

$x^2+3x = \log 100 + \log 2$

$x^2+3x = \log(10)^2 + \log 2$

$x^2+3x = 2\log 10 + \log 2$

$x^2+3x = 2 + \log 2$

$x^2+3x-2-\log 2 = 0$

$a = 1 \qquad b = 3 \qquad c = -2-\log 2$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2-4(1)(-2-\log 2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+8+4\log 2}}{2}$$

$$x = \frac{-3 \pm \sqrt{9+8+\log(2)^4}}{2}$$

$$x = \frac{-3 \pm \sqrt{17+\log 16}}{2}$$

b) $\ln(x+4) = 5y + \ln C$

$\ln(x+4) = \ln(e)^{5y} + \ln C$

$\ln(x+4) = \ln(e^{5y} \cdot C)$

$x+4 = e^{5y} \cdot C$

$x = Ce^{5y} - 4$

117.- a) $\log_2 4 \times \log_4 6 \times \log_6 8 = -3$

$\Rightarrow \cancel{\log_2 4} \times \dfrac{\cancel{\log_2 6}}{\cancel{\log_2 4}} \times \dfrac{\log_2 8}{\cancel{\log_2 6}}$

$\Rightarrow \log_2 8 \Rightarrow \log_2 2^3 \Rightarrow 3\cancel{\log_2 2}^{1}$

$\Rightarrow 3$; FALSO.

b) $(\log x)^n = n\log x$

$\Rightarrow \log x^n$; FALSO

c) $\ln(1+2+3) = \ln 1 + \ln 2 + \ln 3$

$\Rightarrow \ln(6) \neq \ln 1 + \ln 2 + \ln 3$; FALSO

d) $e^{\ln\sqrt{13}} = 13^{1/2}$

$\Rightarrow \cancel{e}^{\cancel{\log_e}\sqrt{13}} = \sqrt{13}$

$\Rightarrow \sqrt{13} = \sqrt{13}$; VERDADERO

e) $2^{\log_2 2} + \log_4\left(\frac{1}{16}\right) = 4$

$2 + \log_4 1^{0} - \log_4 16 \Rightarrow 2 + 0 - \log_4 4^2$

$\Rightarrow 2 - 2\cancel{\log_4 4}^{1}$

$\Rightarrow 2 - 2$

$\Rightarrow 0$

118.- a) $36^{\log_6 5} + 10^{1-\log 2} - 3^{\log_9 36}$

$\Rightarrow 6^{2\log_6 5} + 10 \times 10^{-\log 2} - 3^{2\times\frac{1}{2}\log_9 36}$

$\Rightarrow 6^{\log_6 5^2} + 10 \times 10^{\log 2^{-1}} - 9^{\log_9 36^{1/2}}$

$\Rightarrow 5^2 + 10 \times 2^{-1} - 36^{1/2} \Rightarrow 25 + \frac{10}{2} - \sqrt{36}$

$\Rightarrow 25 + 5 - 6 \quad \Rightarrow 24$ //

b) $81^{\frac{1}{\log_5 3}} - 27^{\log_9 36} - 3^{\frac{4}{\log_7 9}}$

$\frac{1}{\log_5 3} \Rightarrow \frac{1}{\frac{\log_3 3}{\log_3 5}} \Rightarrow \log_3 5$

$\log_9 36 \Rightarrow \frac{\log_3 36}{\log_3 9} \Rightarrow \frac{\log_3 36}{\log_3 3^2} \Rightarrow \frac{\log_3 36}{2\cancel{\log_3 3}} \Rightarrow \frac{\log_3 36}{2}$

$\Rightarrow \frac{1}{2}\log_3 36 \Rightarrow \log_3 36^{1/2} \Rightarrow \log_3 6$

$\frac{4}{\log_7 9} \Rightarrow \frac{4}{\frac{\log_3 9}{\log_3 7}} \Rightarrow \frac{4\log_3 7}{\log_3 9} \Rightarrow \frac{4\log_3 7}{\log_3 3^2} \Rightarrow \frac{\overset{2}{\cancel{4}}\log_3 7}{\underset{1}{\cancel{2}}\cancel{\log_3 3}}$

$\Rightarrow 2\log_3 7 \Rightarrow \log_3 7^2 \Rightarrow \log_3 49$

$\Rightarrow 81^{\log_3 5} - 27^{\log_3 6} - 3^{\log_3 49} \Rightarrow 3^{4\log_3 5} - 3^{3\log_3 6} - 3^{\log_3 49}$

$\Rightarrow 3^{\log_3 5^4} - 3^{\log_3 6^3} - 49 \Rightarrow 5^4 - 6^3 - 49 \Rightarrow 625 - 216 - 49$

$\Rightarrow 360$ //

c) $\log_8 \log_4 (\log_2 16)$

$\log_2 16 \Rightarrow \log_2 2^4 \Rightarrow 4 \cancel{\log_2 2}^1 \Rightarrow 4$

$\Rightarrow \log_8 \cancel{\log_4 4}^1$

$\Rightarrow \log_8 1 \Rightarrow \underline{0}$

d) $2 - \log_2 \log_3 \sqrt{\sqrt[4]{3}}$

$\sqrt{\sqrt[4]{3}} \Rightarrow ((3)^{1/4})^{1/2}$

$\Rightarrow 3^{1/8}$

$\Rightarrow 2 - \log_2 (\log_3 3^{1/8})$

$\log_3 3^{1/8} \Rightarrow \frac{1}{8} \cancel{\log_3 3}^1$

$\Rightarrow 1/8$

$\Rightarrow 2 - \log_2 (\frac{1}{8})$

$\Rightarrow 2 - (\log_2 1 - \log_2 8) \Rightarrow 2 - (0 - \log_2 2^3)$

$\Rightarrow 2 - 0 + 3 \cancel{\log_2 2}^1$

$\Rightarrow 5$

e) $1 + \log(\log \sqrt{\sqrt[5]{10}})$

$\sqrt{\sqrt[5]{10}} \Rightarrow ((10)^{\frac{1}{5}})^{\frac{1}{2}}$

$\Rightarrow 10^{1/10}$

$\Rightarrow 1 + \log(\log 10^{\frac{1}{10}})$

$\Rightarrow \log_{10} 10^{1/10} \Rightarrow \frac{1}{10} \cancel{\log_{10} 10}^1 \Rightarrow \frac{1}{10}$

$\Rightarrow 1 + \log(\frac{1}{10}) \Rightarrow 1 - (\log 1 - \log 10) \Rightarrow 1 - (0 - 1)$

$\Rightarrow 2$

f) $\log\left(11-\log_{\frac{1}{3}}\sqrt{3}\cdot\log_{\sqrt{3}}\left(\frac{1}{\sqrt{3}}\right)\right)$

$\log_{\sqrt{3}}1-\log_{\sqrt{3}}\sqrt{3}$

$\Rightarrow 0-1$

$\Rightarrow -1$

$\log_{\frac{1}{3}}(\sqrt{3})$

$\Rightarrow \dfrac{\log_3\sqrt{3}}{\log_3\left(\frac{1}{3}\right)} \Rightarrow \dfrac{\log_3 3^{1/2}}{\log_3 1-\log_3 3}$

$\Rightarrow \dfrac{\frac{1}{2}\cancel{\log_3 3}^{1}}{0-1} \Rightarrow -\frac{1}{2}$

$\Rightarrow \log\left(11-\left(-\frac{1}{2}\right)(-1)\right) \Rightarrow \log\left(11-\frac{1}{2}\right)$

$\Rightarrow \log\frac{21}{2}$

g) $\log_3 7\cdot\log_7 5\cdot\log_5 4+1$

$\Rightarrow \cancel{\log_3 7}\cdot\dfrac{\cancel{\log_3 5}}{\cancel{\log_3 7}}\cdot\dfrac{\log_3 4}{\cancel{\log_3 5}}+1$

$\Rightarrow \log_3 4+1$

119.- $\dfrac{1}{\log_a n}+\dfrac{1}{\log_{a^2} n}+\dfrac{1}{\log_{a^3} n}+\dfrac{1}{\log_{a^4} n}+\dfrac{1}{\log_{a^5} n}=15\log_n a$

$\Rightarrow \dfrac{\log_n a}{\cancel{\log_n n}\,1}+\dfrac{\log_n a^2}{\cancel{\log_n n}\,1}+\dfrac{\log_n a^3}{\cancel{\log_n n}\,1}+\dfrac{\log_n a^4}{\cancel{\log_n n}\,1}+\dfrac{\log_n a^5}{\cancel{\log_n n}\,1}$

$$\Rightarrow \log_n a + 2\log_n a + 3\log_n a + 4\log_n a + 5\log_n a$$

$$\Rightarrow 15\log_n a$$

120.-

a) $\log_2 5 = a$ $\quad \log_{100} 40 \times \frac{10}{10} \Rightarrow \log_{100} \frac{400}{10}$

$$\Rightarrow \log_{100} 400 - \log_{100} \sqrt{10^2}$$

$$\Rightarrow \log_{100}(4 \times 100) - \log_{100} \sqrt{100}$$

$$\Rightarrow \log_{100} 4 + \cancel{\log_{100}} 100 - \frac{1}{2} \cancel{\log_{100}} 100$$

$$\Rightarrow \log_{100} 4 + 1 - \frac{1}{2}$$

$$\Rightarrow \frac{\log_2 4}{\log_2 100} + \frac{1}{2} \Rightarrow \frac{\log_2 2^2}{\log_2 (25 \times 4)} + \frac{1}{2}$$

$$\Rightarrow \frac{2\log_2 2}{\log_2 25 + \log_2 4} + \frac{1}{2} \Rightarrow \frac{2}{\log_2 5^2 + \log_2 2^2} + \frac{1}{2}$$

$$\Rightarrow \frac{+2}{2\log_2 5 + 2\log_2 2} + \frac{1}{2} \Rightarrow \frac{2}{2a+2} + \frac{1}{2}$$

$$\Rightarrow \frac{2}{2(a+1)} + \frac{1}{2} \Rightarrow \frac{2+a+1}{2(a+1)}$$

$$\Rightarrow \frac{3+a}{2(a+1)}$$

b) $\log_6 2 = a \wedge \log_6 5 = b$

$$\log_3 5 \Rightarrow \frac{\log_6 5}{\log_6 3} \Rightarrow \frac{b}{\log_6\left(\frac{6}{2}\right)} \Rightarrow \frac{b}{\cancel{\log_6 6}^{1} - \log_6 2}$$

$$\Rightarrow \frac{b}{1-a}$$

c) $\log_3 20 \wedge \log_3 15 = b$

$$\log_2 360 \Rightarrow \frac{\log_3(360)}{\log_3 2} \Rightarrow \frac{\log_3(40 \times 9)}{\log_3 2} \Rightarrow \frac{\log_3 40 + \log_3 9}{\log_3 2}$$

$$\Rightarrow \frac{\log_3(2 \times 20) + \log_3 3^2}{\log_3 2} \Rightarrow \frac{\log_3 2 + \overbrace{\log_3 20}^{a} + 2\,\cancel{\log_3 3}^{1}}{\log_3 2} \Rightarrow \frac{\log_3 2 + a + 2}{\log_3 2}$$

$$\Rightarrow \frac{\frac{a-b+1}{2} + a + 2}{\frac{a-b+1}{2}} \Rightarrow \frac{\frac{a-b+1+2a+4}{\cancel{2}}}{\frac{a-b+1}{\cancel{2}}} \Rightarrow \frac{3a-b+5}{a-b+1}$$

$\log_3 225 = \log(25 \times 9)$

$\log_3 15^2 = \log_3 5^2 + \log_3 3^2$

$\cancel{2}\log_3 15 = \cancel{2}\log_3 5 + \cancel{2}\,\cancel{\log_3 3}^{1}$

$b = \log_3 5 + 1$

$\log_3 5 = b - 1$

$a = \log_3 20$

$a = \log_3(5 \times 4)$

$a = \log_3 5 + \log_3 2^2$

$a = b - 1 + 2\log_3 2$

$\frac{a-b+1}{2} = \log_3 2$

d) $\log_6(28(6^{1-2a}))$ $\log_6 2 = \frac{a}{4}$. $\log_6 7 = \frac{3a}{2}$

$\Rightarrow \log_6 28 + \log_6 6^{1-2a}$

$\Rightarrow \log_6(7\times 4) + (1-2a)\cancel{\log_6 6}^{1} \Rightarrow \log_6 7 + \log_6 4 + 1-2a$

$\Rightarrow \frac{3a}{2} + \log_6 2^2 + 1 - 2a \Rightarrow 1 - \frac{a}{2} + 2\log_6 2$

$\Rightarrow 1 - \frac{a}{2} + 2\left(\frac{a}{4}\right) \Rightarrow 1 - \cancel{\frac{a}{2}} + \cancel{\frac{a}{2}}$

$\Rightarrow 1$

e) $\log_{m^2}(mn^3)$ $\therefore \log_a m = x$; $\log_a n = y$

$\Rightarrow \frac{\log_a(mn^3)}{\log_a m^2} \Rightarrow \frac{\log_a m + \log_a n^3}{2\log_a m} \Rightarrow \frac{x+3y}{2x}$

21.- $3^x(4^{2x+1}) = 6^{x+2}$

$\ln(3^x(4^{2x+1})) = \ln 6^{x+2}$

$\ln 3^x + \ln 4^{2x+1} = (x+2)\cdot \ln 6$

$x\cdot \ln 3 + (2x+1)\ln 4 = (x+2)\ln 6$

$x\ln 3 + 2x\ln 4 + \ln 4 = x\ln 6 + 2\ln 6$

$x\ln 3 + x\ln 4^2 - x\ln 6 = 2\ln 6 - \ln 4$

$x(\ln 3 + \ln 16 - \ln 6) = \ln 6^2 - \ln 4$

$x\ln\left(\frac{3\times 16}{6}\right) = \ln\left(\frac{36}{4}\right) \rightarrow x\ln 8 = \ln 9$

$x = \frac{\ln 9}{\ln 8}$

122.- $\log_{a^2}\sqrt{108}$ $\qquad \log_a 2 = \frac{5}{2}$; $\log_a 3 = \frac{1}{3}$

$$\Rightarrow \frac{\log_a \sqrt{108}}{\log_a a^2} \Rightarrow \frac{\log_a 108^{1/2}}{2\cancel{\log_a a}^1} \Rightarrow \frac{\frac{1}{2}\log_a(12 \times 9)}{2}$$

$$\Rightarrow \frac{1}{4}\log_a(3 \times 2^2 \times 3^2) \Rightarrow \frac{1}{4}\log_a(3^3 \times 2^2) \Rightarrow \frac{1}{4}\log_a(3^3) + \frac{1}{4}\log_a(2^2)$$

$$\Rightarrow \frac{3}{4}\log_a 3 + \frac{2}{4}\log_a 2 \Rightarrow \frac{\cancel{3}}{4}\left(\frac{1}{\cancel{3}}\right) + \frac{1}{2}\left(\frac{5}{2}\right)$$

$$\Rightarrow \frac{1}{4} + \frac{5}{4} \Rightarrow \frac{\cancel{6}^3}{\cancel{4}_2} = 3/2$$

123) $2^x + \left(\frac{1}{2}\right)^{2x-3} - 6\left(\frac{1}{2}\right)^x = 1$

$2^x + (2)^{3-2x} - 6(2)^{-x} = 1$

$2^x + \frac{8}{2^{2x}} - \frac{6}{2^x} = 1 \qquad * \boxed{2^x = a} *$

$a + \frac{8}{a^2} - \frac{6}{a} = 1 \qquad a^3 + 8 - 6a = a^2$

$a^3 - a^2 - 6a + 8 = 0$

$$\begin{array}{l|l}
a^3 - a^2 - 6a + 8 & a - 2 \\ \hline
-a^3 + 2a^2 & a^2 + a - 4 \\
\quad a^2 - 6a & \\
\quad -a^2 + 2a & \\
\qquad -4a + 8 & \\
\qquad 4a - 8 & \\
\qquad // \ // &
\end{array}$$

$= (a-2)(a^2 + a - 4) = 0$

$\boxed{a_1 = 2}$

Resolviendo $a^2 + a - 4$ $\qquad a = 1 \quad b = 1 \quad c = -4$

$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad a = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$

$a = \frac{-1 \pm \sqrt{1 + 16}}{2} \qquad a = \frac{-1 \pm \sqrt{17}}{2}$

$\boxed{a_2 = \frac{-1 + \sqrt{17}}{2}} \qquad \boxed{a_3 = \frac{-1 - \sqrt{17}}{2}}$

Reemplazo en $* \boxed{2^x = a} *$

Con $a = \frac{-1+\sqrt{17}}{2}$

$2^x = a \qquad 2^x = \frac{-1+\sqrt{17}}{2} \qquad \log_2(2)^x = \log_2\left(\frac{-1+\sqrt{17}}{2}\right)$

$x \log_2(2) = \log_2\left(\frac{-1+\sqrt{17}}{2}\right) \qquad \boxed{x = \log_2\left(\frac{-1+\sqrt{17}}{2}\right)}$

Con $a = \frac{-1-\sqrt{17}}{2}$

$2^x = a \qquad 2^x = \frac{-1-\sqrt{17}}{2} \qquad \log_2(2)^x = \log_2\left(\frac{-1-\sqrt{17}}{2}\right)$ Imposible

124.- una de las alternativas no es correcta, identifiquela

a) $F(x) = \ln(x-1)$

$\text{dom } F = (1, \infty)$

verdadero ya que $x-1 > 0 \longrightarrow x > 1$

b) $F(x) = \frac{1}{\log_{x+1} 2}$ $\quad \text{dom } F = (-1, 0) \cup (0, \infty)$

verdadero ya que $x+1 > 0$ y $x \neq 0$

c) $F(x) = \frac{1}{|x| - x}$ $\quad \text{dom } F = \mathbb{R} - \{0\}$

$\boxed{\text{Falso}}$ ya que $|x| - x = 0$

d) $F(x) = \frac{x^8 - x^3 + x - \sqrt{2}}{\log(x^2+1)}$; $\text{dom } F = \mathbb{R} - \{0\}$

verdadero ya que $x^2+1 \neq 1$

e) $F(x) = \log(x[u(x)])$; $\text{dom } F = \mathbb{R}^+$

verdadero ya que $u(x) = 1 \wedge x > 0$

125.-

a) $5 > 3$

b) $5 \ln\left(\frac{1}{2}\right) > 3 \ln(1/2)$

ERROR; En una desigualdad al multiplicar por un número negativo $(\ln 1/2)$; entonces varía el símbolo.

126.-

$f(x) = 2e^{x-3}$

$y = 2e^{x-3}$

$\Rightarrow \ln y = \ln(2e^{x-3})$

$\Rightarrow \ln y = \ln 2 + \ln e^{x-3}$

$\Rightarrow \ln y - \ln 2 = (x-3)\cancel{\ln e}^{1}$

$\Rightarrow \ln\left(\frac{y}{2}\right) + 3 = x$

$\Rightarrow f(x)^{-1} = \ln\left(\frac{x}{2}\right) + 3$

127.-

$f(x) = \begin{cases} e^x & ; x > 0 \\ 1 - x^2 & ; x \leq 0 \end{cases}$ rango: $(1, +\infty)$

$y = e^x$

$\ln y = \ln(e^x)$

$\ln y = x \cdot \cancel{\ln e} \Rightarrow f(x)^{-1} = \ln x \; ; x > 1$

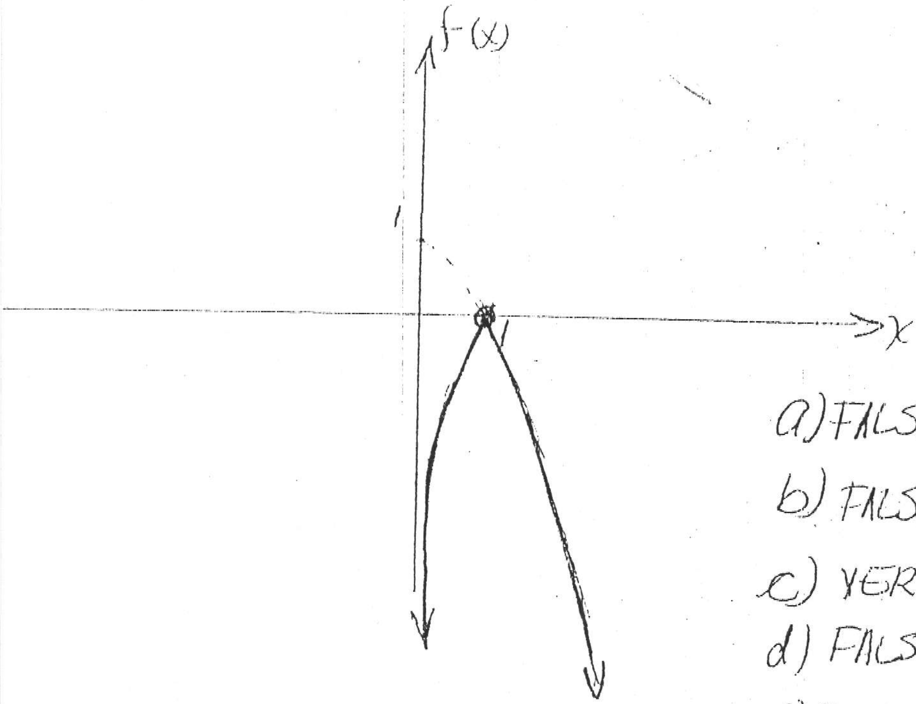
$y = 1 - x^2$

$x^2 = 1 - y$

$x = \sqrt{1-y} \Rightarrow f(x)^{-1} = \sqrt{1-y} \; ; x \leq$

c) correcto

128.- $f(x)=\begin{cases} 1-x^2 & ; x\geq 1 \\ \ln x & ; x<1 \end{cases}$



a) FALSO

b) FALSO

c) VERDADERO

d) FALSO

e) FALSO

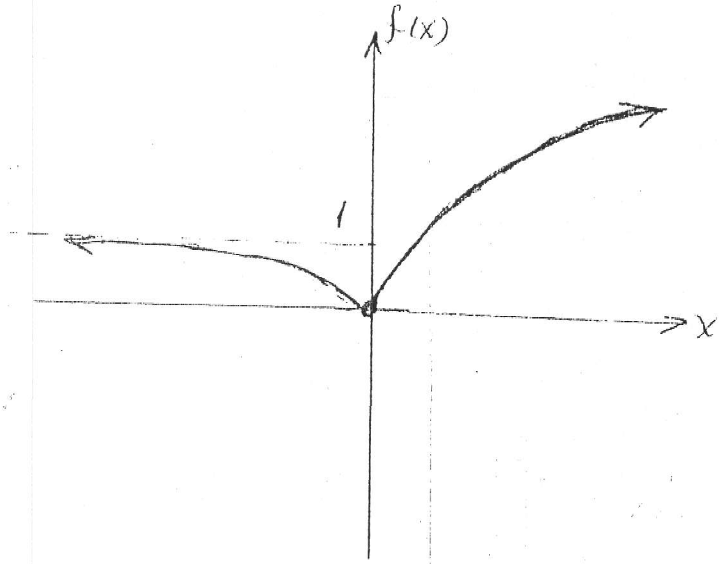
129.- $f(x)=\begin{cases} \ln(x+1) & ; x\geq 0 \\ 1-e^x & ; x<0 \end{cases}$

$y=\ln(x+1)$

asíntota

$x+1=0$

$x=-1$



130.- $\log_a(MN) = \log_a(M) + \log_a(N)$

$x = \log_a M \quad ; \quad y = \log_a N$

$M = a^x \quad ; \quad N = a^y$

$M \times N = a^x \cdot a^y$

$MN = a^{x+y}$

$\log_a(MN) = x+y \Rightarrow \log_a(MN) = \log_a M + \log_a N$

131.- a) $p(x)$: $\log(x+4) + \log(2x+3) = \log(1-2x)$

$\cancel{\log}[(x+4)(2x+3)] = \cancel{\log}(1-2x)$

$2x^2+3x+8x+12 = 1-2x$

$2x^2+11x+2x+12-1=0$

$2x^2+13x+11=0$

$(2x+11)(2x+2)=0$

$\cancel{x=-11/2}$ $\quad x=-1$

NO

b) $p(x)$: $\ln\left(\frac{x}{x-1}\right) + \ln\left(\frac{x+1}{x}\right) - \ln(x^2-1) + 2 = 0$

$\ln\left(\frac{\frac{\cancel{x}}{x-1} \times \frac{x+1}{\cancel{x}}}{\frac{x^2-1}{1}}\right) = -2 \Rightarrow \ln\left(\frac{(x+1)}{(x-1)(x^2-1)}\right) = -2$

$\Rightarrow \ln\left(\frac{\cancel{x+1}}{(x-1)(x-1)\cancel{(x+1)}}\right) = -2 \Rightarrow e^{-2} = \frac{1}{(x-1)^2}$

$\Rightarrow \frac{1}{e^2} = \frac{1}{(x-1)^2} \Rightarrow \sqrt{(x-1)^2} = \sqrt{e^2}$

$x-1 = {}^{+}e$

$x = 1^{+}e$

c) $r(x): (\log_2 x)^2 = \log_2 x^2$

$a = \log_2 x$

$a^2 = 2a$

$a^2 - 2a = 0$

$a(a-2) = 0$

$\{a = 0 \vee a = 2\}$

$\log_2 x = 0 \qquad \log_2 x = 2$

$x = 2^0 \qquad x = 2^2$

$x = 1 \qquad x = 4$

d) $h(x): \cancel{\log_3}(x^2 - 3x - 5) = \cancel{\log_3}(7 - 2x)$

$x^2 - 3x - 5 = 7 - 2x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\cancel{x = 4} \vee x = -3$

e) $m(x) : \log_2 (9^{x-1}+7) = 2 + \log_2 (3^{x-1}+1)$

$\log_2 (9^{x-1}+7) - \log_2 (3^{x-1}+1) = 2$

$\Rightarrow \log_2 \left(\frac{9^{x-1}+7}{3^{x-1}+1}\right) = 2$

$\frac{9^{x-1}+7}{3^{x-1}+1} = 4 \Rightarrow 9^{x-1}+7 = 4(3^{x-1}+1)$

$\Rightarrow 9^{x-1}+7 = 4(3^{x-1})+4$

$\Rightarrow 9^{x-1} - 4(3^{x-1}) + 3 = 0$

$\Rightarrow 9^{x} \cdot 9^{-1} - 4(3^{x-1}) + 3 = 0 \Rightarrow 3^{2x} \cdot 3^{-2} - 4(3^{x-1}) + 3 = 0$

$\Rightarrow 3^{2x-2} - 4(3^{x-1}) + 3 = 0 \Rightarrow 3^{2(x-1)} - 4(3^{x-1}) + 3 = 0$

$a = 3^{x-1}$

$\Rightarrow a^2 - 4a + 3 = 0 \Rightarrow (a-3)(a-1) = 0 \Rightarrow \boxed{a=3} \wedge \boxed{a=1}$

$\cancel{3}^{x-1} = \cancel{3}^1$

$x-1 = 1$

$x = 2$

$3^{x-1} = 1$

$\Rightarrow \cancel{3}^{x-1} = \cancel{3}^0$

$x-1 = 0$

$x = 1$

f) $p(x): \log_5(5^{1/x}+125) = \log_5 6 + 1 + \frac{1}{2x}$

$\Rightarrow \log_5(5^{1/x}+125) - \log_5 6 = \frac{2x+1}{2x}$

$\Rightarrow \log_5\left(\frac{5^{1/x}+125}{6}\right) = \frac{2x+1}{2x}$

$\Rightarrow 5^{\frac{2x+1}{2x}} = \frac{5^{1/x}+125}{6} \Rightarrow 6 \times 5^{1+\frac{1}{2x}} = 5^{\frac{1}{x}}+125$

$\Rightarrow (6 \times 5)\, 5^{\frac{1}{2x}} = 5^{\frac{1}{x}}+125 \Rightarrow 30(5^{\frac{1}{x}})^{\frac{1}{2}} - 5^{\frac{1}{x}} - 125 = 0$

$a = 5^{1/x}$

$b^2 = a$

$b = a^{1/2}$

$30a^{1/2} - a - 125 = 0$

$-b^2 + 30b - 125 = 0$

$b^2 - 30b + 125 = 0 \Rightarrow (b-25)(b-5) = 0$

$b = 25 \quad \wedge \quad b = 5$

$(a^{1/2})^2 = (25)^2 \quad \wedge \quad (a^{1/2})^2 = (5)^2$

$a = 625 \quad \wedge \quad a = 25$

$\cancel{5}^{\frac{1}{x}} = \cancel{5}^4 \qquad \cancel{5}^{\frac{1}{x}} = \cancel{5}^2$

$\frac{1}{x} = 4 \qquad \frac{1}{x} = 2$

$x = \frac{1}{4} \qquad x = \frac{1}{2}$

g) $q(x)$: $\log^2 x + \log x + 1 = \dfrac{7}{\log\left(\frac{x}{10}\right)}$

$$\Rightarrow \log^2(x) + \log x + 1 - \frac{7}{\log x - \cancel{\log} 10} = 0$$

$$\Rightarrow \log^2(x) + \log x + 1 - \frac{7}{\log x - 1} = 0 \qquad a = \log x$$

$$\Rightarrow a^2 + a + 1 - \frac{7}{a-1} = 0 \Rightarrow \frac{a^2(a-1) + (a+1)(a-1) - 7}{(a-1)} = 0$$

$$\Rightarrow a^3 \cancel{-a^2} \cancel{+a^2} - 1 - 7 = 0 \Rightarrow a^3 - 8 = 0$$

$$a^3 = 8$$

$$\boxed{a = 2}$$

$$\log x = 2$$

$$x = 10^2$$

$$x = 100$$

h) $r(x)$: $\log^2 x^3 - \log(0{,}1\, x^{10}) = 0$

$$\Rightarrow (\log x^3)^2 - \log\left(\frac{x^{10}}{10}\right) = 0$$

$$\Rightarrow (3\log x)^2 - (\log x^{10} - \cancel{\log} 10) = 0$$

$$\Rightarrow 9\log^2 x - 10\log x + 1 = 0$$

$$a = \log x$$

$$\Rightarrow 9a^2 - 10a + 1 = 0$$

$$(9a - 9)(9a - 1) = 0$$

$$\boxed{a = 1} \qquad \boxed{a = 1/9}$$

$$\log x = 1 \qquad \log x = 1/9$$

$$x = 10 \qquad x = 10^{1/9}$$

i.) $m(x): \log_{\frac{1}{2}x} x^2 - 14\log_{16x} x^3 + 40\log_{4x}\sqrt{x} = 0$

$$\Rightarrow \frac{\log_x x^2}{\log_x\left(\frac{x}{2}\right)} - 14\,\frac{\log_x x^3}{\log_x(16x)} + 40\,\frac{\log_x\sqrt{x}}{\log_x(4x)} = 0$$

$$\Rightarrow \frac{\log_x x^2}{\log_x x - \log_x 2} - \frac{14\log_x x^3}{\log_x 16 + \log_x x} + 40\,\frac{\log_x x^{1/2}}{\log_x 4 + \log_x x} = 0$$

$$\Rightarrow \frac{2\log_x x}{\log_x x - \log_x 2} - \frac{42\log_x x}{\log_x 2^4 + \log_x x} + 40\cdot\frac{1}{2}\,\frac{\log_x x}{\log_x 2^2 + \log_x x} = 0$$

$$a = \log_x x \quad ; \quad b = \log_x 2$$

$$\Rightarrow \frac{2a}{a-b} - \frac{42a}{a+4b} + \frac{20a}{a+2b} = 0$$

$$\Rightarrow \frac{2a(a+2b)(a+4b) - 42a(a-b)(a+2b) + 20a(a-b)(a+4b)}{(a-b)(a+4b)(a+2b)} = 0$$

$$2a\left[(a+2b)(a+4b) - 21(a-b)(a+2b) + 10(a-b)(a+4b)\right] = 0$$

$$2a\left[a^2+4ab+2ab+8b^2 - 21(a^2+2ab-ab-2b^2) + 10(a^2+4ab-ab-4b^2)\right] = 0$$

$$2a\left[a^2+6ab+8b^2 - 21a^2 - 21ab + 42b^2 + 10a^2 + 30ab - 40b^2\right] = 0$$

$$2a(-10a^2+15ab+10b^2) = 0$$

$$-10a(2a^2-3ab-2b^2) = 0 \Rightarrow \boxed{a=0} \quad (2a-4b)(2a+b) = 0$$

$$\boxed{a=2b} \quad \boxed{a=-b/2}$$

$\log_x x = 0$

$x = 1$

$\log_x x = 2\log_x 2$

$\log_x x = \log_x 2^2$

$x = 4$

$\log_x x = -\frac{1}{2}\log_x 2$

$\cancel{\log_x} x = \cancel{\log_x} 2^{-1/2}$

$x = 2^{-1/2}$

$x = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\sqrt{2}}{2}$

132.- $p(x): \log_{\frac{1}{4}} x - \dfrac{1}{\log_{\frac{1}{4}} x} - \dfrac{3}{2} = 0$

$a = \log_{\frac{1}{4}} x$

$\Rightarrow a - \dfrac{1}{a} - \dfrac{3}{2} = 0 \Rightarrow \dfrac{2a^2 - 2 - 3a}{2a} = 0$

$\Rightarrow 2a^2 - 3a - 2 = 0 \Rightarrow (2a - 4)(2a + 1) = 0$

$2a = 4 \qquad 2a = -1$

$\boxed{a = 2} \qquad \boxed{a = -1/2}$

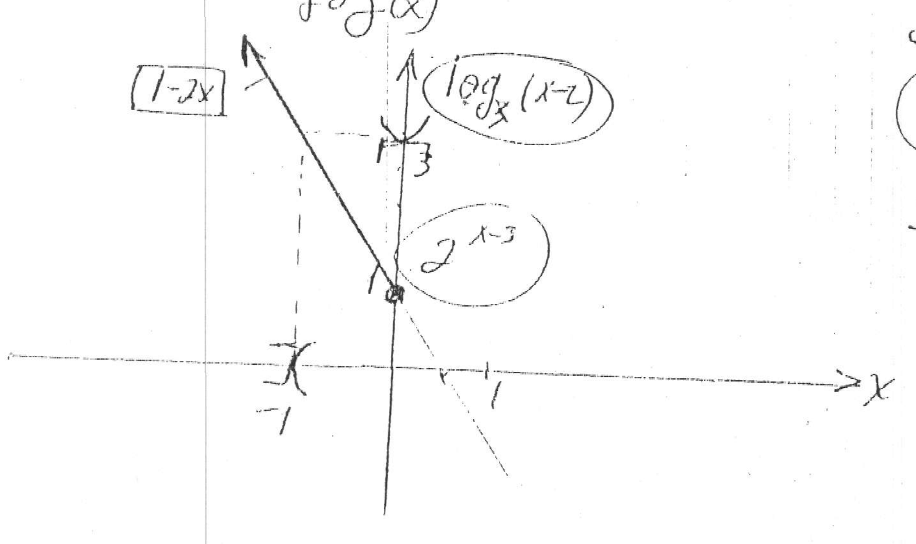
$\log_{\frac{1}{4}} x = 2 \qquad \log_{\frac{1}{4}} x = -\frac{1}{2}$

$x = \left(\frac{1}{4}\right)^2 \qquad x = \left(\frac{1}{4}\right)^{-1/2} \Rightarrow x = (4)^{1/2}$

$x = \dfrac{1}{16} \qquad x = 2$

Suma : $\dfrac{33}{16}$

133.- $f(x) = \begin{cases} 2^{x-3} & ; x \leq 3 \\ \log_3(x-2) & ; x > 3 \end{cases} \qquad g(x) = 1 - 2x \quad ; x \leq 0$



$y = 1 - 2x$

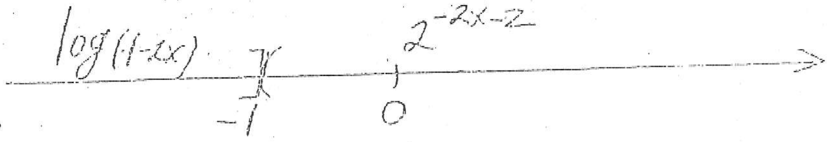
$x = 0$
$y = 1$

$y = 0$
$x = 1/2$

$3 = 1 - 2x$

$2x = 1 - 3$

$x = -1$



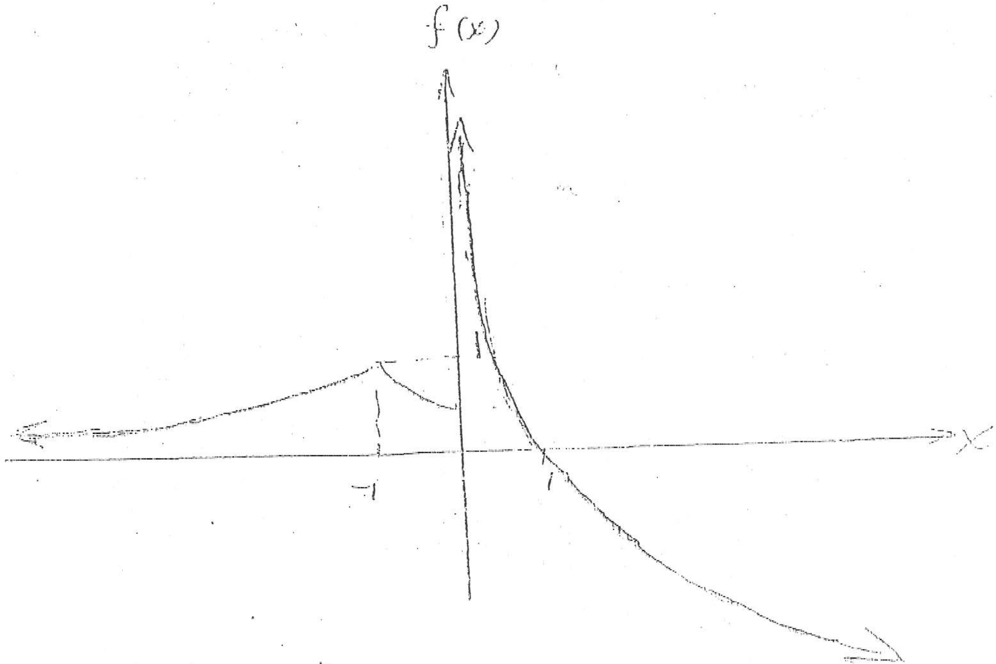
$\log_3(1-2x-2) \Rightarrow 2^{1-2x-3}$

$\log_3(-2x-1) \Rightarrow 2^{-2x-2}$

$$(f\circ g)(x) = \begin{cases} \log_3(-1-2x) & ; x \le -1 \\ 2^{-2x-2} & ; -1 < x \le 0 \end{cases}$$

b) correcto

134.- $f(x) = \begin{cases} 2^{-|x+1|} & ; x \le 0 \\ \log_{\frac{1}{2}} |x| & ; x > 0 \end{cases}$



135.- $p(x)$: $\text{sgn}(\underbrace{\ln(||x|-1|)}) = -1$

dominio signo $(x<0)$

$y = \ln||x|-1|$

Asíntotas

$||x|-1| = 0$

$|x| = 1$

$\boxed{x = \pm 1}$

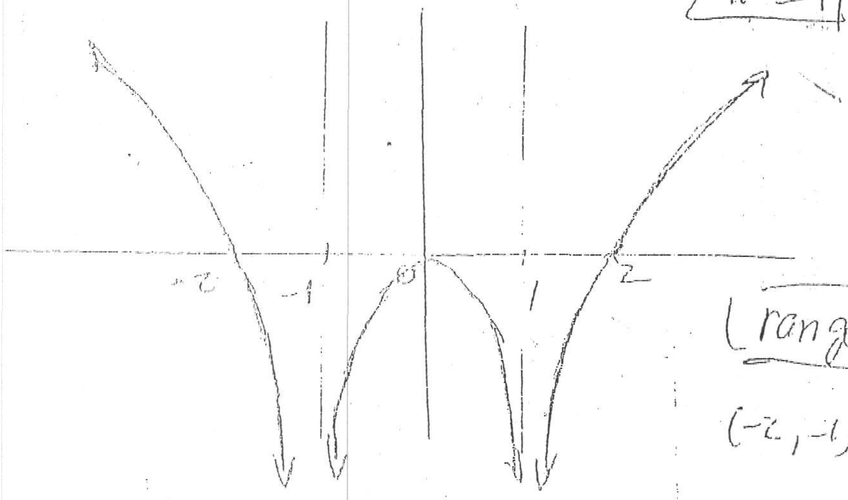
Raíces

$||x|-1| = 1$

$|x| - 1 = 1 \qquad -|x| + 1 = 1$

$|x| = 2 \qquad |x| = 0$

$\boxed{x = \pm 2} \qquad \boxed{x = 0}$



rango negativo

$(-2,-1) \cup (-1,0) \cup (0,1) \cup (1,2)$

136.- $\dfrac{f(-2) - f(0)}{f^{-1}(1)} \Rightarrow \dfrac{e^{-2} + 1 - \ln 3}{-\frac{1}{2}}$

$y = 2x$

$x = \frac{y}{2}$

$f(x)^{-1} = \frac{x}{2}$

$\Rightarrow -2(e^{-2} + 1 - \ln 3)$

$\Rightarrow 2\ln 3 - 2 - 2e^{-2}$

$\Rightarrow \ln 3^2 - 2 - 2e^{-2} \Rightarrow \ln 9 - 2 - 2e^{-2}$

137.- a) $\log_{\frac{1}{2}}\left(\dfrac{2x^2 - 4x - 6}{4x - 11}\right) \leq -1$

$\dfrac{2x^2 - 4x - 6}{(4x - 11)} \leq \left(\frac{1}{2}\right)^{-1}$

$\dfrac{2x^2 - 4x - 6}{(4x - 11)} \leq 2$

$2x^2 - 4x - 6 \leq 2(4x - 11)$

Pto. críticos

$4x - 11 = 0$

$4x = 11$

$x = \frac{11}{4}$

$2x^2 - 4x - 6 - 8x + 22 \leq 0$

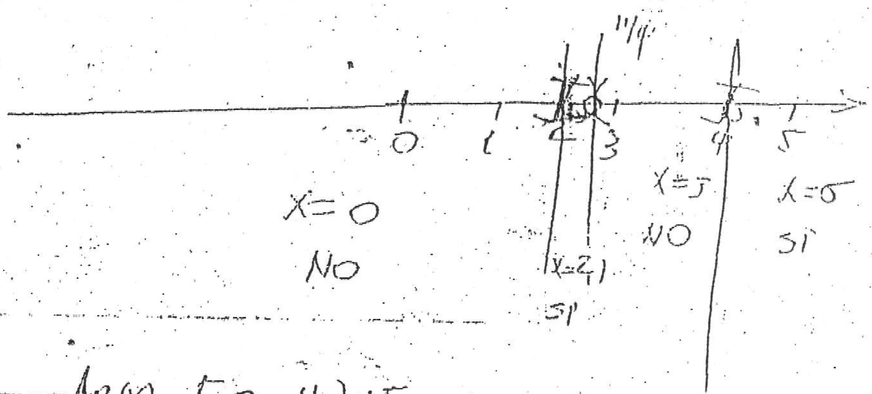
$2x^2 - 12x + 16 \leq 0$

$2(x^2 - 6x + 8) \leq 0$

$x^2 - 6x + 8 \leq 0$

$(x - 4)(x - 2) \leq 0$

$\boxed{x \leq 4} \quad \boxed{x \leq 2}$



$Dp(x) = [2, \frac{11}{4}) \cup [4, +\infty)$

b) $\cancel{\log_2}\left(\frac{4}{x+3}\right) > \cancel{\log_2}(2-x)$

$\frac{4}{x+3} > 2-x \Rightarrow \frac{4}{x+3} - 2 + x > 0 \Rightarrow \frac{4+(-2+x)(x+3)}{x+3} > 0$

$\frac{4-2x-6+x^2+3x}{x+3} > 0 \Rightarrow \frac{x^2+x-2}{x+3} > 0$

Punto crítico

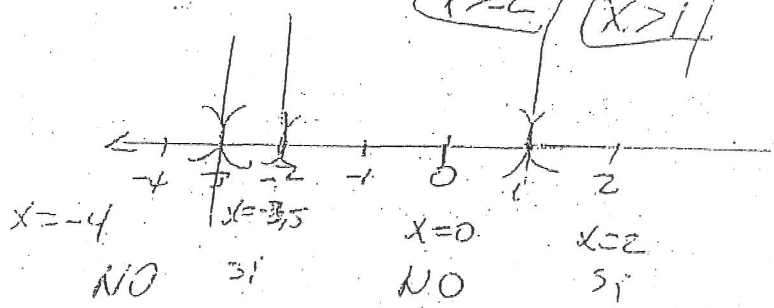
$x+3=0$

$x=-3$

$x^2+x-2>0$

$(x+2)(x-1)>0$

$\boxed{x>-2}$ / $\boxed{x>1}$



$Dp(x): (-3,2) \cup (1,+\infty)$

) $\log_{\frac{1}{2}}(x^3+8) - \frac{1}{2}\log_{\frac{1}{2}}(x^2+4x+4) \leq \log_{\frac{1}{2}}(x+58)$

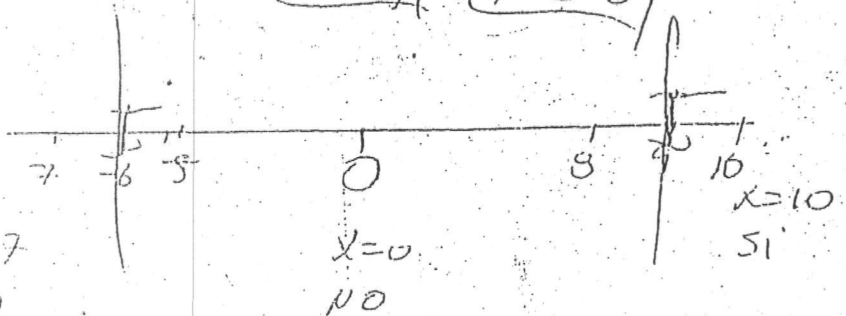
$\log_{\frac{1}{2}}(x^3+8) - \log_{\frac{1}{2}}\sqrt{(x+2)^2} \leq \log_{\frac{1}{2}}(x+58)$

$\cancel{\log_{\frac{1}{2}}}\left(\frac{x^3+8}{x+2}\right) \leq \cancel{\log_{\frac{1}{2}}}(x+58)$

$\Rightarrow \frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}} \leq x+58 \Rightarrow x^2-2x+4-58 \leq 0$

$\Rightarrow x^2-3x-54 \leq 0 \Rightarrow (x-9)(x+6) \leq 0$

$(x \leq 9) \; (x \leq -6)$



$Ap(x): [9, +\infty)$

) $\cancel{\log_{x-2}}(2x-3) \geq \cancel{\log_{x-2}}(24-6x)$

Punto critico

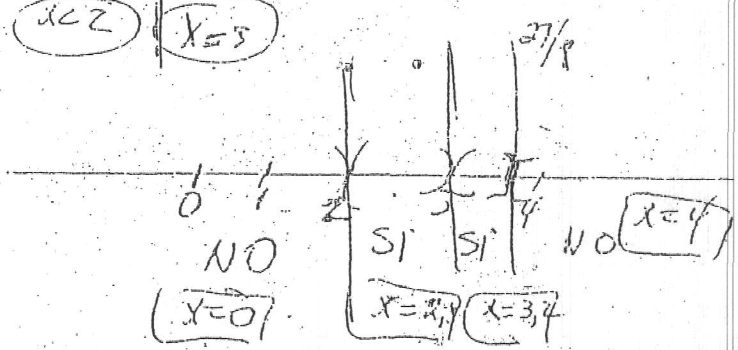
$x-2<0$	$x-2=1$
$x<2$	$x=3$

$2x-3 \geq 24-6x$

$2x+6x \geq 24+3$

$8x \geq 27$

$x \geq 27/8$



Sol $(2, 3) \cup (\frac{27}{8}, 4]$

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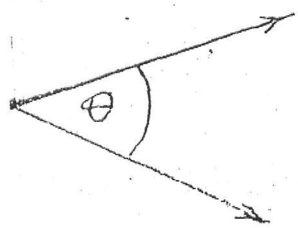
SOLUCIÓN EJERCICIOS PROPUESTOS

CAPÍTULO CUATRO

TRIGONOMETRÍA

1.- 

ES LA REGION DE UN PLANO COMPRENDIDO ENTRE LOS SEGMENTOS DE RECTA.

b) CORRECTO

2.- b) CORRECTO, ESTA DEFINIDO POR EL VERTICE Y LOS SEGMENTOS DE RECTA.

3.- a) CORRECTO

4.- $\alpha+\theta=180°$; NO PUEDEN SER AGUDOS.

b) CORRECTO

5.- DOS ANGULOS OPUESTOS POR EL VERTICE SON IGUALES

b) CORRECTO

6.- a) $\cancel{30°} \times \frac{\pi\, rad}{\cancel{180°}_{6}} = \frac{\pi}{6}$ radian

b) $\cancel{135°}^{3} \times \frac{\pi\, rad}{\cancel{180°}_{4}} = \frac{3\pi}{4}$ radian

c) $-\cancel{120°}^{2} \times \frac{\pi\, rad}{\cancel{180°}_{3}} = -\frac{2\pi}{3}$ radian

d) $\overset{5}{\cancel{480}}° \times \dfrac{\pi\,rad}{\underset{2}{\cancel{180°}}} = \dfrac{5\pi}{2}$ radian

e) $-\overset{3}{\cancel{540}}° \times \dfrac{\pi\,rad}{\underset{1}{\cancel{180°}}} = 3\pi$ radian

f) $\overset{1}{\cancel{60}}° \times \dfrac{\pi\,rad}{\underset{3}{\cancel{180°}}} = \dfrac{\pi}{3}$ radian

7.-

a) $\dfrac{\pi}{\underset{1}{\cancel{6}}} \times \dfrac{\overset{30}{\cancel{180°}}}{\cancel{\pi\,rad}} = 30°$

b) $-\dfrac{5\pi}{\underset{1}{\cancel{4}}} \times \dfrac{\overset{45}{\cancel{180°}}}{\cancel{\pi\,rad}} = -225°$

c) $\dfrac{4\pi}{\underset{1}{\cancel{3}}} \times \dfrac{\overset{60}{\cancel{180°}}}{\cancel{\pi\,rad}} = 240°$

d) $\dfrac{\pi}{\underset{1}{\cancel{2}}} \times \dfrac{\overset{90}{\cancel{180°}}}{\cancel{\pi\,rad}} = 90°$

e) $\dfrac{\pi}{\underset{1}{\cancel{12}}} \times \dfrac{\overset{15}{\cancel{180°}}}{\underset{1}{\cancel{\pi\,rad}}} = 15°$

f) $4\pi \times \dfrac{180°}{\cancel{\pi\,rad}} = 720°$

8.-

radian	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$\frac{2\pi}{3}$	$3\pi/4$	$\frac{28\pi}{45}$	$\frac{5\pi}{6}$	$\frac{\pi}{12}$
grado	0	30	45	60°	90	120	135	112°	150°	15°

$\overset{28}{\cancel{112}}° \times \dfrac{\pi\,rad}{\underset{45}{\cancel{180°}}} = \dfrac{28\pi}{45}$ | $\overset{5}{\cancel{150}}° \times \dfrac{\pi\,rad}{\underset{6}{\cancel{180°}}} = \dfrac{5\pi}{6}$

9.-

$1\text{ hora} \Rightarrow \frac{\cancel{360}^{30}}{\cancel{12}_{1}} = 30°$

ángulo $\Rightarrow$ 1 hora $=$ 5 minutos $= 30°$

1 minuto $= \frac{30}{5} = 6°$

$3\cancel{\text{minutos}} \times \frac{6°}{1\cancel{\text{minutos}}} = \cancel{18}^{1}° \times \frac{\pi \text{ rad}}{\cancel{180°}_{10}} = \frac{\pi}{10}\text{ rad}$

$l = \theta R$

$\frac{7\cancel{\pi}}{\cancel{10}} = \frac{\cancel{\pi}}{\cancel{10}} R$

$R = 7\text{ cm}$

10.-

Si x y y son ángulos suplementarios

$\boxed{x + y = 180°}$ (-1)

x → ángulo
y → suplemento

$y = 4(90 - x)$

$y = 360 - 4x$

$\boxed{4x + y = 360}$

$$\begin{array}{r} -x - y = -180 \\ 4x + y = 360 \\ \hline 3x = 180 \end{array}$$

$x = \frac{180}{3}$

$x = 60°$

11.- ángulos congruentes (iguales)

$$8X = 180^\circ$$

$$\boxed{X = \frac{45^\circ}{2}}$$

$$\frac{\cancel{45^\circ}^{\,1}}{2} \times \frac{\pi rad}{\cancel{180^\circ}_{\,4}} = \frac{\pi}{8} rad$$

12.-

$$180 - X = 123$$

$$180 - 123 = X$$

a) $X = 57^\circ$

b) ~~ángulo~~ complementario ~~ángulo complementario~~ $90 - 57$

$\Rightarrow 33^\circ$

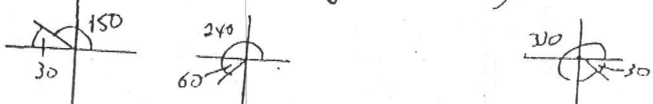
13.-

a) $\operatorname{sen} 30^\circ \times \cancel{\cos} 90^{\,0} \left(-\cos \frac{7\pi}{6}\right) \tan \frac{3\pi}{4}$

$\Rightarrow 0$

b) $\operatorname{sen} \frac{5\pi}{6} \times \cos \frac{4\pi}{3} \left(-\tan \frac{\pi}{6}\right) \tan 330$

$\operatorname{sen} 150 \times \cos 240 (-\tan 30) \tan 330$



$\operatorname{sen} 30 \times (-\cos 60)(-\tan 30(-\tan 30)$

$\frac{1}{2} \times \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{3}\right)\left(-\frac{\sqrt{3}}{3}\right)$

$\Rightarrow -\frac{3}{36} \Rightarrow -\frac{1}{12}$

$\frac{\pi}{6} = 30^\circ$

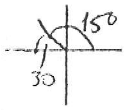
$\frac{5\pi}{6} = 150$

$\frac{\pi}{3} = 60$

$4\left(\frac{\pi}{3}\right) = 240$

c) $3\cos\left(\frac{\pi}{6}\right) + \operatorname{sen}\left(\frac{5\pi}{6}\right)\tan\frac{\pi}{3}$ $\qquad \frac{\pi}{6} = 30$

$3\cos 30 + \operatorname{sen} 150 \cdot \tan 60$ $\qquad 5\left(\frac{\pi}{6}\right) = 150$



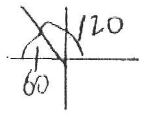
$3\cos 30 + \operatorname{sen} 30 \tan 60$

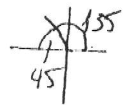
$3\frac{\sqrt{3}}{2} + \frac{1}{2} \times \sqrt{3} \Rightarrow \frac{4}{2}\sqrt{3}$

$\Rightarrow 2\sqrt{3}$

d) $\tan\left(\frac{\pi}{6}\right) - \cos\frac{2\pi}{3} - \tan\frac{3\pi}{4}$ $\qquad \frac{\pi}{3} = 60^\circ$

$\tan 30 - \cos 120 - \tan 135$ $\qquad 2\left(\frac{\pi}{3}\right) = 120^\circ$





$\frac{\pi}{4} = 45$

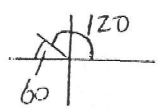
$3\left(\frac{\pi}{4}\right) = 135^\circ$

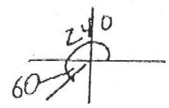
$\tan 30 - (-\cos 60) - (-\tan 45)$

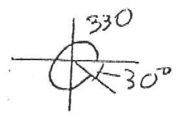
$\frac{\sqrt{3}}{3} + \frac{1}{2} + 1$

$\Rightarrow \frac{2\sqrt{3}+3+6}{6} \Rightarrow \frac{9+2\sqrt{3}}{6}$

e) $\frac{\operatorname{sen} 120 + \cos 240}{\tan 60 + \tan 330}$





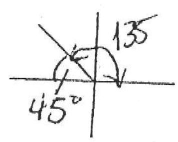


$\Rightarrow \frac{\operatorname{sen} 60 + (-\cos 60)}{\tan 60 + (-\tan 30)} \Rightarrow \frac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\sqrt{3} - \frac{\sqrt{3}}{3}} \Rightarrow \frac{\frac{\sqrt{3}-1}{2}}{\frac{3\sqrt{3}-\sqrt{3}}{3}}$

$\Rightarrow \frac{3(\sqrt{3}-1)}{2(2\sqrt{3})} \Rightarrow \frac{3(\sqrt{3}-1)}{4\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{3(\sqrt{3})^2 - \sqrt{3}}{4(\sqrt{3})^2} \Rightarrow \frac{9-\sqrt{3}}{12}$

f) $\frac{2\operatorname{sen}^2\left(\frac{\pi}{6}\right)\cos^2\pi}{4\tan\left(\frac{\pi}{4}\right)\operatorname{sen}^2\left(\frac{3\pi}{4}\right)}$ $\qquad \frac{\pi}{4} = 45$

$3\left(\frac{\pi}{4}\right) = 135$



$$\frac{2\sin^2 30\ \cos^2 180}{4\tan 45\ \sin^2 135} \Rightarrow \frac{2(\sin 30)^2\,(\cos 180)^2}{4\times(\sin 135)^2}$$

$$\Rightarrow \frac{2\left(\frac{1}{2}\right)^2(-1)^2}{4(1)\times\left(\frac{\sqrt{2}}{2}\right)^2} \Rightarrow \frac{\frac{\cancel{2}^{1}}{\cancel{4}_{1}}}{\frac{\cancel{4}^{1}\times\cancel{2}^{1}}{\cancel{4}_{1}}} \Rightarrow \frac{1}{4}$$

14:-

a) $\tan\pi + \sin\pi$

$\Rightarrow \cancel{\tan 180}^{\to 0} + \cancel{\sin 180}^{\to 0}$

$\Rightarrow 0$

b) $\frac{\sin 50}{\cos 40} \Rightarrow \frac{\cancel{\sin 50}}{\cancel{\sin 50}} \Rightarrow 1$

c) $3\sin 45 - 4\tan\left(\frac{\pi}{6}\right)$

$3\sin 45 - 4\tan(30)$

$3\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{3}}{3}\right) \Rightarrow \frac{3\sqrt{2}}{2} - \frac{4\sqrt{3}}{3}$

$\Rightarrow \frac{9\sqrt{2} - 8\sqrt{3}}{6}$

d) $\frac{\sin(-40)}{\cos 50} \Rightarrow \frac{-\sin 40}{\cos 50}$

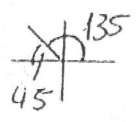
$\Rightarrow \frac{-\cancel{\cos 50}}{\cancel{\cos 50}}$

$\Rightarrow -1$

e) $6\cos\left(\frac{3\pi}{4}\right) + 2\tan\left(-\frac{\pi}{3}\right)$

$\frac{\pi}{4} = 45°$

$3\left(\frac{\pi}{4}\right) = 135°$



$\Rightarrow 6\cos 135 + 2(-\tan 60)$

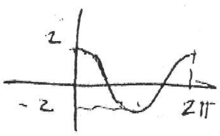
$\Rightarrow 6(-\cos 45) - 2\tan 60$

$\Rightarrow -6\cdot\frac{\sqrt{2}}{2} - 2\sqrt{3} \Rightarrow 3\sqrt{2} - 2\sqrt{3}$

6

15.— ~~dom~~ rango $\underbrace{[-1,3]}_{4}$

Amplitud: $\frac{4}{2} \Rightarrow 2$; $q=2$



$p=1$

$y = 1+2\cos x$

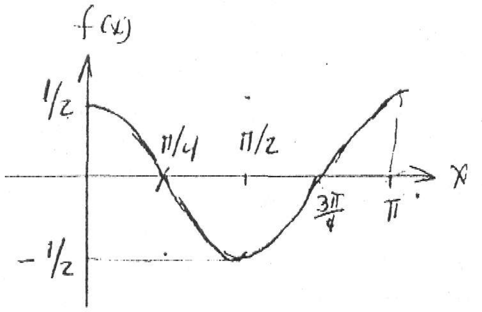
$p^2 - q^2 \Rightarrow -3$ e) CORRECTO

16.— $f(x) = \frac{1}{2}\cos(2x)$

$T = \frac{2\pi}{a} \Rightarrow \frac{\cancel{2}\pi}{\cancel{2}}$

$T=\pi$

$A = 1/2$



e) CORRECTO

17.— $f(x) = p + q\,\text{sen}(kx)$;

$T = 4\pi$

$T = \frac{2\cancel{\pi}}{K} = 4\cancel{\pi}$

$K = 1/2$

Amplitud $= \frac{(11-e)}{2}$

$q = \frac{11-e}{2}$

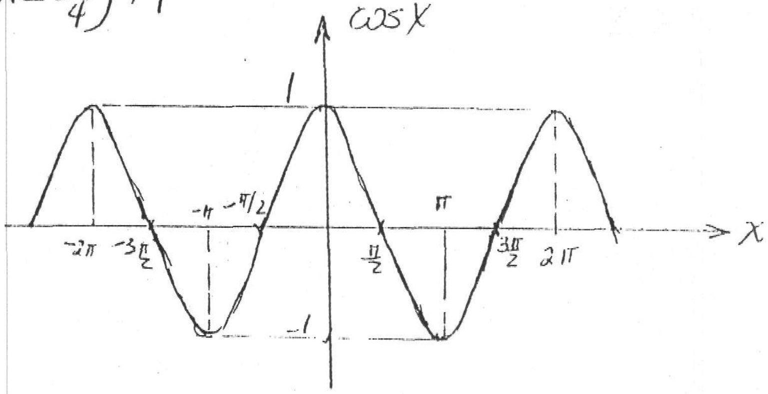
$p = e + \text{Amplitud}$

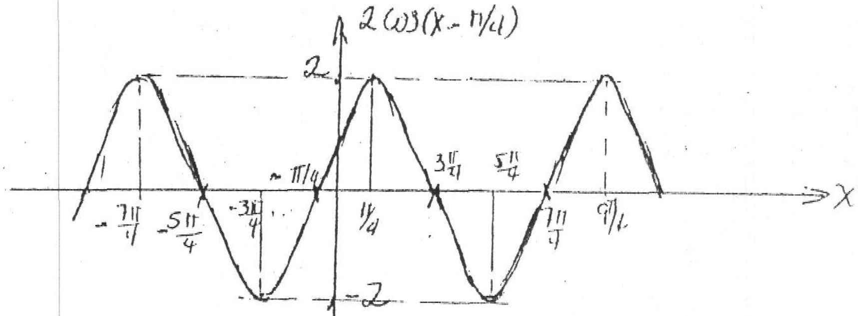
$P = e + \frac{11-e}{2} \Rightarrow \frac{2e+11-e}{2}$

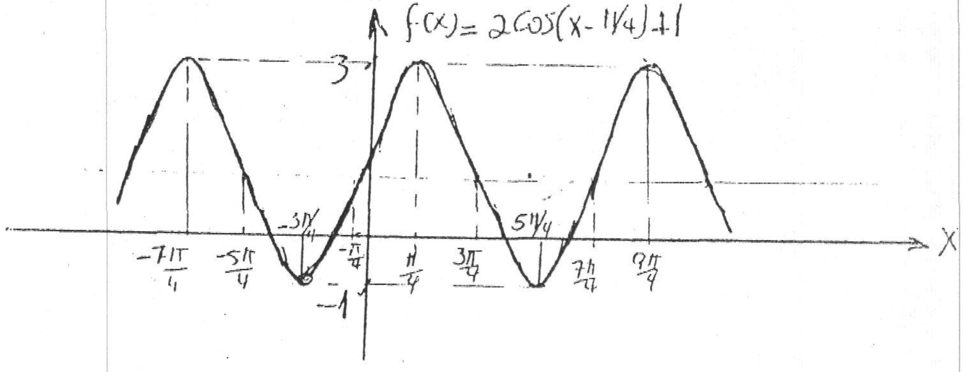
$P = \frac{e+11}{2}$

18.—

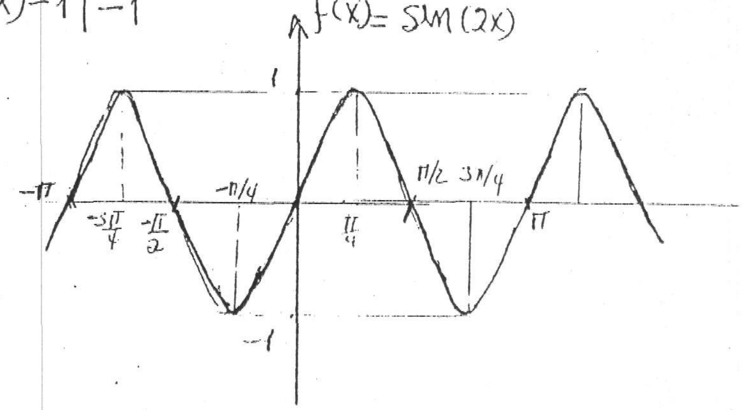
a) $y = 2\cos\left(x - \frac{\pi}{4}\right) + 1$







b) $y = |\sin(2x) - 1| - 1$

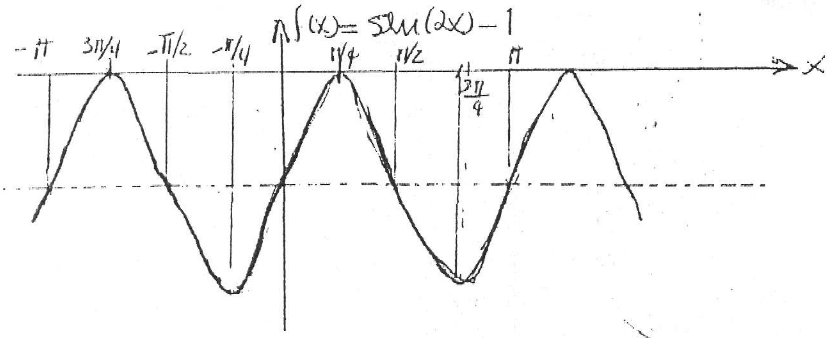


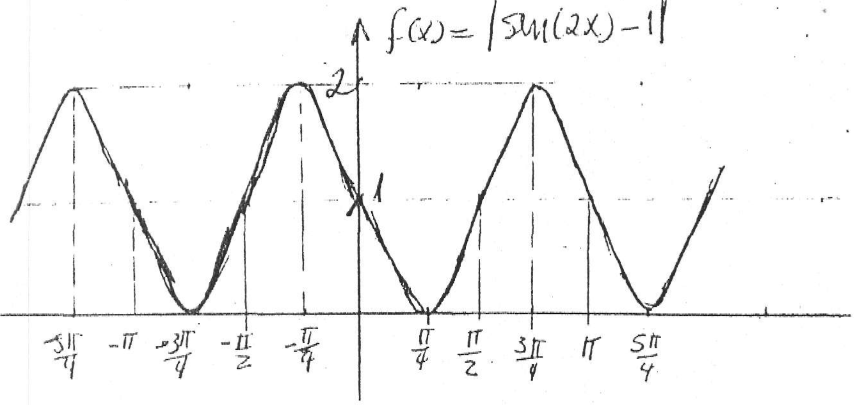
$T = \frac{2\pi}{a}$

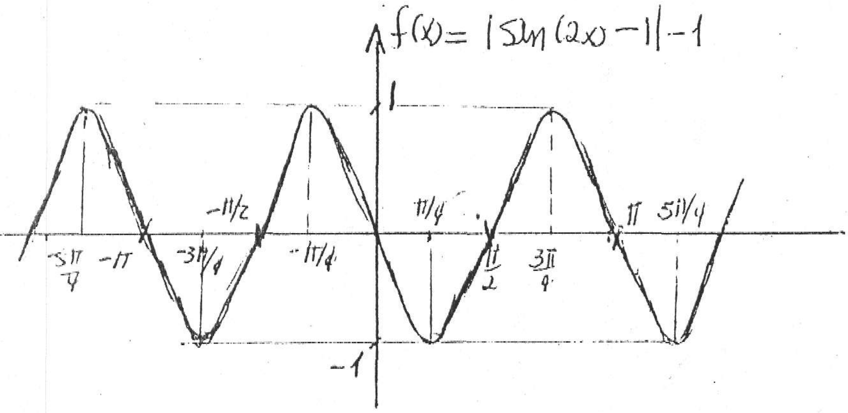
$T = \frac{2\pi}{2}$

$T = \pi$

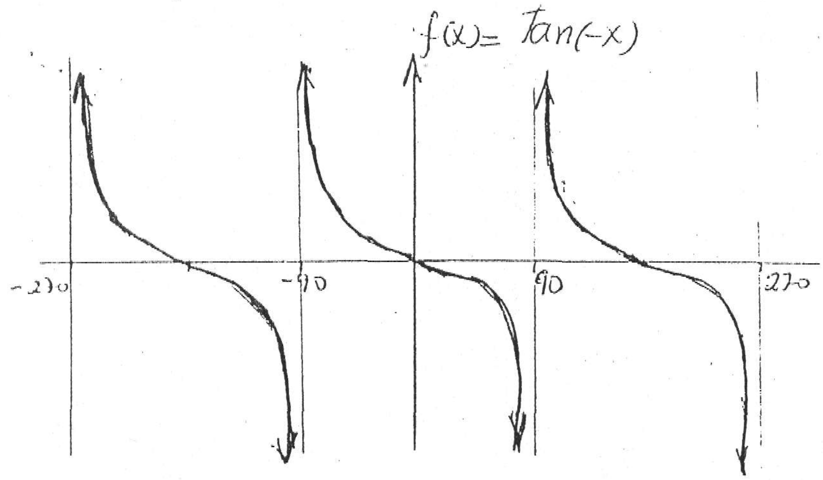
8


f(x)= sin(2x) − 1
−π
3π/4
−π/2
−π/4
π/4
π/2
3π/4
π
x


f(x)= |sin(2x) − 1|
2
1
−5π/4
−π
−3π/4
−π/2
−π/4
π/4
π/2
3π/4
π
5π/4


f(x)= |sin(2x) − 1| − 1
1
−1
−π/2
π/4
π
5π/4
−5π/4
−π
−3π/4
−π/4
π/2
3π/4

c) $y = 1 - \tan(\pi - x)$

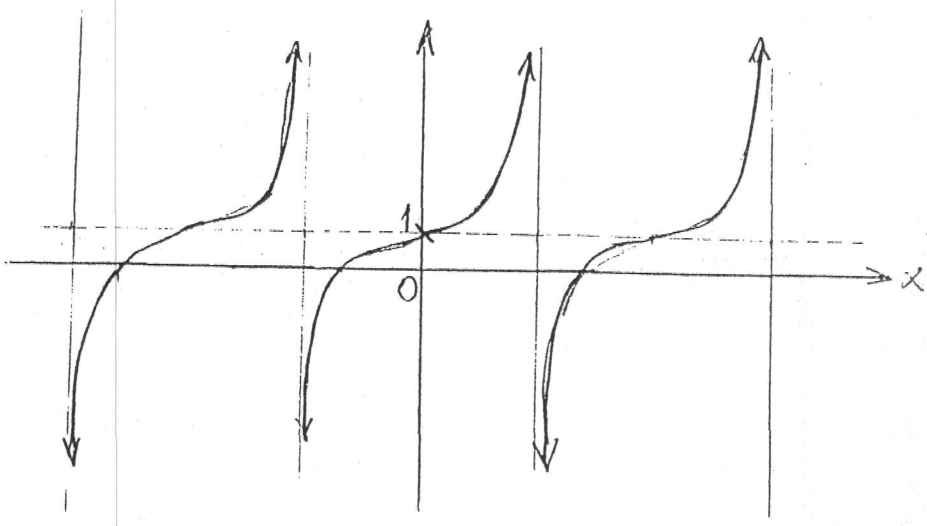

f(x)= tan(−x)
−270
−90
90
270

$$f(x) = \tan(180 - x) \Rightarrow \frac{\cancel{\tan 180^{\circ}} - \tan x}{1 + \cancel{\tan 180^{\circ}} \tan x}$$

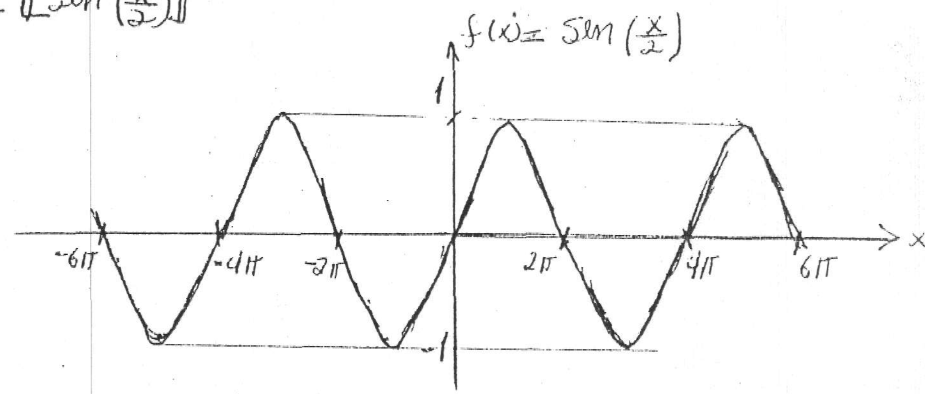
$$f(x) = \tan(180 - x) = -\tan x = \tan(-x)$$

$$f(x) = 1 - \underbrace{\tan(180 - x)}_{-\tan x}$$

$$f(x) = 1 + \tan x$$



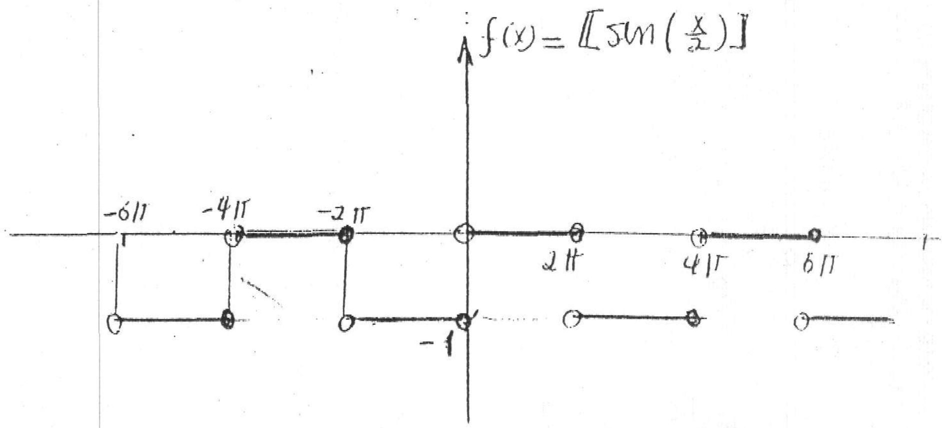
d) $y = \frac{1}{2} - [\![\operatorname{sen}(\frac{x}{2})]\!]$

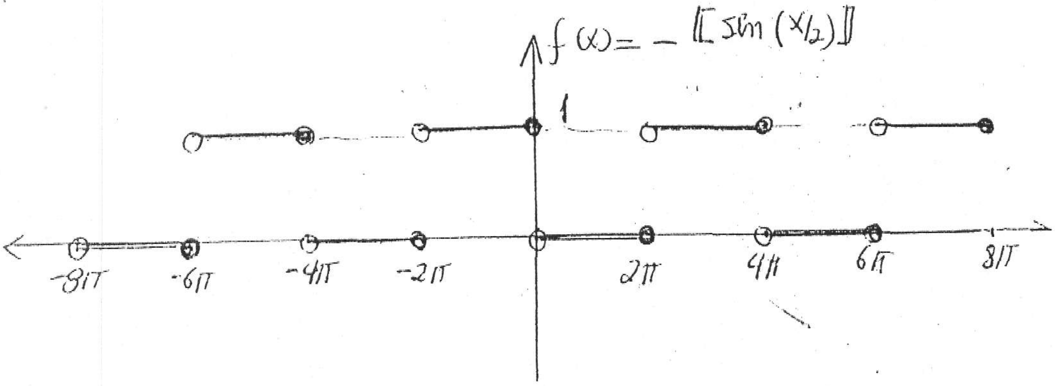


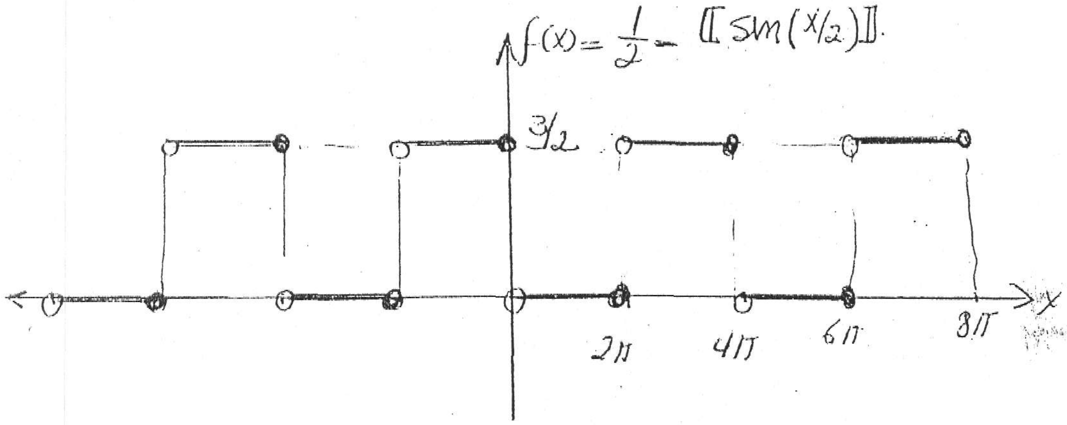
$T = \frac{2\pi}{a}$

$T = \frac{2\pi}{\frac{1}{2}}$

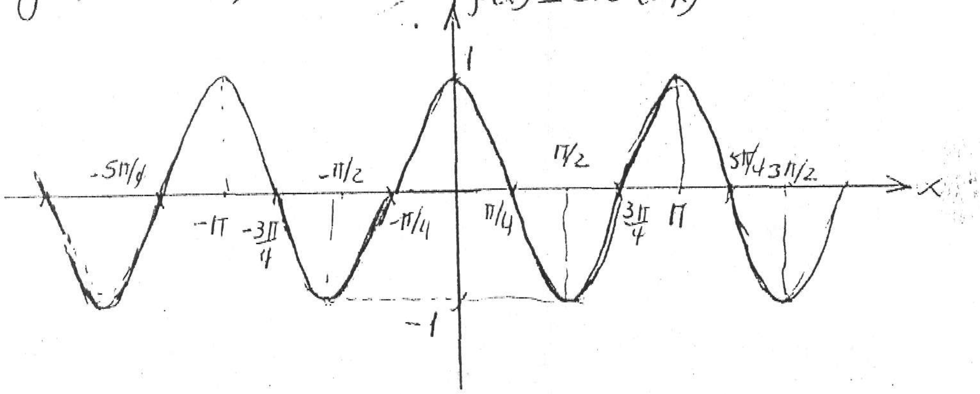
$T = 4\pi$

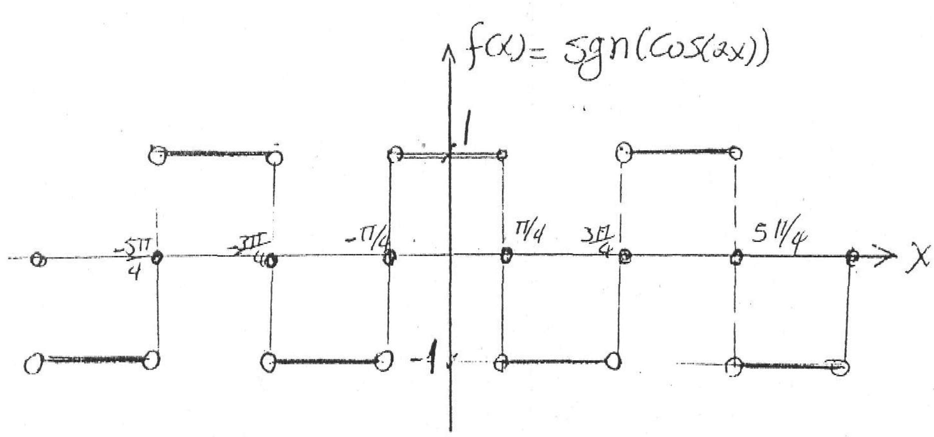






e) $y = \mathrm{sgn}(\cos(2x))$





19.– Aproximadamente $T = 7{,}5h \Rightarrow \frac{15}{2}$

a) $T = \frac{15}{2} \Rightarrow \frac{2\pi}{b} = \frac{15}{2}$

$b = \frac{4\pi}{15}$

2Amplitud: $(4{,}5 - 1{,}5)$

Amplitud $= \frac{3}{2}$

$a = 3/2$

b) $h(t) = \frac{3}{2}\cos\left(\frac{4\pi}{15}t\right)$

$h(13) = \frac{3}{2}\cos\left(\frac{4\pi}{15}(13)\right)$

c) mínimo

$\cos\left(\frac{4\pi}{15}t\right) = -1$

$\left(\frac{4\pi}{15}t\right) = \cos^{-1}(-1)$

$\frac{4\cancel{\pi}}{15}t = \cancel{\pi}$

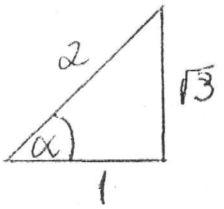
$t = \frac{15}{4} + 7{,}5n \;;\; n \in \mathbb{N}$

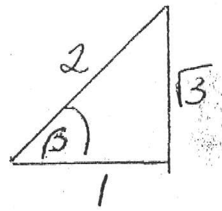
$t_1 = \frac{15}{4} + \frac{15}{2} \Rightarrow \cancel{\frac{15 + 30}{4}} \Rightarrow \frac{45}{4}h$

12

20.- $\alpha = \arccos(-1/2)$; $\frac{\pi}{2} < \alpha < \pi$ $\beta = \arcsin(-\sqrt{3}/2)$; $\frac{3\pi}{2} < \beta < 2\pi$

$\cos\alpha = -\frac{1}{2}$ $(2^{do}\ C)$ $\sin\beta = -\frac{\sqrt{3}}{2}$ $(4^{to}\ C)$





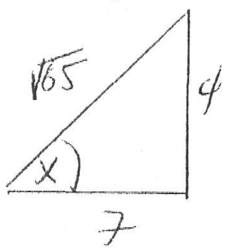
$\sin\alpha + \tan\beta$.

$$\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{1}\right)$$

$$\frac{\sqrt{3} - 2\sqrt{3}}{2} \Rightarrow -\frac{\sqrt{3}}{2}$$

21.- $x = \arctan\left(\frac{4}{7}\right)$; $\pi \leq x \leq \frac{3\pi}{2}$

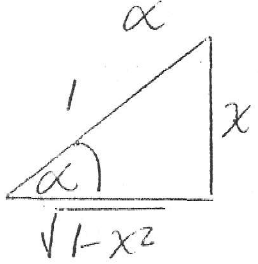
$\tan x = \frac{4}{7}$ $(3^{ro}\ C)$



$$\cos x = \frac{-7}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}}$$

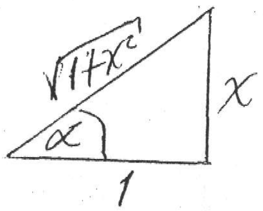
$$\cos x = \frac{-7\sqrt{65}}{65}$$

22.- a) $\cos(\underbrace{\arcsin(x)}_{\alpha})$ $\Rightarrow \sin\alpha = x$



$$\cos\alpha = \frac{\sqrt{1-x^2}}{1}$$

b) $\underbrace{\cos(\arctan(x))}_{\alpha} \Rightarrow \tan(\alpha) = \frac{x}{1}$



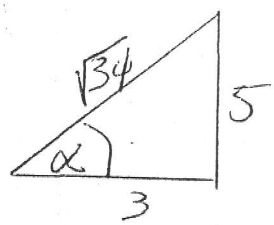
$$\cos\alpha = \frac{1}{\sqrt{1+x^2}}$$

c) $\arccos\left[\cos\left(-\frac{17}{5}\pi\right)\right]$

$$\cancel{\cos^{-1}}\left[\cancel{\cos}\left(-\frac{17}{5}\pi\right)\right]$$

$$\Rightarrow -\frac{17}{5}\pi$$

d) $\underbrace{\operatorname{sen}\left[\arctan\left(-\frac{5}{3}\right)\right]}_{\alpha}$ $\quad \tan\alpha = -\frac{5}{3}$



$$\operatorname{sen}\alpha = \frac{5}{\sqrt{34}}$$

23.- $\forall x,y \in \mathbb{R} : (\operatorname{sen}^{-1}(x+y)) = \operatorname{sen}^{-1}x + \operatorname{sen}^{-1}y$

b) CORRECTO

24.- $8\cos 10 . \cos 20 . \cos 40$

$\Rightarrow \cos 20 = \cos(30-10)$

$\Rightarrow \cos 30 \cos 10 + \operatorname{sen} 30 \operatorname{sen} 10$

$\Rightarrow \boxed{\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\operatorname{sen} 10}$

$\Rightarrow \cos 40 = \cos(30+10)$

$\Rightarrow \cos 30 \cos 10 - \operatorname{sen} 30 \operatorname{sen} 10$

$\Rightarrow \boxed{\frac{\sqrt{3}}{2}\cos 10 - \frac{1}{2}\operatorname{sen} 10°}$

$\therefore 8\cos 10 \times \cos 20 \times \cos 40$

$\Rightarrow 8\cos 10\left(\frac{\sqrt{3}}{2}\cos 10 + \frac{1}{2}\operatorname{sen} 10\right)\left(\frac{\sqrt{3}}{2}\cos 10 - \frac{1}{2}\operatorname{sen} 10\right)$

$\Rightarrow 8\cos 10\left(\frac{3}{4}\cos^2 10 - \frac{1}{4}\operatorname{sen}^2 10\right) \Rightarrow \frac{8}{\cancel{4}}\cos 10\left(3\cos^2 10 - \operatorname{sen}^2 10\right)$

$\Rightarrow 2\cos 10(3\cos^2 10 - (1 - \cos^2 10)) \Rightarrow 2\cos 10\underbrace{(4\cos^2 10 - 1)}_{\frac{1}{2\operatorname{sen} 10}}$

$\Rightarrow \frac{\cancel{2}\cos 10}{\cancel{2}\operatorname{sen} 10} \Rightarrow \cot 10°$

d) correcto

15

25.– $\cos\left(\frac{\pi}{12}\right) \Rightarrow \frac{\cancel{\pi}}{\cancel{12}} \times \frac{\cancel{180}}{\cancel{\pi}\,\text{rad}} = 15^\circ$

$\cos(15) \Rightarrow \cos(45-30)$

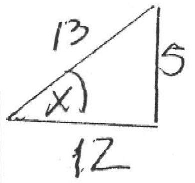
$\Rightarrow \cos 45 \cos 30 + \text{sen} 45 \,\text{sen} 30$

$\Rightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$\Rightarrow \frac{\sqrt{6} + \sqrt{2}}{4}$

d) CORRECTO

26.– $\sin x = \frac{5}{13}$ 2do cuadrante



$c = \sqrt{13^2 - 5^2}$

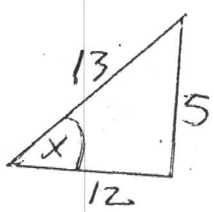
$c = 12$

$\cos(x+60) \Rightarrow \cos x \cos 60 - \sin x \,\text{sen} 60$

$\Rightarrow -\frac{12}{13} \times \frac{1}{2} - \frac{5}{13} \times \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{-12 + 5\sqrt{3}}{26}$

27.– $\text{sen} x = \frac{5}{13}$ 2do cuadrante



$\sin(2x) = 2 \,\text{sen} x \cos x$

$\Rightarrow 2 \times \frac{5}{13} \times -\frac{12}{13}$

$\Rightarrow -\frac{120}{169}$

es correcto

√

28.— $\sqrt{\dfrac{2\sec(3x)}{1+\sec(3x)}}$

$\Rightarrow \sqrt{\dfrac{\dfrac{2}{\cos(3x)}}{1+\dfrac{1}{\cos(3x)}}} \Rightarrow \sqrt{\dfrac{\dfrac{2}{\cancel{\cos(3x)}}}{\dfrac{\cos(3x)+1}{\cancel{\cos(3x)}}}} \Rightarrow \sqrt{\dfrac{2}{1+\cos(3x)}}$

$\Rightarrow \left(\sqrt{\dfrac{2}{1+\cos(3x)} \times \dfrac{1-\cos(3x)}{1-\cos(3x)}} \Rightarrow \sqrt{\dfrac{2(1-\cos(3x))}{1-\cos^2(3x)}} \right.$

$\left. \sqrt{\dfrac{2(1-\cos(3x))}{\sin^2(3x)}} \Rightarrow \dfrac{\sqrt{2(1-\cos(3x))}}{\sqrt{\sin^2(3x)}} \Rightarrow \dfrac{\sqrt{2(1-\cos(3x))}}{\sin(3x)} \right)$

$\Rightarrow \left(\sqrt{\dfrac{1+\cos(3x)}{2}}\right)^{-1} = \left(\cos\left(\dfrac{3x}{2}\right)\right)^{-1} \Rightarrow \dfrac{1}{\cos\left(\frac{3x}{2}\right)}$

$\Rightarrow \sec\left(\dfrac{3x}{2}\right)$ e) correcto

29.— a) $\sin\left(\frac{x}{2}+\frac{x}{2}\right) = 2\sin\frac{x}{2}\cos\frac{x}{2}$ CORRECTO

$\frac{1}{2}\sin x = \sin\frac{x}{2}\cdot\cos\frac{x}{2}$

b) $\cos(4x) = \cos^2(2x) - \sin^2(2x)$ CORRECTO

c) $\tan x \cdot \cos x = \dfrac{1}{\csc x}$

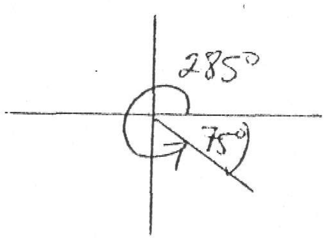
$\dfrac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x} \Rightarrow \sin x \Rightarrow \left(\dfrac{1}{\sin x}\right)^{-1} \Rightarrow (\csc x)^{-1}$

$\Rightarrow \dfrac{1}{\csc x}$ CORRECTO

d) $sen^2(2x) + cos(2x) = 1$; FALSO

30. $tan\left(\frac{19\pi}{12}\right)$ $\qquad \frac{19\cancel{\pi}}{\cancel{12}_{1}} \times \frac{\cancel{180°}^{15}}{\cancel{\pi rad}_{1}} = 285$

$\Rightarrow +tan\, 285$



$\Rightarrow -\,tan\, 75$

$\Rightarrow -\,tan(45+30)$

$$\Rightarrow -\frac{sen(45+30)}{cos(45+30)} \Rightarrow -\frac{sen45cos30 + sen30\,cos45}{cos45cos30 - sen45\,sen30}$$

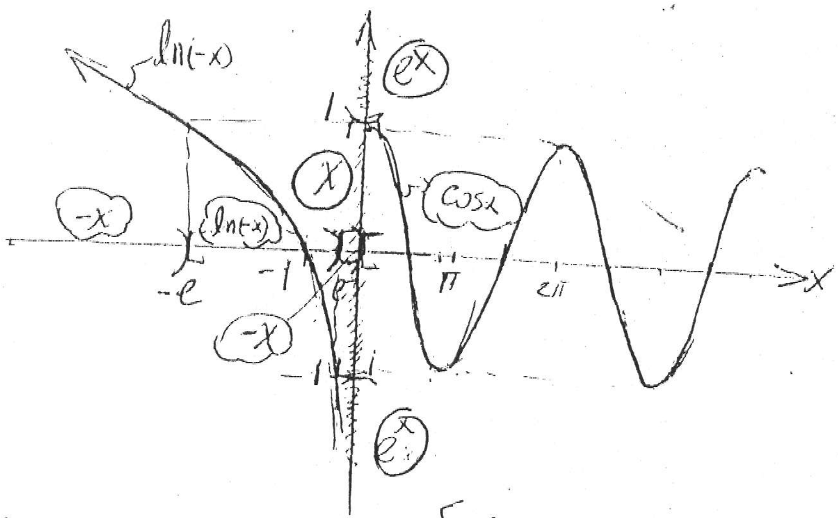
$$\Rightarrow -\frac{\frac{\sqrt{2}}{\cancel{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{2}\times\frac{\sqrt{2}}{\cancel{2}}}{\frac{\sqrt{2}}{\cancel{2}}\times\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{\cancel{2}}\times\frac{1}{2}} \Rightarrow -\frac{\frac{\sqrt{3}}{2}+\frac{1}{2}}{\frac{\sqrt{3}}{2}-\frac{1}{2}} \Rightarrow$$

$$\frac{\frac{\sqrt{3}+1}{\cancel{2}}}{\frac{1-\sqrt{3}}{\cancel{2}}} \Rightarrow \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \Rightarrow \frac{1+2\sqrt{3}+3}{1-(\sqrt{3})^2} \Rightarrow$$

$$\frac{4+2\sqrt{3}}{-2} \Rightarrow \frac{\cancel{2}(2+\sqrt{3})}{-\cancel{2}} \Rightarrow -(2+\sqrt{3})$$

31.- $f(x) = \begin{cases} cos x & ; x \geq 0 \\ ln(-x) & ; x < 0 \end{cases}$ $\qquad g(x) = \begin{cases} x & ; |x| \leq 1 \\ e^x & ; |x| \geq 1 \end{cases}$

$g \circ f$



$e^{\ln(-x)} \Rightarrow -x$

$$(g \circ f)(x) \begin{cases} -x & ; x < -e \\ \ln(-x) & ; -e \le x < -e^{-1} \\ -x & ; -e^{-1} \le x < 0 \\ \cos x & ; x \ge 0 \end{cases}$$

32.– $a = \tan 25°$

$$\frac{\tan 205 - \tan 115}{\tan 245 + \tan 335}$$

$$\tan 205 = \tan(180+25) \Rightarrow \frac{\overset{0}{\tan 180} + \overset{a}{\tan 25}}{1 - \overset{0}{\tan 180} \tan 25} \Rightarrow -a$$

$$\tan 115 = \frac{\sin(90+25)}{\cos(90+25)} \Rightarrow \frac{\overset{1}{\sin 90}\cos 25 + \sin 25 \overset{0}{\cos 90}}{\overset{0}{\cos 90}\cos 25 - \overset{1}{\sin 90}\sin 25}$$

$$\Rightarrow \frac{\cos 25}{-\sin 25} \Rightarrow \left(-\frac{\sin 25}{\cos 25}\right)^{-1} \Rightarrow -(a)^{-1} \Rightarrow -\frac{1}{a}$$

$$\tan 245 = \tan(270-25) \Rightarrow \frac{\sin(270-25)}{\cos(270-25)}$$

$$\Rightarrow \frac{\overset{-1}{\sin 270}\cos 25 - \sin 25 \overset{0}{\cos 270}}{\overset{0}{\cos 270}\cos 25 + \overset{-1}{\sin 270}\sin 25} \Rightarrow \frac{-\cos 25}{-\sin 25} \Rightarrow \frac{1}{a}$$

$$\tan 335 = \tan(360-25) \Rightarrow \frac{\tan 360 - \tan 25}{1+\tan 360 \tan 25} = -a$$

$$\Rightarrow \frac{-a-\left(-\frac{1}{a}\right)}{\frac{1}{a}+(-a)} \Rightarrow \frac{\frac{1}{a}-a}{\frac{1}{a}-a} = 1$$

33.—

a) $\frac{1}{2\sin 10} - 2\sin 70$

$\sin 10 = \sin(30-20) \Rightarrow \sin 30 \cos 20 - \sin 20 \cos 30$

$\left(\frac{1}{2}\cos 20 - \frac{\sqrt{3}}{2}\operatorname{sen} 20\right)$

$\frac{\cos 20 - \sqrt{3}\operatorname{sen} 20}{2}$

$\sin 10 \sin 70 \Rightarrow \frac{1}{2}(\cos(60) - \cos 80)$

$\therefore \frac{1}{2\sin 10} - 2\sin 70 \quad \frac{1-4\operatorname{sen} 10 \sin 70}{2\operatorname{sen} 10}$

$$\Rightarrow \frac{1-4\left(\frac{1}{2}(\cos 60 - \cos 80)\right)}{2\operatorname{sen} 10} \Rightarrow \frac{1-2\cos 60 + 2\cos 80}{2\operatorname{sen} 10} \Rightarrow \frac{1-1+2\sin 10}{2\operatorname{sen} 10} = 1$$

b.) $\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$ $\qquad \frac{\pi}{12} \times \frac{180}{\pi} = 15^\circ$

$\Rightarrow \sin 15^\circ \times \cos(15)$

$\Rightarrow \sin(45-30) \times \cos(45-30)$

$\Rightarrow (\sin 45 \cos 30 - \sin 30 \cos 45)(\cos 45 \cos 30 + \sin 45 \sin 30)$

$\Rightarrow \left(\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}\,\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}\right)$

$\Rightarrow \left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right)\left(\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\right) \Rightarrow \left(\frac{\sqrt{6}}{4}\right)^2 - \left(\frac{\sqrt{2}}{4}\right)^2$

$\Rightarrow \frac{6}{16} - \frac{2}{16} \Rightarrow \frac{4}{16} \Rightarrow \frac{1}{4}$

c.) $\tan 55 - \tan 35$

$\tan(55-35) = \frac{\tan 55 - \tan 35}{1 + \tan 55 \tan 35}$

$\Rightarrow \tan 20 = \frac{\tan 55 - \tan 35}{1 + \tan 55 \times \cot 55} \Rightarrow \tan 20 = \frac{\tan 55 - \tan 35}{1 + \cancel{\frac{\sin 55}{\cos 55}} \times \cancel{\frac{\cos 55}{\sin 55}}}$ $\quad 1$

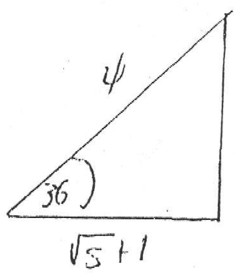
$\tan 55 - \tan 35 = 2 \tan 20$

d.) $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{3\pi}{5}\right)$ $\qquad \frac{\pi}{5} \times \frac{180}{\pi} = 36^\circ$

$3\left(\frac{\pi}{5}\right) = 108^\circ$

$\Rightarrow \cos(36)\cos(108)$

$\Rightarrow \cos 36\,(4\cos^3(36) - 3\cos 36)$



$\cos(2x) = 2\cos^2 x - 1$

$\cos(3x) = 4\cos^3 x - 3\cos x$

$\Rightarrow 4\cos^4 36 - 3\cos^2 36$

$\Rightarrow 4\left(\frac{\sqrt{5}+1}{4}\right)^4 - 3\left(\frac{\sqrt{5}+1}{4}\right)^2$

$\Rightarrow -1/4$

e.) $$\frac{\sin\left(\frac{3\pi}{2}+\alpha\right)\tan\left(\frac{\pi}{2}+\beta\right)\ \sin\left(\frac{3\pi}{2}-\beta\right)\cot\left(\frac{\pi}{2}+\alpha\right)}{\cos(\pi-\alpha)\cot\left(\frac{3\pi}{2}-\beta\right)\ \cos(2\pi-\beta)\tan(\pi-\alpha)}$$

$\frac{\pi}{2} = 90$

$3\left(\frac{\pi}{2}\right) = 270.$

$$\Rightarrow \frac{\sin(90+\alpha)\tan(90+\beta)\ \sin(270-\beta)\cot(90+\alpha)}{\cos(180-\alpha)\cot(270-\beta)\ \cos(360-\beta)\tan(\pi-\alpha)}$$

$\sin(90+\alpha) = \cancel{\sin 90}^{1}\cos\alpha + \sin\alpha\,\cancel{\cos 90}^{0} \Rightarrow \cos\alpha$

$\tan(90+\beta) = \frac{\sin(90+\beta)}{\cos(90+\beta)} = \frac{\cancel{\sin 90}^{1}\cos\beta + \sin\beta\,\cancel{\cos 90}^{0}}{\cancel{\cos 90}^{0}\cos\beta - \cancel{\sin 90}^{1}\sin\beta} \Rightarrow \frac{\cos\beta}{-\sin\beta} \Rightarrow -\cot\beta.$

$\sin(270-\beta) = \cancel{\sin 270}^{-1}\cos\beta - \sin\beta\,\cancel{\cos 270}^{0} = -\cos\beta$

$\cot(90+\alpha) = \frac{\cos(90+\alpha)}{\sin(90+\alpha)} = \frac{\cancel{\cos 90}^{0}\cos\alpha - \cancel{\sin 90}^{1}\sin\alpha}{\cancel{\sin 90}\cos\alpha + \sin\alpha\,\cancel{\cos 90}^{0}} = \frac{-\sin\alpha}{\cos\alpha} \Rightarrow -\tan\alpha$

$\cos(180-\alpha) = \cancel{\cos 180}^{-1}\cos\alpha + \cancel{\sin 180}^{0}\sin\alpha \Rightarrow -\cos\alpha$

$\cot(270-\beta) = \frac{\cos(270-\beta)}{\sin(270-\beta)} = \frac{\cancel{\cos 270}^{0}\cos\beta + \cancel{\sin 270}^{-1}\sin\beta}{\cancel{\sin 270}^{-1}\cos\beta - \sin\beta\,\cancel{\cos 270}^{0}} \Rightarrow \frac{-\sin\beta}{-\cos\beta} = \tan\beta.$

$\cos(360-\beta) = \cancel{\cos 360}^{1}\cos\beta + \cancel{\sin 360}^{0}\sin\beta \Rightarrow \cos\beta$

$\tan(180-\alpha) = \frac{\cancel{\tan 180}^{0} - \tan\alpha}{1 + \cancel{\tan 180}^{0}\tan\alpha} \Rightarrow -\tan\alpha$

$$\Rightarrow \frac{\cancel{\cos\alpha}(-\cot\beta)\ (+\cancel{\cos\beta})(-\cancel{\tan\alpha})}{+\cancel{\cos\alpha}\ \tan\beta\ (\cancel{\cos\beta})(-\cancel{\tan\alpha})} \Rightarrow -\frac{\cot\beta}{\tan\beta} \Rightarrow -\frac{\frac{\cos\beta}{\sin\beta}}{\frac{\sin\beta}{\cos\beta}} \Rightarrow -\frac{\cos^2\beta}{\sin^2\beta}$$

F.) $\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(\frac{4\pi}{65}\right)\cos\left(\frac{8\pi}{65}\right)\cos\left(\frac{16\pi}{65}\right)\cos\left(\frac{32\pi}{65}\right)$

$$a = \frac{\pi}{65}$$

$\Rightarrow \cos(a)\cdot\cos(2\cdot a)\cos(4a)\cos(8\cdot a)\cos(16a)\cos(32a)$

$\Rightarrow E = \cos(a)\cos(2a)\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(a)\cos(a)}_{\operatorname{sen}(2a)}\cdot\cos(2a)\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\cdot 2\operatorname{sen}(a)E = \underbrace{2\cdot\operatorname{sen}(2a)\cdot\cos(2a)}_{\operatorname{sen}(4a)}\cos(4a)\cos(8a)\cos(16a)\cos(32a)$

$2\cdot 4\operatorname{sen}(a)E = \underbrace{2\cdot\operatorname{sen}(4a)\cos(4a)}_{\operatorname{sen}(8a)}\cos(16a)\cos(32a)\cos(8a)$

$2\cdot 8\operatorname{sen}(a)E = \underbrace{2\operatorname{sen}(8a)\cos(8a)}_{\operatorname{sen}(16a)}\cos(16a)\cos(32a)$

$2\cdot 16\operatorname{sen}(a)E = \underbrace{2\cdot\operatorname{sen}(16a)\cos(16a)}\cos(32a)$

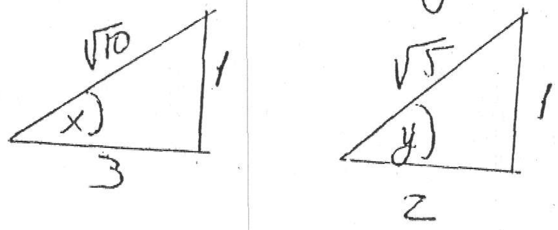
$2\cdot 32\operatorname{sen}(a)E = 2\cdot\operatorname{sen}(32a)\cos(32a)$

$64\operatorname{sen}(a)E = \operatorname{sen}(64a)$

$$E = \frac{\operatorname{sen}(64a)}{64\operatorname{sen} a} \Rightarrow \frac{\operatorname{sen}\left(\frac{64\pi}{65}\right)}{64\operatorname{sen}\left(\frac{\pi}{65}\right)}$$

$$\cos\left(\frac{\pi}{65}\right)\cos\left(\frac{2\pi}{65}\right)\cos\left(4\frac{\pi}{65}\right)\cos\left(8\frac{\pi}{65}\right)\cos\left(16\frac{\pi}{65}\right)\cos\left(32\frac{\pi}{65}\right) \Rightarrow \frac{\operatorname{sen}\left(\frac{64\pi}{65}\right)}{64\operatorname{sen}\left(\frac{\pi}{65}\right)}$$

34.– $\underbrace{\cos^{-1}\left(\frac{3}{\sqrt{10}}\right)}_{x} + \underbrace{\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)}_{y} = \frac{\pi}{4}$



$$x + y = \pi/4$$

$$\cos(x+y) = \cos \pi/4$$

$$\cos x \cos y - \sin x \sin y = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{3}{\sqrt{10}} \cdot \frac{2}{\sqrt{5}} - \frac{1}{\sqrt{10}} \cdot \frac{1}{\sqrt{5}}$$

$$\frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}} \Rightarrow \frac{5}{\sqrt{50}} \Rightarrow \frac{\cancel{5}}{\cancel{5}\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}$$ demostrado

35.–

a) $$\frac{\sin^3(\alpha - 270)\cos(360 - \alpha)}{\tan^3\left(\alpha - \frac{\pi}{2}\right)\cos^3\left(\alpha - \frac{3\pi}{2}\right)} = \cos\alpha$$

$$\sin(\alpha - 270) = \sin\alpha \overset{0}{\cancel{\cos 270}} - \overset{-1}{\cancel{\sin 270}} \cos\alpha = \cos\alpha$$

$$\cos(360-\alpha) = \cancel{\cos 360}^{1}\cos\alpha + \cancel{\sin 360}^{0}\sin\alpha = \cos\alpha$$

$$\tan(\alpha-90) = \frac{\sin(\alpha-90)}{\cos(\alpha-90)} = \frac{\sin\alpha \cancel{\cos 90}^{0} - \cancel{\sin 90}^{1}\cos\alpha}{\cos\alpha\cancel{\cos 90}^{0} + \cancel{\sin 90}^{1}\sin\alpha} \Rightarrow -\frac{\cos\alpha}{\sin\alpha}$$

$$\cos(\alpha-270) = \cos\alpha\cancel{\cos 270}^{0} + \sin\alpha\cancel{\sin 270}^{-1} = -\sin\alpha$$

$$\Rightarrow \frac{(\cos\alpha)^3\cdot\cos\alpha}{\left(-\frac{\cos\alpha}{\sin\alpha}\right)^3(-\sin\alpha)^3} \Rightarrow \frac{\cos^4\alpha}{\frac{\cancel{\cos^3\alpha}}{\cancel{\sin^3\alpha}}\cdot\cancel{\sin^3\alpha}} \Rightarrow \cos\alpha$$

demostrado

b) $$\frac{\cos^2\alpha}{\cot\left(\frac{\alpha}{2}\right)-\tan\left(\frac{\alpha}{2}\right)} = \frac{1}{4}\sin(2\alpha)$$

$$\cot\left(\frac{\alpha}{2}\right) = \frac{\cos(\alpha/2)}{\sin(\alpha/2)} \Rightarrow \frac{\sqrt{\frac{\cos x+1}{2}}}{\sqrt{\frac{-\cos\alpha+1}{2}}} \Rightarrow \sqrt{\frac{\frac{\cos x+1}{\cancel{2}}}{\frac{-\cos x+1}{\cancel{2}}}} \Rightarrow \frac{\sqrt{\cos x+1}}{\sqrt{-\cos x+1}}$$

$$\Rightarrow \frac{\cos^2\alpha}{\sqrt{\frac{\cos\alpha+1}{-\cos x+1}} - \sqrt{\frac{-\cos\alpha+1}{\cos\alpha+1}}} \Rightarrow \frac{\cos^2\alpha}{\frac{\sqrt{\cos\alpha+1}}{\sqrt{\cos\alpha+1}} - \frac{\sqrt{-\cos\alpha+1}}{\sqrt{\cos\alpha+1}}} \Rightarrow \frac{\cos^2\alpha}{\frac{(\sqrt{\cos\alpha+1})^2-(\sqrt{-\cos\alpha+1})^2}{\sqrt{-\cos\alpha+1}\sqrt{\cos\alpha+1}}}$$

$$\Rightarrow \frac{\cos^2\alpha}{\frac{\cos\alpha+\cancel{1}+\cos\alpha-\cancel{1}}{\sqrt{(1-\cos\alpha)(\cos\alpha+1)}}} \Rightarrow \frac{\cos^2\alpha}{\frac{2\cos\alpha}{\sqrt{1-\cos^2\alpha}}} \Rightarrow \frac{\cos^2\alpha}{\frac{2\cos\alpha}{\sqrt{1-\cos^2\alpha}}} \Rightarrow \frac{\cos\alpha\sqrt{1-\cos^2\alpha}}{2}$$

$$\Rightarrow \frac{\cos\alpha\sqrt{\sin^2\alpha}}{2} \Rightarrow \frac{1}{2}\sin\alpha\cdot\cos\alpha \Rightarrow \frac{1}{2}\cdot\frac{\overbrace{2\sin\alpha\cos\alpha}^{\sin(2\alpha)}}{2}$$

$$\Rightarrow \frac{1}{4}\sin(2\alpha)$$ demostrado

c) $\dfrac{\tan^2(2x)-\tan^2(x)}{1-\tan^2(2x)\tan^2(x)} = \tan(3x)\tan x.$

$\Rightarrow \dfrac{(\tan(2x)+\tan x)(\tan(2x)-\tan x)}{(1+\tan(2x)\tan x)(1-\tan(2x)\tan x)} \Rightarrow \underbrace{\left(\dfrac{\tan(2x)+\tan x}{1-\tan(2x)\tan x}\right)}_{\tan(2x+x)} \underbrace{\left(\dfrac{\tan(2x)-\tan x}{1+\tan(2x)\tan x}\right)}_{\tan(2x-x)}$

$\Rightarrow \tan(3x)\tan x$

d) $\sin w \sin(60-w)\sin\left(\overset{60}{\frac{\pi}{3}}+w\right) = \sin(3w)$

$\sin w \cdot (\sin 60\cos w - \sin w\cos 60)(\sin 60\cos w + \sin w\cos 60)$

$\sin w\left(\frac{\sqrt{3}}{2}\cos w - \frac{1}{2}\sin w\right)\left(\frac{\sqrt{3}}{2}\cos w + \frac{1}{2}\sin w\right)$

$\sin w\left(\frac{3}{4}\cos^2 w - \frac{1}{4}\sin^2 w\right) \Rightarrow \frac{1}{4}\sin w(3\cos^2 w - \sin^2 w)$

$\Rightarrow \frac{1}{4}\underbrace{(3\sin w\cos^2 w - \sin^3 w)}_{\sin(3w)} \Rightarrow \frac{1}{4}\sin(3w)$

$\sin(3w) = \sin(2w+w) \Rightarrow \sin(2w)\cos w + \sin w\cos(2w)$

$\Rightarrow 2\sin w\cos w \cdot \cos w + \sin w(\cos^2 w - \sin^2 w)$

$\Rightarrow 2\sin w\cos^2 w + \sin w\cos^2 w - \sin^3 w$

$\Rightarrow \boxed{3\sin w\cos^2 w - \sin^3 w.}$

e) $\sin 47 + \sin 61 - \sin 11 - \sin 25 = \cos 7$

$\sin 47 + \sin 61 - (\sin 25 + \sin 11)$

$2\sin 54\cos 7 - (2\sin 18\cos 7)$

$\cos 7(2\sin 54 - 2\sin 18)$

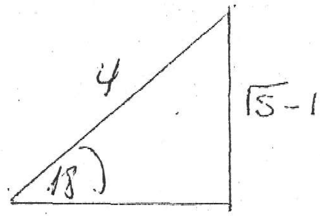
$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\sin(3x) = 3\sin x - 4\sin^3 x$

$\Rightarrow \cos 7\,(2(3\sin 18 - 4\sin^3 18) - 2\sin 18)$

$\Rightarrow \cos 7\,(6\sin 18 - 8\sin^3 18 - 2\sin 18)$

$\Rightarrow \cos 7\,(4\sin 18 - 8\sin^3 18)$



$\Rightarrow \cos 7\left(\cancel{4}\,\frac{(\sqrt{5}-1)}{\cancel{4}} - 8\left(\frac{\sqrt{5}-1}{4}\right)^3\right)$

$\Rightarrow \cos 7\left(\sqrt{5}-1 - \frac{\cancel{8}(\cancel{8}\sqrt{5}-\cancel{16})}{\cancel{64}}\right) \Rightarrow \cos 7\,(\cancel{\sqrt{5}}-1-\cancel{\sqrt{5}}+2)$

$\Rightarrow \cos 7$ demostrado

f) $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x} = \dfrac{1+\cos x}{\sin x}$

$\Rightarrow \dfrac{1+(\sin x+\cos x)}{1+(\sin x-\cos x)} \times \dfrac{1-(\sin x-\cos x)}{1-(\sin x-\cos x)} \Rightarrow \dfrac{1-(\sin x-\cos x)+(\sin x+\cos x)+\sin^2 x+\cos^2 x}{1-(\sin x-\cos x)^2}$

$\Rightarrow \dfrac{1-\cancel{\sin x}+\cos x+\cancel{\sin x}+\cos x+\sin^2 x+\cos^2 x}{1-(\sin^2 x+2\sin x\cos x+\cos^2 x)} \Rightarrow \dfrac{1+2\cos x+\sin^2 x+\cos^2 x}{1-(1-2\sin x\cos x)}$

$\Rightarrow \dfrac{1+2\cos x-(1-\cos^2 x)+\cos^2 x}{\cancel{1}-\cancel{1}+2\sin x\cos x} \Rightarrow \dfrac{\cancel{1}+2\cos x-\cancel{1}+\cos^2 x+\cos^2 x}{2\sin x\cos x}$

$\Rightarrow \dfrac{2\cos x+2\cos^2 x}{2\sin x\cos x} \Rightarrow \dfrac{\cancel{2}\cancel{\cos x}(1+\cos x)}{\cancel{2}\sin x\cancel{\cos x}}$

$\Rightarrow \dfrac{1+\cos x}{\sin x}$ demostrado

36.—

a) $\sin^2\theta(1+\cot^2\theta)=1$

$$\sin^2\theta\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right) \Rightarrow \cancel{\sin^2\theta}\left(\frac{\sin^2\theta+\cos^2\theta}{\cancel{\sin^2\theta}}\right)$$

$$\Rightarrow \underbrace{\sin^2\theta+\cos^2\theta}_{1}$$

$$\Rightarrow 1 \quad \text{VERDADERO}$$

b) $1-\csc^2\theta=-\cot^2\theta$

$$\Rightarrow 1-\frac{1}{\sin^2\theta} \Rightarrow \frac{\sin^2\theta-1}{\sin^2\theta} \Rightarrow \frac{-(1-\sin^2\theta)}{\sin^2\theta}$$

$$\Rightarrow -\frac{\cos^2\theta}{\sin^2\theta} \Rightarrow -\cot^2\theta \quad \text{VERDADERO}$$

c) $\sin\theta(\cot\theta+\tan\theta)=\sec\theta$

$$\Rightarrow \sin\theta\left(\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right) \Rightarrow \cancel{\sin\theta}\left(\frac{\underbrace{\cos^2\theta+\sin^2\theta}_{1}}{\cancel{\sin\theta}\cos\theta}\right)$$

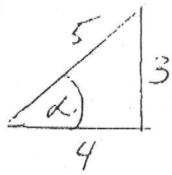
$$\Rightarrow \frac{1}{\cos\theta} \Rightarrow \sec\theta \quad \text{VERDADERO}$$

d.) $(1-\operatorname{sen}^2\theta)(1+\tan^2\theta)=-1$

$$\cos^2\theta\left(1+\frac{\operatorname{sen}^2\theta}{\cos^2\theta}\right) \Rightarrow \cancel{\cos^2\theta}\left(\frac{\cos^2\theta+\operatorname{sen}^2\theta}{\cancel{\cos^2\theta}}\right)$$

$\Rightarrow 1$ VERDADERO

37.— $\operatorname{sen}\alpha=-\frac{3}{5}$ 3er cuadrante

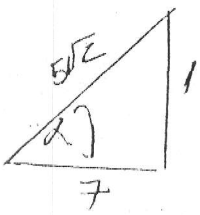


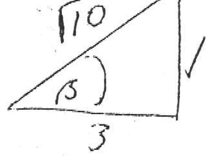
$$\tan(2\alpha)=\frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\cdot\tan\alpha}$$

$$\tan(2\alpha)=\frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} \Rightarrow \frac{\frac{3}{2}}{1-\frac{9}{16}} \Rightarrow \frac{\frac{3}{2}}{\frac{16-9}{16}}$$

$$\Rightarrow \frac{3}{\frac{7}{8}} \Rightarrow \frac{24}{7}$$

38.— $\tan\alpha=\frac{1}{7}$ $\operatorname{sen}\beta=\frac{1}{\sqrt{10}}$ 1er cuadrante





$$(\alpha+2\beta)=\pi/4$$

$$\cos(\alpha+2\beta)=\cos\pi/4$$

$$\Rightarrow \cos\alpha\cos(2\beta)-\operatorname{sen}\alpha\operatorname{sen}(2\beta)$$

$$\Rightarrow \cos\alpha\cdot(\cos^2\beta-\operatorname{sen}^2\beta)-\operatorname{sen}\alpha\cdot 2\operatorname{sen}\beta\cos\beta$$

$$\Rightarrow \cos\alpha(\cos^2\beta-\operatorname{sen}^2\beta)-2\operatorname{sen}\alpha\cos\beta\operatorname{sen}\beta$$

$$\Rightarrow \frac{7}{5\sqrt{2}}\left(\left(\frac{3}{\sqrt{10}}\right)^2-\left(\frac{1}{\sqrt{10}}\right)^2\right)-2\times\frac{1}{5\sqrt{2}}\,\frac{3}{\sqrt{10}}\cdot\frac{1}{\sqrt{10}}$$

$$\Rightarrow \frac{7}{5\sqrt{2}}\left(\frac{9}{10}-\frac{1}{10}\right)-\frac{6}{50\sqrt{2}} \Rightarrow \frac{7}{5\sqrt{2}}\left(\frac{\cancel{8}^{\,4}}{\cancel{10}_{\,5}}\right)-\frac{3}{25\sqrt{2}}$$

$$\Rightarrow \frac{28}{25\sqrt{2}}-\frac{3}{25\sqrt{2}} \Rightarrow \frac{\cancel{25}^{\,1}}{\cancel{25}_{\,1}\sqrt{2}} \Rightarrow \frac{\sqrt{2}}{2}=\cos\left(\frac{\pi}{4}\right)$$

demostrado.

39.— $2\tan\left(\frac{x}{2}\right)$ $x\in[0,90]$

$$\frac{2\cancel{\pi}}{\cancel{3}}\times\frac{\cancel{180^\circ}^{\,60}}{\cancel{\pi\,rad}} \Rightarrow 120^\circ$$

$$\frac{\pi}{2}=90^\circ$$

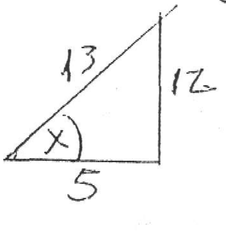
$$2\tan\left(\frac{120}{2}\right)-2\tan\left(\frac{90}{2}\right)$$

$$2\tan 60-\cancel{2\tan 45}^{\,1}$$

$$2\sqrt{3}-1$$

40.— $\operatorname{sen}(x)=-\frac{12}{13}$ 4º cuadrante.

$$\cos(x+60^\circ)=\cos x\cos 60-\operatorname{sen}x\operatorname{sen}60$$



$$\Rightarrow \frac{5}{13}\times\frac{1}{2}-\frac{12}{13}\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{5}{26}-\frac{12\sqrt{3}}{26}$$

$$\Rightarrow \frac{5-12\sqrt{3}}{26}$$

41.- $\tan(3x) = ?$

$$\tan(2x) = \tan(x+x)$$

$$\Rightarrow \frac{\tan x + \tan x}{1 - \tan x \cdot \tan x}$$

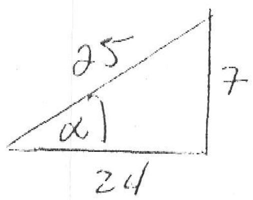
$$\Rightarrow \left(\frac{2\tan x}{1-\tan^2 x} \right)$$

$$\tan(2x+x) = \frac{\tan(2x) + \tan x}{1 - \tan(2x) \cdot \tan x}$$

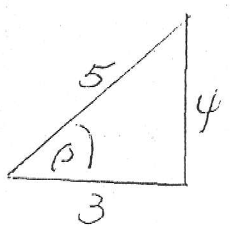
$$\Rightarrow \frac{\frac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \frac{2\tan x}{1-\tan^2 x} \cdot \tan x} \Rightarrow \frac{\frac{2\tan x + \tan x(1-\tan^2 x)}{1-\tan^2 x}}{1 - \frac{2\tan^2 x}{1-\tan^2 x}}$$

$$\Rightarrow \frac{\frac{2\tan x + \tan x - \tan^3 x}{\cancel{1-\tan^2 x}}}{\frac{1-\tan^2 x - 2\tan^2 x}{\cancel{1-\tan^2 x}}} \Rightarrow \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$$

42.-



(2do cuadrante)



(3er cuadrante)

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\Rightarrow \frac{24}{25} \times \frac{3}{5} - \frac{7}{25} \times \frac{4}{5}$$

$$\Rightarrow \frac{72}{75} - \frac{28}{75} \Rightarrow \frac{44}{75}$$

43.- a) $\frac{(2\operatorname{sen}^2\theta - 1)^2}{\operatorname{sen}^4\theta - \cos^4\theta} = 1 - 2\cos^2\theta$

$$\Rightarrow \frac{(2(1-\cos^2\theta)-1)^2}{\cancel{(\operatorname{sen}^2\theta+\cos^2\theta)}(\operatorname{sen}^2\theta - \cos^2\theta)} \Rightarrow \frac{(2-2\cos^2\theta-1)^2}{\operatorname{sen}^2\theta - \cos^2\theta}$$

$$\Rightarrow \frac{(1-2\cos^2\theta)^2}{1-\cos^2\theta-\cos^2\theta} \Rightarrow \frac{(1-2\cos^2\theta)^2}{\cancel{(1-2\cos^2\theta)}} \Rightarrow 1-2\cos^2\theta$$

b.) $\dfrac{2\tan x}{1-\tan^2 x} + \dfrac{1}{2\cos^2 x - 1} = \dfrac{\cos x + \operatorname{sen} x}{\cos x - \operatorname{sen} x}$

$\Rightarrow \dfrac{2\,\dfrac{\operatorname{sen} x}{\cos x}}{1-\dfrac{\operatorname{sen}^2 x}{\cos^2 x}} + \dfrac{1}{2\cos^2 x - 1} \Rightarrow \dfrac{\dfrac{2\operatorname{sen} x}{\cos x}}{\dfrac{\cos^2 x - \operatorname{sen}^2 x}{\cos^2 x}} + \dfrac{1}{2\cos^2 x - 1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x}{\cos^2 x - \operatorname{sen}^2 x} + \dfrac{1}{2\cos^2 x - 1} \Rightarrow \dfrac{2\operatorname{sen} x \cos x}{\cos^2 x - (1-\cos^2 x)} + \dfrac{1}{2\cos^2 x - 1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x}{\cos^2 x - 1 + \cos^2 x} + \dfrac{1}{2\cos^2 x - 1} \Rightarrow \dfrac{2\operatorname{sen} x\cos x}{2\cos^2 x - 1} + \dfrac{1}{2\cos^2 x - 1}$

$\Rightarrow \dfrac{2\operatorname{sen} x\cos x + 1}{2\cos^2 x - 1} \Rightarrow \dfrac{2\operatorname{sen} x\cos x + \operatorname{sen}^2 x + \cos^2 x}{\cos^2 x - \operatorname{sen}^2 x} \Rightarrow \dfrac{(\operatorname{sen} x + \cos x)^{\cancel{2}}}{\cancel{(\cos x + \operatorname{sen} x)}(\cos x - \operatorname{sen} x)}$

$\dfrac{\operatorname{sen} x + \cos x}{\cos x - \operatorname{sen} x}$

c.) $\tan\alpha + \dfrac{1}{\cos^3 x} - \dfrac{1}{\sec\alpha - \tan\alpha} = \dfrac{\operatorname{sen}^2\alpha}{\cos^3\alpha}$

$\Rightarrow \dfrac{\operatorname{sen}\alpha}{\cos\alpha} + \dfrac{1}{\cos^3 x} - \dfrac{1}{\dfrac{1}{\cos\alpha} - \dfrac{\operatorname{sen}\alpha}{\cos\alpha}} \Rightarrow$

$\Rightarrow \dfrac{\operatorname{sen}\alpha}{\cos\alpha} + \dfrac{1}{\cos^3\alpha} - \dfrac{1}{\dfrac{1-\operatorname{sen}\alpha}{\cos\alpha}}$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} + \frac{1}{\cos^3\alpha} - \frac{\cos\alpha}{1-\sin\alpha}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha(1-\sin\alpha) + 1-\sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha - \sin^2\cos^2\alpha + 1-\sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha\cos^2\alpha - \sin^2\alpha\cos^2\alpha + \sin^2\alpha + \cos^2\alpha - \sin\alpha - \cos^4\alpha}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{(\sin\alpha\cos^2\alpha - \sin\alpha) - (\sin^2\alpha\cos^2\alpha - \sin^2\alpha) + (\cos^2\alpha - \cos^4\alpha)}{(\cos^3\alpha)(1-\sin\alpha)}$$

$$\Rightarrow \frac{\sin\alpha(\cos^2\alpha-1) - \sin^2\alpha(\cos^2\alpha-1) + \cos^2\alpha(1-\cos^2\alpha)}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{-\sin\alpha(1-\cos^2\alpha) + \sin^2\alpha(1-\cos^2\alpha) + \cos^2\alpha(1-\cos^2\alpha)}{\cos^3\alpha(1-\sin\alpha)}$$

$$\Rightarrow \frac{(1-\cos^2\alpha)(-\sin\alpha + \cancel{\sin^2\alpha + \cos^2\alpha}^{\,1})}{\cos^3\alpha(1-\sin\alpha)} \Rightarrow \frac{\sin^2\alpha\,(\cancel{1-\sin\alpha})}{\cos^3\alpha\,(\cancel{1-\sin\alpha})}$$

$$\Rightarrow \frac{\sin^2\alpha}{\cos^3\alpha}$$

d) $$\frac{3\cos^2 z + 5\sin z - 5}{\cos^2 z} = \frac{3\sin z - 2}{1+\sin z}$$

$$\Rightarrow \frac{3(1-\sin^2 z) + 5(\sin z - 1)}{1-\sin^2 z} \Rightarrow \frac{3(1-\sin z)(1+\sin z) - 5(1-\sin z)}{(1+\sin z)(1-\sin z)}$$

$$\Rightarrow \frac{\cancel{(1-\operatorname{sen} z)}(3(1+\operatorname{sen} z)-5)}{(1+\operatorname{sen} z)\cancel{(1-\operatorname{sen} z)}} \Rightarrow \frac{3+3\operatorname{sen} z-5}{1+\operatorname{sen} z}$$

$$\Rightarrow \frac{3\operatorname{sen} z-2}{1+\operatorname{sen} z}$$

e.) $$\frac{2\operatorname{sen}^2 w+3\cos w-3}{\operatorname{sen}^2 w}=\frac{2\cos w-1}{1+\cos w}$$

$$\Rightarrow \frac{2(1-\cos^2 w)+3(\cos w-1)}{1-\cos^2 w} \Rightarrow \frac{2(1+\cos w)(1-\cos w)-3(1-\cos w)}{(1+\cos w)(1-\cos w)}$$

$$\Rightarrow \frac{\cancel{(1-\cos w)}(2(1+\cos w)-3)}{(1+\cos w)\cancel{(1-\cos w)}} \Rightarrow \frac{2+2\cos w-3}{1+\cos w}$$

$$\Rightarrow \frac{2\cos w-1}{1+\cos w}$$

F.) $$\frac{\operatorname{sen}^2 t+4\operatorname{sen} t+3}{\cos^2 t}=\frac{3+\operatorname{sen} t}{1-\operatorname{sen} t}$$

$$\Rightarrow \frac{\operatorname{sen}^2 t+\operatorname{sen} t+3\operatorname{sen} t+3}{1-\operatorname{sen}^2 t} \Rightarrow \frac{(\operatorname{sen}^2 t+\operatorname{sen} t)+(3\operatorname{sen} t+3)}{(1+\operatorname{sen} t)(1-\cos t)}$$

$$\Rightarrow \frac{\operatorname{sen} t(\operatorname{sen} t+1)+3(1+\operatorname{sen} t)}{(1+\operatorname{sen} t)(1-\operatorname{sen} t)} \Rightarrow \frac{\cancel{(1+\operatorname{sen} t)}(\operatorname{sen} t+3)}{\cancel{(1+\operatorname{sen} t)}(1-\operatorname{sen} t)}$$

$$\Rightarrow \frac{\operatorname{sen} t+3}{1-\operatorname{sen} t}$$

9.) $\sec y - \dfrac{\cos y}{1+\text{sen}\, y} = \tan y$

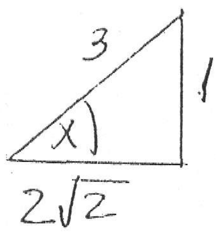
$$\frac{1}{\cos y} - \frac{\cos y}{1+\text{sen}\, y} \Rightarrow \frac{1+\text{sen}\, y-\cos^2 y}{(1+\text{sen}\, y)\cos y} \Rightarrow \frac{1+\text{sen}\, y-(1-\text{sen}^2 y)}{(1+\text{sen}\, y)\cos y}$$

$$\Rightarrow \frac{(1+\text{sen}\, y)-(1+\text{sen}\, y)(1-\text{sen}\, y)}{\cos y\,(1+\text{sen}\, y)} \Rightarrow \frac{\cancel{(1+\text{sen}\, y)}(\cancel{1}-\cancel{1}+\text{sen}\, y)}{\cos y\,\cancel{(1+\text{sen}\, y)}}$$

$$\Rightarrow \frac{\text{sen}\, y}{\cos y}$$

$$\Rightarrow \tan y$$

44.–



$C = \sqrt{3^2-1}$

$C = \sqrt{8}$

$C = 2\sqrt{2}$

a) $\cos x = \dfrac{2\sqrt{2}}{3}$

b) $\cos(2x) = \cos^2 x - \text{sen}^2 x$

$$\Rightarrow \left(\frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{3}\right)^2 \Rightarrow \frac{8}{9} - \frac{1}{9}$$

$$\Rightarrow \frac{7}{9}$$

c) $\text{sen}(2x) = 2\,\text{sen}\,x\cos x$

$$\Rightarrow 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\Rightarrow \frac{4\sqrt{2}}{9}$$

45.- $p(x): 2\operatorname{sen}^2(x) - 7\operatorname{sen}(x) + 3 = 0 \qquad x \in [0, \pi]$

$a = \operatorname{sen} x$

$2a^2 - 7a + 3 = 0$

$(2a - 6)(2a - 1) = 0$

$2a - 6 = 0 \qquad 2a - 1 = 0$

$2a = 6 \qquad 2a = 1$

$a = 3 \qquad a = 1/2$

$\Downarrow \qquad \Downarrow$

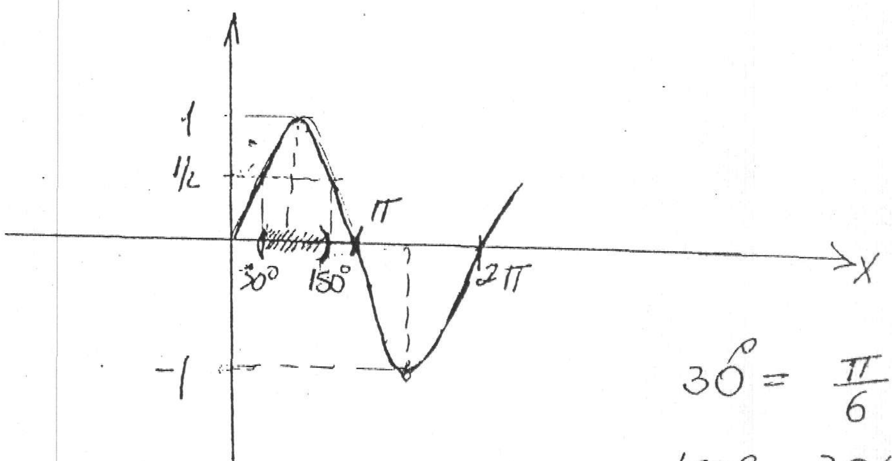
$\operatorname{sen} x \neq 3$ NO $\qquad \operatorname{sen} x = 1/2$

$x = 30°, 150°$

Suma: $30 + 150$

$\Rightarrow 180° \Rightarrow \pi$ rad $\quad$ a) CORRECTO

46.- $p(x): \operatorname{sen} x > 1/2 \quad ; x \in [0, 2\pi)$



$30° = \frac{\pi}{6}$

$150° = 30(5)$

$\Rightarrow \frac{\pi}{6}(5)$

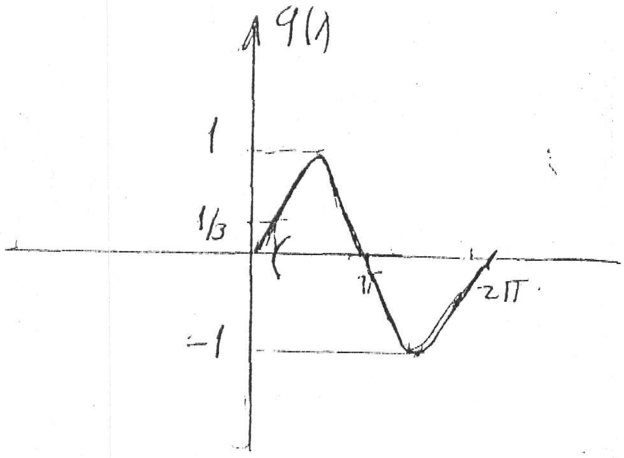
$\Rightarrow \frac{5\pi}{6}$

$Ap(x): (30°, 150°)$

$\Rightarrow \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

✓

47.- $g(x): \cos x < 1/3$; $x \in (0, 2\pi)$



$\cos x = 1/3$

$x = 70{,}52^\circ$

$A_{D}(x): (70{,}52^\circ, 360^\circ)$

48.- $f(x) = \cos x + \sqrt{3}\,\operatorname{sen} x$

a) $f(x) = R\cos(x - \alpha) \Rightarrow R\cos\alpha\cancel{\cos x} + R\cancel{\operatorname{sen} x}\operatorname{sen}\alpha$

$\cancel{\cos x} + \sqrt{3}\,\cancel{\operatorname{sen} x}$

$R\cos\alpha = 1$ $\qquad$ $R\operatorname{sen}\alpha = \sqrt{3}$

$R = \dfrac{1}{\cos\alpha}$ $\qquad$ $R = \dfrac{\sqrt{3}}{\operatorname{sen}\alpha}$

$$\frac{1}{\cos\alpha} = \frac{\sqrt{3}}{\operatorname{sen}\alpha}$$

$$\frac{\operatorname{sen}\alpha}{\cos\alpha} = \sqrt{3}$$

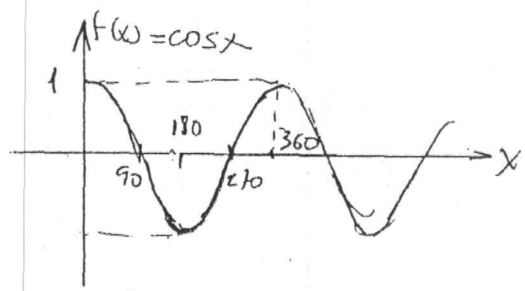
$$\tan\alpha = \sqrt{3}$$

$$\alpha = 60^\circ$$

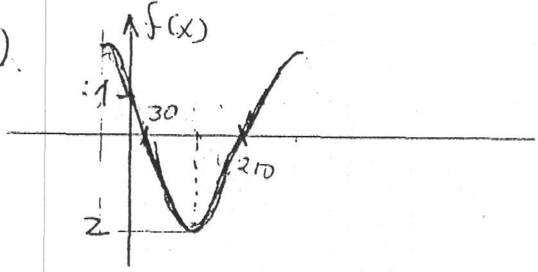
$$R = \frac{1}{\cos\alpha} = \sec 60^\circ = 2$$

b) Rango: $[-2, 2]$

c) $f(x) = 2\cos(x - 60)$



$f(x) = 2\cos(x-60)$



Si es inversible, porque es biyectiva en el intervalo [0, 90]

d) $f(x) = 2\cos(x-60) = \sqrt{2}$

$\cos(x-60) = \frac{\sqrt{2}}{2}$

$(x-60) = 45°$

$x = 105°$

49.- $P(x) = 2\operatorname{sen}^2 x = 1 - \cos x$

$\Rightarrow 2(1-\cos^2 x) = 1 - \cos x$

$\Rightarrow 2 - 2\cos^2 x = \cancel{1} - \cos x$

$\Rightarrow 2\cos^2 x - \cos x - 1 = 0$

$a = \cos x$

$\Rightarrow 2a^2 - a - 1 = 0$

$(2a-2)(2a+1) = 0$

$2a - 2 = 0 \qquad 2a + 1 = 0$

$2a = 2 \qquad 2a = -1$

$a = 1 \qquad a = -1/2$

$\cos x = 1$

$x = 0°, 360°$

$\cos x = -1/2$

$x = 135°, 225°$

Suma $= 0 + 135 + 225 + 360 = 720° \Rightarrow 2\pi$

d) CORRECTO

50.- $2\cos^2 x = \operatorname{sen}(2x)$; $x \in [0, \pi]$

$\Rightarrow 2\cos^2 x = 2\operatorname{sen}x\cos x$

$\Rightarrow 2\cos^2 x - 2\operatorname{sen}x\cos x = 0$

$\Rightarrow 2\cos x(\cos x - \operatorname{sen}x) = 0$

$\Rightarrow 2\cos x = 0$ $\qquad$ $\cos x - \operatorname{sen}x = 0$

$x = 0$ $\qquad$ $\cos x = \operatorname{sen}x$

$\dfrac{\operatorname{sen}x}{\cos x} = 1$

$\tan x = 1$

$x = 45°$

4.15

Solución

Ejercicios Propuestos

Capítulo cinco

MATEMATICAS

MATRICES y SISTEMAS DE ECUACIONES E INECUACIONES

1:- $A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad B=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

$$A+B=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$(A+B)^2=\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}\times\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

$$(A+B)^2 \Rightarrow \begin{pmatrix} (a+e)^2+(b+f)(c+g) & \text{—} \\ \text{—} & \text{—} \end{pmatrix}$$

$$(a+e)^2+(b+f)(c+g)=\underline{a^2+2ae+e^2+bc+bg+cf+fg}$$

$$A^2=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\times\begin{pmatrix} a & b \\ c & d \end{pmatrix}=\begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix}$$

$$B^2=\begin{pmatrix} e & f \\ g & h \end{pmatrix}\times\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} e^2+fg & ef+fh \\ eg+hg & fg+h^2 \end{pmatrix}$$

$$2AB=2\begin{pmatrix} a & b \\ c & d \end{pmatrix}\times\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} 2ae+2bg & 2af+2bh \\ 2ce+2dg & 2cf+2dh \end{pmatrix}$$

$$A^2+2AB+B^2=a^2+bc+2ae+2bg+e^2+fg$$

$$(A+B)^2 \neq A^2+B^2+2AB \qquad b)\ \text{FALSO}$$

2-

$$\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}_{2\times2} \cdot \underset{2\times3}{X} = \begin{pmatrix} 8 & -1 & 7 \\ 4 & 8 & -3 \end{pmatrix}_{2\times3}$$

$$\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = \begin{pmatrix} 8 & -1 & 7 \\ 4 & 8 & -3 \end{pmatrix}$$

$$\begin{cases} 4a+5d = 8 \quad (3) \\ 3a+4d = 4 \quad (-4) \end{cases}$$

$$\begin{cases} 4b+5e = -1 \quad (3) \\ 3b+4e = 8 \quad (-4) \end{cases}$$

$$\begin{cases} 3c+4f = -3 \quad (-4) \\ 4c+5f = 7 \quad (3) \end{cases}$$

$$\begin{array}{l} 12a+15d=24 \\ -12a-16d=-16 \\ \hline // \quad -d=8 \end{array}$$

$d=-8$

$4a+5d=8$

$4a=8-5(-8)$

$a=\frac{48}{4}$

$a=12$

$$\begin{array}{l} 12b+15e=-3 \\ -12b-16e=-32 \\ \hline -e=-35 \end{array}$$

$e=35$

$4b+5e=-1$

$4b=-1+5e$

$b=\frac{-1+-5(35)}{4}$

$b=-44$

$$\begin{pmatrix} 12 & -44 & 43 \\ -8 & 35 & -33 \end{pmatrix}$$

$$\begin{array}{l} -12c-16f=12 \\ 12c+15f=21 \\ \hline -f=33 \Rightarrow f=-33 \end{array}$$

$3c+4f=-3$

$3c=-3-4f$

$c=\frac{-3-4(-33)}{3}$

$c=43$

3.- a)

$$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \qquad a_{ij}=i+j-2$$

$$A=\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

b) $$B=\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}$$

Si es matriz diagonal

c) $A+B=C$

$$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}+\begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

NO ES MATRIZ IDENTIDAD

d) $$\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \\ d_{31} & d_{32} \end{pmatrix} \qquad d_{ij}=2i+3j-4$$

$$\Rightarrow \begin{pmatrix} 2(1)+3(1)-4 & 2(1)+3(2)-4 \\ 2(2)+3(1)-4 & 2(2)+3(2)-4 \\ 2(3)+3(1)-4 & 3(2)+3(2)-4 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix}$$

e) $$\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix}\times\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}=\begin{pmatrix} 4 & 9 \\ 6 & 15 \\ 8 & 21 \end{pmatrix}$$

ii) $\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix} \times \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & -8 \\ 0 & -12 \\ 0 & -16 \end{pmatrix}$

iii) $\begin{pmatrix} 1 & 4 \\ 3 & 6 \\ 5 & 8 \end{pmatrix} \times \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 6 & 3 \\ 8 & 5 \end{pmatrix}$

4.- $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$

$A = \begin{pmatrix} 0 & 3 & 4 & 5 \\ 3 & 0 & 5 & 6 \\ 4 & 5 & 0 & 7 \end{pmatrix}$

5.- $f(x,y) = x + y \in M_{3\times3}$ $\quad A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 1 & -1 \\ 1 & 1 & 4 \end{pmatrix}$

$A^t = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix}$

$\begin{array}{c} (-2) \\ (3) \\ (-3) \end{array} \left(\begin{array}{ccc|ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right) \Rightarrow (+1)(2) \left(\begin{array}{ccc|ccc} 3 & 2 & 4 & 1 & 0 & 0 \\ 0 & -1 & -11 & -2 & 3 & 0 \\ 0 & -1 & -8 & 1 & 0 & -3 \end{array}\right)$

$\left(\begin{array}{ccc|ccc} 3 & 0 & -18 & -3 & 6 & 0 \\ 0 & -1 & -11 & -2 & 3 & 0 \\ 0 & 0 & 3 & 3 & -3 & -3 \end{array}\right) (-11)(6) \left(\begin{array}{ccc|ccc} 1 & 0 & -6 & -1 & 2 & 0 \\ 0 & 1 & 11 & 2 & -3 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right)$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 5 & -4 & -6 \\ 0 & 1 & 0 & -9 & 8 & 11 \\ 0 & 0 & 1 & 1 & -1 & -1 \end{array}\right) \qquad A=\begin{pmatrix} 3 & 2 & 4 \\ 2 & 1 & -1 \\ 1 & 1 & 4 \end{pmatrix}$$

$$A^t=\begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix} \qquad A^{-1}=\begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix}$$

a) $f(A, A^t)=\begin{pmatrix} 6 & 4 & 5 \\ 4 & 2 & 0 \\ 5 & 0 & 8 \end{pmatrix}$

b) $f(A^T, A^{-1})=\begin{pmatrix} 8 & -2 & -5 \\ -7 & 9 & 12 \\ 5 & -2 & 3 \end{pmatrix}$

c) $f(A^{-1}, \underbrace{A \cdot A^{-1}}_{I}) \cdot \begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} 6 & -4 & -6 \\ -9 & 9 & 11 \\ 1 & -1 & 0 \end{pmatrix}$$

d) $f(A^{-1}, A^{-1}A^t)$

$$\begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} \times \begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -17 & 12 & -23 \\ 33 & -21 & 43 \\ -3 & 2 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 & -4 & -6 \\ -9 & 8 & 11 \\ 1 & -1 & -1 \end{pmatrix} + \begin{pmatrix} -17 & 12 & -23 \\ 33 & -21 & 43 \\ -3 & 2 & -4 \end{pmatrix} = \begin{pmatrix} -12 & 8 & -29 \\ 24 & -13 & 54 \\ -2 & 1 & -5 \end{pmatrix}$$

6.-

a) $B-2C = \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} -14 & -8 \\ 6 & 2 \\ 12 & -4 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -17 & -3 \\ 0 & 4 \\ 12 & -5 \end{pmatrix}$

b.) $9 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 9 & 0 \\ 0 & 9 \end{pmatrix}$

c.-) $BC^t \Rightarrow \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 7 & -3 & -6 \\ 4 & -1 & 2 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} -1 & 4 & 28 \\ -34 & 16 & 40 \\ -4 & 1 & -2 \end{pmatrix}$

d.-) $C^t \times B \Rightarrow \begin{pmatrix} 7 & -3 & -6 \\ 4 & -1 & 2 \end{pmatrix} \begin{pmatrix} -3 & 5 \\ -6 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 35 \\ -6 & 16 \end{pmatrix}$

$$7.- \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 4 \\ -1 & 0 & -2 \end{pmatrix}_{3\times 3} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}_{3\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}_{3\times 2}$$

$$\left.\begin{array}{l} a+2c+3e=0 \\ a+4c+4e=0 \\ -a \quad\quad -2e=0 \end{array}\right\}$$

SISTEMA HOMOGENEO,

SOLUCION TRIVIAL

$a=0$

$c=0$

$e=0$

$b+2d+3f=0$

$b+4d+4f=1$

$-b \quad 0 \quad -2f=1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 1 & 4 & 4 & 1 \\ -1 & 0 & -2 & 1 \end{array}\right)^{(-2)} \left(\begin{array}{ccc|c} 0 & 2 & 1 & 1 \\ 0 & 4 & 2 & 2 \\ -1 & 0 & -2 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 2 \\ -1 & 0 & -2 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 2 \\ 1 & 0 & 2 & -1 \end{array}\right)$$

$4d+2f=2$

$\boxed{2d+f=1} \qquad b+2f=-1$

$2d=1-f \qquad \boxed{b=-1-2f}$

$\boxed{d=\frac{1-f}{2}} \qquad f=-1$

$d=1$

$b=1$

$$X=\begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & -1 \end{pmatrix}$$

6) $\begin{pmatrix} 0 & -2 & 2 \\ 3 & 1 & 3 \\ 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 6 & 12 \\ 0 & 0 \end{pmatrix}$

$-2c + 2e = 3$

$3a + c + 3e = 6$

$a - 2c + 3e = 0$

$(-3)\left(\begin{array}{ccc|c} 0 & -2 & 2 & 3 \\ 3 & 1 & 3 & 6 \\ 1 & -2 & 3 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -2 & 2 & 3 \\ 0 & 7 & -6 & 6 \\ 1 & -2 & 3 & 0 \end{array}\right) \overset{(7)(-1)}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -2 & 2 & 3 \\ 0 & 7 & -6 & 6 \end{array}\right)(2)$

$\Rightarrow 2\left(\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & -2 & 2 & 3 \\ 0 & 0 & -2 & 33 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 0 & 27 \\ 0 & -2 & 0 & 36 \\ 0 & 0 & -2 & 33 \end{array}\right)$

$2a = 27$

$\boxed{a = \frac{27}{2}}$

$-2c = 36$

$\boxed{c = -18}$

$-2e = 33$

$\boxed{e = -33/2}$

$-2d + 2f = 6$

$3b + d + 3f = 12$

$b - 2d + 3f = 0$

$\Rightarrow (-3)\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & 1 & 3 & 12 \\ 0 & -2 & 2 & 6 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 7 & -6 & 12 \\ 0 & -1 & 1 & 3 \end{array}\right) \begin{matrix}(7) \\ (-2)\end{matrix} \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & -1 & 1 & 3 \\ 0 & 7 & -6 & 12 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & -6 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & -1 & -33 \end{array}\right)$

$\Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -39 \\ 0 & -1 & 0 & -30 \\ 0 & 0 & 1 & 33 \end{array}\right)$

$\boxed{b = -39}$

$\boxed{d = 30}$

$\boxed{f = 33}$

$X = \frac{1}{2}\begin{pmatrix} 27 & -78 \\ -36 & 60 \\ -33 & 66 \end{pmatrix}$

8

c) $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 6 & 9 & 8 \\ 0 & 1 & 6 \end{pmatrix}$

$a + 2b + 3c = 6$
$2a + 3b + 4c = 9$
$3a + 4b + c = 8$

$$\begin{matrix}(-3)(-2)\end{matrix}\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 3 & 4 & 9 \\ 3 & 4 & 1 & 8 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -8 & -10 \end{array}\right) \Rightarrow^{(2)} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -2 & -3 \\ 0 & +1 & +4 & +5 \end{array}\right)$$

$$(2)\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 2 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

$a = 1$
$b = 1$
$c = 1$

$d + 2e + 3f = 0$
$2d + 3e + 4f = 1$
$3d + 4e + f = 6$

$$(-3)(-2)\left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 6 \end{array}\right) \Rightarrow^{(2)} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 1 \\ 0 & +2 & +8 & -6 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 4 & -4 \end{array}\right)$$

$$(2)\left(\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$d = 1$
$e = 1$
$f = -1$

$$X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

d) $X\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -1 & 1\end{pmatrix}$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}\times\begin{pmatrix}3 & 2\\ -2 & -1\end{pmatrix}=\begin{pmatrix}-1 & 2\\ -1 & 1\end{pmatrix}$$

$$(-2)\begin{cases}3a-2b=-1\\ 2a-b=2\end{cases}$$

$\Rightarrow -b=2-2a$

$b=2a-2$

$$\begin{array}{r}3a-2b=-1\\ -4a+2b=-4\\ \hline -a=-5\end{array}$$

$a=5$

$b=2(5)-2$

$b=8$

$$(-2)\begin{cases}3c-2d=-1\\ 2c-d=1\end{cases}$$

$$\begin{array}{r}3c-2d=-1\\ -4c+2d=-2\\ \hline -c=-3\end{array}$$

$c=3$

$b=2c-1$

$b=2(3)-1$

$b=5$

$$X=\begin{pmatrix}5 & 8\\ 3 & 5\end{pmatrix}$$

8.-

$$A=\begin{pmatrix}4 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}\quad A^2=\begin{pmatrix}16 & 0 & 0\\ 4 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$A^3=\begin{pmatrix}16 & 0 & 0\\ 4 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}\times\begin{pmatrix}4 & 0 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}=\begin{pmatrix}64 & 0 & 0\\ 16 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 - 3B^2$$

$$\begin{pmatrix} 64 & 0 & 0 \\ 16 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} -6 & -3 & -3 \\ -3 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 58 & -3 & -3 \\ 13 & -3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

b)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad B = \begin{pmatrix} x & y \\ z & w \end{pmatrix} \quad A^t = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \quad B^t = \begin{pmatrix} x & z \\ y & w \end{pmatrix}$$

$$A \times B = \begin{pmatrix} ax + bz & ay + bw \\ cx + dz & cy + dw \end{pmatrix} \quad B^t \times A^t = \begin{pmatrix} ax + bz & cx + zd \\ ay + bw & cy + dw \end{pmatrix}$$

$$(A \times B)^t = \begin{pmatrix} ax + bz & cx + dz \\ ay + bw & cy + dw \end{pmatrix}$$

demostrado.

9.-

$$A = \begin{pmatrix} \cos\beta & 0 & \operatorname{sen}\beta \\ 0 & 1 & 0 \\ -\operatorname{sen}\beta & 0 & \cos\beta \end{pmatrix}$$

$$\begin{array}{c} (\operatorname{sen}\beta) \\ \\ (\cos\beta) \end{array} \left(\begin{array}{ccc|ccc} \cos\beta & 0 & \operatorname{sen}\beta & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -\operatorname{sen}\beta & 0 & \cos\beta & 0 & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} \\ \\ (-\operatorname{sen}\beta) \end{array} \left(\begin{array}{ccc|ccc} \cos\beta & 0 & \operatorname{sen}\beta & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} \cos\beta & 0 & 0 & 1-\operatorname{sen}^2\beta & 0 & -\operatorname{sen}\beta\cos\beta \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} \frac{\cos\beta}{\cos\beta} & 0 & 0 & \frac{\cos^2\beta}{\cos\beta} & 0 & -\frac{\operatorname{sen}\beta\cos\beta}{\cos\beta} \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \cos\beta & 0 & -\operatorname{sen}\beta \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \operatorname{sen}\beta & 0 & \cos\beta \end{array}\right) \qquad A^T = \begin{pmatrix} \cos\beta & 0 & -\operatorname{sen}\beta \\ 0 & 1 & 0 \\ \operatorname{sen}\beta & 0 & \cos\beta \end{pmatrix}$$

A^{-1}

DEMOSTRADO

10.- $A = \begin{pmatrix} 3 & 1 \\ -5 & 6 \end{pmatrix}$ $\quad B = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$ $\quad X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$AX + X = B$$

$$AX = \begin{pmatrix} 3a+c & 3b+d \\ -5a+6c & -5b+6d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 0 & -3 \end{pmatrix}$$

$$\begin{cases} 4a+c = 4 & (-7) \\ -5a+7c = 0 \end{cases} \qquad \begin{cases} 4b+d = 8 & (-7) \\ -5b+7d = -3 \end{cases}$$

$$\begin{array}{r} -28b-7d = -56 \\ -5b+7d = -3 \\ \hline -33b \quad = -59 \end{array}$$

$$b = \frac{59}{33}$$

$$\begin{array}{r} -28a-7c = -28 \\ -5a+7c = 0 \\ \hline -33a \quad = -28 \end{array}$$

$$a = \frac{28}{33}$$

$c = 4-4a$

$c = 4-4\left(\frac{28}{33}\right)$

$$c = \frac{20}{33}$$

$$\Rightarrow \frac{1}{33}\begin{pmatrix} 28 & 59 \\ 20 & 28 \end{pmatrix}$$

$d = 8-4b \Rightarrow 4\left(2-\frac{59}{33}\right)$

$$d = \frac{28}{33}$$

11.— $A=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$ $B=\begin{pmatrix}1 & 0\\ 0 & 1\\ 1 & 0\end{pmatrix}$ $C=\begin{pmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}$

$A\times B=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $B\times C=$ (NO SE PUEDE REALIZAR LA OPERACION.)

12.— $M=\begin{pmatrix}a & 2\\ 2 & -1\end{pmatrix}$

a)

$$M^2=\begin{pmatrix}a & 2\\ 2 & -1\end{pmatrix}\times\begin{pmatrix}a & 2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}a^2+4 & 2a-2\\ 2a-2 & 5\end{pmatrix}$$

b) $M^2=\begin{pmatrix}5 & -4\\ -4 & 5\end{pmatrix}$

$2a-2=-4$

$2a=-2$

$a=-1$

13.— $A=\begin{pmatrix}3 & 2\\ 4 & 3\end{pmatrix}$ $B=\begin{pmatrix}5 & 1 & 0 & -3\end{pmatrix}$ $C=\begin{pmatrix}2 & 0\\ 1 & -4\\ 3 & 1\\ 0 & -1\end{pmatrix}$ $D=\begin{pmatrix}10 & 15 & -5\\ 11 & 10 & 10\end{pmatrix}$

a)

$$A\cdot D=\begin{pmatrix}52 & 65 & 5\\ 73 & 90 & 10\end{pmatrix}$$

b) $A^t=\begin{pmatrix}3 & 4\\ 2 & 3\end{pmatrix}\times\begin{pmatrix}2 & 1 & 3 & 0\\ 0 & -4 & 1 & -1\end{pmatrix}$

$$\Rightarrow\begin{pmatrix}6 & -13 & 7 & -4\\ 4 & -8 & 9 & -3\end{pmatrix}$$

c) $CD = \begin{pmatrix} 2 & 0 \\ 1 & -4 \\ 3 & 1 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 10 & 15 & -5 \\ 11 & 10 & 10 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 20 & 30 & -10 \\ -34 & -25 & -45 \\ 41 & 55 & -5 \\ -11 & -10 & -10 \end{pmatrix}$

d) $C^t B^t$

$\begin{pmatrix} 2 & 1 & 3 & 0 \\ 0 & -4 & 1 & -1 \end{pmatrix} \times \begin{pmatrix} 8 \\ 1 \\ 0 \\ -3 \end{pmatrix} = \begin{pmatrix} 11 \\ -1 \end{pmatrix}$

14.- $B = \begin{pmatrix} 2 & 1 \\ 3 & 1 \\ 0 & 6 \end{pmatrix}$ $A = \begin{pmatrix} 1 & 2 \\ 5 & 3 \end{pmatrix}$

$B \times A = \begin{pmatrix} 7 & 7 \\ 8 & 9 \\ 30 & 18 \end{pmatrix} \Rightarrow (B \times A)^t = \begin{pmatrix} 7 & 8 & 30 \\ 7 & 9 & 18 \end{pmatrix}$ DEMOSTRADO

$\begin{pmatrix} 1 & 5 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 3 & 0 \\ 1 & 1 & 6 \end{pmatrix} = \begin{pmatrix} 7 & 8 & 30 \\ 7 & 9 & 18 \end{pmatrix}$

15.- $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

$\left(\begin{array}{cc|cc} a & 0 & 1 & 0 \\ 0 & b & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1/a & 0 \\ 0 & 1 & 0 & 1/b \end{array}\right)$

$A^{-1} = \begin{pmatrix} 1/a & 0 \\ 0 & 1/b \end{pmatrix} \Rightarrow$ MATRIZ IDENTIDAD, (I) (-I)

16.—

$$A^2 = 2A + I$$

$$A^2 - 2A - I = 0$$

$$(A - 2I)(A + I) = 0$$

$$A - 2I = 0 \qquad A = -I$$

$$A = 2I$$

17.—

$$B = \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix}$$

$$B^2 = 2B + I$$

$$B^2 = \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix} \times \begin{pmatrix} 1+m & 0 \\ 0 & 1-m \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (1+m)^2 & 0 \\ 0 & (1-m)^2 \end{pmatrix} = \begin{pmatrix} 2+2m & 0 \\ 0 & 2-2m \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} (1+m)^2 & 0 \\ 0 & (1-m)^2 \end{pmatrix} = \begin{pmatrix} 3+2m & 0 \\ 0 & 3-2m \end{pmatrix}$$

$$(1+m)^2 = 3 + 2m$$

$$1 + \cancel{2m} + m^2 = 3 + \cancel{2m}$$

$$m^2 = 2$$

$$m = \pm\sqrt{2}$$

$ad - \cancel{bc} - ab + \cancel{bc}$

18.—

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\cancel{\begin{array}{c} (-a) \\ (a) \end{array}\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} (ad-bc) \\ -b \end{array}\left(\begin{array}{cc|cc} a & b & 1 & 0 \\ 0 & ad-bc & a-c & a \end{array}\right) \left(\begin{array}{cc|cc} a^2d-abc & 0 & ad-ab & ab \\ 0 & ad-bc & a-c & a \end{array}\right)}$$

$A \sim B$

$$\underset{(-2)}{}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

NO SON SEMEJANTES

19.- $\det(A) = 0$

$2|x| - |x-2| = 0$

$2|x| = |x-2|$

$2x = |x-2| \qquad\qquad -2x = |x-2|$

$2x = x-2 \wedge 2x = -x+2 \qquad -2x = x-2 \wedge -2x = x+2$

$\boxed{x=-2}$ ✓ $\quad 3x = 2 \qquad -3x = -2 \qquad \boxed{x=-2}$

$\boxed{x = 2/3} \qquad \boxed{x = 2/3}$

Sol: $\{-2, 2/3\}$

20.-

a) $\left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 4 & -8 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$

$$\left(\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -4 & 8 & 4 & 1 & 0 \\ 0 & 4 & 6 & 3 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} -4 & 0 & 16 & 8 & 1 & 0 \\ 0 & -4 & 8 & 4 & 1 & 0 \\ 0 & 0 & 14 & 7 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} -28 & 0 & 0 & 0 & -1 & -8 \\ 0 & -28 & 0 & 0 & 3 & -4 \\ 0 & 0 & 14 & 7 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/28 & 8/28 \\ 0 & 0 & 0 & 0 & -3/28 & 4/28 \\ 0 & 0 & 1 & 7/14 & 1/14 & 1/14 \end{array}\right)$$

$$A^{-1}=\begin{pmatrix} 0 & 1/28 & 8/28 \\ 0 & -3/28 & 4/28 \\ 7/14 & 1/14 & 1/14 \end{pmatrix}$$

$$A^{-1}=\frac{1}{28}\begin{pmatrix} 0 & 1 & 8 \\ 0 & -3 & 4 \\ 14 & 2 & 2 \end{pmatrix}$$

b)

$$B=\begin{pmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$B^{-1} \overset{(-2)}{=} \left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\overset{(-2)}{\Rightarrow} (-2)\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 1 & -2 & 0 \\ 0 & 0 & 4 & 1 & -2 & 2 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} -8 & 0 & 0 & -1 & -2 & 2 \\ 0 & 4 & 0 & -1 & 2 & 2 \\ 0 & 0 & 4 & 1 & -2 & 2 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & +1/8 & +1/4 & -1/4 \\ 0 & 1 & 0 & -1/4 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/4 & -1/2 & 1/2 \end{array}\right)$$

$$AB^{-1} \Rightarrow \begin{pmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 1/8 & 1/4 & -1/4 \\ -1/4 & 1/2 & 1/2 \\ 1/4 & -1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1/8 & -3/4 & 7/4 \\ 5/2 & -3 & -5 \\ 1/8 & 5/4 & -1/4 \end{pmatrix}$$

$\frac{1}{8}-\frac{1}{4}+\frac{1}{2}=\frac{1}{8}$ $\quad \frac{1}{2}+2=5/2$ $\quad \frac{3}{8}-\frac{1}{4}=\frac{1}{8}$ $\quad -\frac{3}{4}+\frac{1}{2}=-\frac{1}{4}$

$\frac{1}{4}+\frac{1}{2}-1=-3/4$ $\quad 1-4=-3$ $\quad \frac{3}{4}+\frac{1}{2}=5/4$

$\frac{1}{4}+\frac{1}{2}+1=7/4$ $\quad -1-4=-5$

c)

$$A \times B \Rightarrow \begin{pmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 3 & 0 \\ 0 & -8 & 8 \\ 14 & 1 & 6 \end{pmatrix}$$

$$-2 \begin{pmatrix} 4 & 1 & 3 \\ 0 & -4 & 4 \\ 2 & 8 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} -8 & -2 & -6 \\ 0 & 8 & -8 \\ -4 & -16 & -4 \end{pmatrix}$$

$$A \times B - 2C^t \Rightarrow \begin{pmatrix} -2 & 3 & 0 \\ 0 & -8 & 8 \\ 14 & 1 & 6 \end{pmatrix} + \begin{pmatrix} -8 & -2 & -6 \\ 0 & 8 & -8 \\ -4 & -16 & -4 \end{pmatrix}$$

$$\det. \begin{pmatrix} -10 & 1 & -6 \\ 0 & 0 & 0 \\ 10 & -15 & 2 \end{pmatrix} = 0$$

d)

$$\det. \begin{vmatrix} -1 & 1 & 2 \\ 4 & -8 & 0 \\ 3 & 1 & 0 \end{vmatrix} = 2 \begin{vmatrix} 4 & -8 \\ 3 & 1 \end{vmatrix} \Rightarrow 2(4 - (-24))$$

$$\Rightarrow 56$$

$$\det \begin{vmatrix} 4 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow 4 \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix}$$

$$\Rightarrow 4(1-0) + 2(2-0)$$

$$\Rightarrow 4 + 4$$

$$\Rightarrow 8$$

$$\det \begin{vmatrix} 4 & 0 & 2 \\ 1 & -4 & 8 \\ 3 & 4 & 2 \end{vmatrix} \Rightarrow 4 \begin{vmatrix} -4 & 8 \\ 4 & 2 \end{vmatrix} + 2 \begin{vmatrix} 1 & -4 \\ 3 & 4 \end{vmatrix}$$

$\Rightarrow 4\left(-8-32\right)+2\left(4-(-12)\right)$

$\Rightarrow -160+32$

$\Rightarrow -128.$

det A + det B + det C

56 + 8 + 128

$\Rightarrow 192$

e) $C = \begin{pmatrix} 4 & 0 & 2 \\ 1 & -4 & 8 \\ 3 & 4 & 2 \end{pmatrix}$

$$\Rightarrow C^{-1} = \begin{array}{c} (-3) \\ (-4) \\ (4) \end{array}\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 1 & -4 & 8 & 0 & 1 & 0 \\ 3 & 4 & 2 & 0 & 0 & 1 \end{array}\right) \Rightarrow (-1)\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & 16 & -30 & 1 & -4 & 0 \\ 0 & 16 & 2 & -3 & 0 & 4 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & 16 & -30 & 1 & -4 & 0 \\ 0 & 0 & 32 & -4 & 4 & 4 \end{array}\right) \Rightarrow \begin{array}{c} (-4) \\ (4) \\ (15) \end{array}\left(\begin{array}{ccc|ccc} 4 & 0 & 2 & 1 & 0 & 0 \\ 0 & 16 & -30 & 1 & -4 & 0 \\ 0 & 0 & 8 & -1 & 1 & 1 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 16 & 0 & 0 & -5 & 1 & 1 \\ 0 & 64 & 0 & -11 & -1 & 15 \\ 0 & 0 & 8 & -1 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5/16 & 1/16 & 1/16 \\ 0 & 1 & 0 & -11/64 & -1/64 & 15/64 \\ 0 & 0 & 1 & -1/8 & 1/8 & 1/8 \end{array}\right)$$

$$B^{-1} \times C^{-1} \Rightarrow \begin{pmatrix} 1/8 & 1/4 & -1/4 \\ -1/4 & 1/2 & 1/2 \\ 1/4 & -1/2 & 1/2 \end{pmatrix} \times \begin{pmatrix} -5/16 & 1/16 & 1/16 \\ -11/64 & -1/64 & 15/64 \\ -1/8 & 1/8 & 1/8 \end{pmatrix} = \begin{pmatrix} -\frac{13}{256} & -7/256 & 9/256 \\ -58/256 & 19/256 & 42/256 \\ -14/256 & 22/256 & -10/256 \end{pmatrix}$$

21.- $\begin{pmatrix} -1 & 0 & -2 \\ k & -k & 3 \\ \frac{-k^2}{2} & -3 & -2 \end{pmatrix} \times \begin{pmatrix} -2 & -10 & 1 \\ -k & \frac{-k^3}{3} & k^5 \\ -1 & -2k & 3 \end{pmatrix}$

$-2k + k^2 - 3$
$-10k + \frac{k^4}{3} - 6k$
$k - k^6 - 6$

$\Rightarrow \begin{pmatrix} 4 & 10+4k & -7 \\ k^2-2k-3 & \frac{k^4}{3}-16k & -k^6+k-6 \\ k^2+3k+2 & k^3+5k^2+4k & -3k^5-\frac{k^2}{2}-6 \end{pmatrix}$

k^2+3k+2
$5k^2+k^3+4k$
$-\frac{k^2}{2}-3k^5-6$

$k^2-2k-3=0$	$k^2+3k+2=0$	$k^3+5k^2+4k=0$
$(k-3)(k+1)=0$	$(k+2)(k+1)=0$	$k(k^2+5k+4)=0$
$k=3$ / $k=-1$	$k=-2$ / $k=-1$	$k(k+4)(k+1)=0$
		$k=0$ / $k=-4$ / $k=-1$

Sol : $k=-1$

22.-

a) $\begin{vmatrix} x & 1 \\ -1 & 1/x \end{vmatrix}$ $1-(-1) \Rightarrow 2$; si es inversible

$\begin{vmatrix} e^x & e^{-2x} \\ e^{2x} & e^{3x} \end{vmatrix} \Rightarrow e^{4x} - e^{-4x} \neq 0$; si es inversible

$\begin{vmatrix} 1 & e^x & 0 \\ e^x & -e^{2x} & 0 \\ 0 & 0 & 0 \end{vmatrix}$; det=0 ; NO es inversible

$\begin{vmatrix} \operatorname{sen} x & \cos x \\ -\cos x & \operatorname{sen} x \end{vmatrix}$ det= $\operatorname{sen}^2 x + \cos^2 x$ $\Rightarrow 1$; si es inversible

b) i)
$$\begin{array}{c} \\ (x) \end{array}\left(\begin{array}{cc|cc} x & 1 & 1 & 0 \\ -1 & 1/x & 0 & 1 \end{array}\right) \Rightarrow \begin{array}{c} (-2) \\ \\ \end{array}\left(\begin{array}{cc|cc} x & 1 & 1 & 0 \\ 0 & 2 & 1 & x \end{array}\right)$$

$$\left(\begin{array}{cc|cc} -2x & 0 & -1 & x \\ 0 & 2 & 1 & x \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 1/2x & -1/2 \\ 0 & 1 & 1/2 & x/2 \end{array}\right) \Rightarrow \begin{pmatrix} \frac{1}{2x} & -\frac{1}{2} \\ \frac{1}{2} & \frac{x}{2} \end{pmatrix}$$

ii)
$$\begin{array}{c} -e^x \\ \\ \end{array}\left(\begin{array}{cc|cc} e^x & e^{-2x} & 1 & 0 \\ e^{2x} & e^{3x} & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} e^x & e^{-2x} & 1 & 0 \\ 0 & e^{3x}-e^{-x} & -e^x & 1 \end{array}\right)\begin{array}{c} (e^{3x}-e^{-x}) \\ (-e^{-2x}) \end{array}$$

$$\left(\begin{array}{cc|cc} e^{4x}-1 & 0 & e^{3x}-e^{-x} & -e^{-2x} \\ 0 & e^{3x}-e^{-x} & -e^x & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{e^{3x}-e^{-x}}{e^{4x}-1} & -\frac{e^{-2x}}{e^{4x}-1} \\ 0 & 1 & \frac{-e^x}{e^{3x}-e^{-x}} & \frac{1}{e^{3x}-e^{-x}} \end{array}\right)$$

$$\frac{-e^x}{e^{3x}-e^{-x}} \rightarrow \frac{\frac{-e^x}{e^{-x}}}{\frac{e^{3x}}{e^{-x}}-\frac{e^{-x}}{e^{-x}}} \Rightarrow \frac{-e^{2x}}{e^{4x}-1}$$

$$\frac{1}{e^{3x}-e^{-x}} \rightarrow \frac{\frac{1}{e^{-x}}}{\frac{e^{3x}}{e^{-x}}-\frac{e^{-x}}{e^{-x}}} \Rightarrow \frac{e^x}{e^{4x}-1}$$

$$A^{-1} = \frac{1}{e^{4x}-1}\begin{pmatrix} e^{3x}-e^{-x} & -e^{-2x} \\ -e^{2x} & e^x \end{pmatrix}$$

iii)
$$\left(\begin{array}{cc|cc} \operatorname{sen} x & \cos x & 1 & 0 \\ -\cos x & \operatorname{sen} x & 0 & 1 \end{array}\right)\begin{array}{c} (\cos x) \\ (\operatorname{sen} x) \end{array}$$

$$\Rightarrow \begin{array}{c} \\ -\cos x \end{array}\left(\begin{array}{cc|cc} \operatorname{sen} x & \cos x & 1 & 0 \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} \operatorname{sen} x & 0 & -\cos^2 x & -\operatorname{sen} x \cos x \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & -\frac{\cos^2 x}{\operatorname{sen} x} & -\cos x \\ 0 & 1 & \cos x & \operatorname{sen} x \end{array}\right) \Rightarrow \begin{pmatrix} -\operatorname{cotg} x \cdot \cos x & -\cos x \\ \cos x & \operatorname{sen} x \end{pmatrix}$$

23. a) $\det(A) = \begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0$

$\det(B) = \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$

b)

$A \times B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad$ $B \times A = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$

$A \times B \neq B \times A$.

c) $C \times D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad$ $D \times C = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$C \times D \neq D \times C$

24.

$$\begin{pmatrix} a & 0 \\ -a & a \end{pmatrix} + \begin{pmatrix} b & 0 \\ b & b \end{pmatrix} + \begin{pmatrix} -2c & c \\ 2c & 3c \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ -7 & -6 \end{pmatrix}$$

$c = -2$

$a + b - 2c = 8$

$a + b = 8 + 2c$

$\boxed{a + b = 4}$ $\Rightarrow$ $\underbrace{a+b}+c$

$4 - 2$

$\Rightarrow 2$

$-a + b + 2c = -7$

$-a + b = -7 - 2c$

$\boxed{-a + b = -3}$

25.-

$$A=\begin{pmatrix}3 & -2\\ -3 & 4\end{pmatrix} \qquad -\lambda I=\begin{pmatrix}-\lambda & 0\\ 0 & -\lambda\end{pmatrix}$$

$$A-\lambda I=\begin{pmatrix}3-\lambda & -2\\ -3 & 4-\lambda\end{pmatrix}$$

Matriz singular $\Rightarrow$ $\det(M)=0$

$$\begin{vmatrix}3-\lambda & -2\\ -3 & 4-\lambda\end{vmatrix}=0$$

$$(3-\lambda)(4-\lambda)-6=0$$

$$12-3\lambda-4\lambda+\lambda^2-6=0$$

$$\lambda^2-7\lambda+6=0 \quad \Rightarrow \text{suma de raíces } -\frac{b}{a}$$

$$(\lambda-6)(\lambda-1)=0$$

$\boxed{\lambda=6}$ $\boxed{\lambda=1}$

$\Rightarrow 7$

C) correcto

26.-

$$\begin{vmatrix}\log_2 8 & \log_2 4 & -1\\ \log_3 81 & 3 & -1\\ \log_2(1/2) & 2 & -4\end{vmatrix}$$

$$\log_2 8=\log_2 2^3 \Rightarrow 3\cancel{\log_2 2}^{\,1}$$

$$\log_2(4) \Rightarrow \log_2 2^2 \Rightarrow 2\cancel{\log_2 2}^{\,1}$$

$$\log_3 81 \Rightarrow \log_3 3^4 \Rightarrow 4\cancel{\log_3 3}^{\,1}$$

$$\log_2(1/2)=\cancel{\log_2 1}^{\,0}-\cancel{\log_2 2}^{\,1} \Rightarrow -1$$

$$\begin{array}{c}\\ \\ (3X4)\end{array}\begin{vmatrix}3 & 2 & -1\\ 4 & 3 & -1\\ -1 & 2 & -4\end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 8 & -13 \\ 0 & 11 & -17 \\ -1 & 2 & -4 \end{vmatrix} \Rightarrow - \begin{vmatrix} 8 & -13 \\ 11 & -17 \end{vmatrix} \Rightarrow -(-130-(-143))$$

$\Rightarrow -7$

e) CORRECTO

27.- $XA + B = C$

$$X=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} 5 & -2 \\ 7 & 1 \end{pmatrix} + \begin{pmatrix} 6 & 7 \\ 5 & -2 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix}$$

$$\begin{pmatrix} 5a+7b & -2a+b \\ 5c+7d & -2c+d \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ -8 & 7 \end{pmatrix} + \begin{pmatrix} -6 & -7 \\ -5 & 2 \end{pmatrix}$$

$$(-7)\cdot\begin{cases} 5a+7b=-11 \\ -2a+b=-7 \end{cases} \qquad (-7)\begin{cases} 5c+7d=-13 \\ -2c+d=9 \end{cases}$$

$$\begin{array}{r} 5a+7b=-11 \\ 14a-7b=49 \\ \hline 19a=38 \end{array}$$

$a=2$

$-2a+b=-7$

$b=-7+2a$

$b=-3$

$$\begin{array}{r} 5c+7d=-13 \\ +14c-7d=-63 \\ \hline 19c=-76 \end{array}$$

$c=-4$

$-2c+d=9$

$d=9+2c$

$d=1$

$$X=\begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}$$

$\det(X)=(2-12)$

$\Rightarrow -10$

28! $p(x) = \begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 8x & 2 \\ -1 & x \end{vmatrix} - \begin{vmatrix} 4 & 1 \\ -2 & x \end{vmatrix} = 0$

$\Rightarrow (4-6) + (8x^2+2) - (4x+2) = 0$

$\Rightarrow -2 + 8x^2 \cancel{+2} - 4x - 2 = 0$

$\Rightarrow 8x^2 - 4x - 2 = 0$

$\Rightarrow 4x^2 - 2x - 1 = 0$ suma de raices : $\frac{-b}{a} \Rightarrow \frac{-(-2)}{4}$

$\Rightarrow \frac{1}{2}$ a) CORRECTO

29.-

$A = \begin{pmatrix} \sin 90 & \sec 60 & \sin 90 \\ \cos 360 & \sin 180 & \cos 60 \\ -\cos 180 & \sin 360 & \cos 360 \end{pmatrix}$ $B = \begin{pmatrix} \log 1 & \log_2\left(\frac{1}{4}\right) & 0 \\ \log_2\left(\frac{1}{4}\right) & \log_3 9 & 0 \\ \log_3 9 & \ln e & \ln e \end{pmatrix}$

$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1/2 \\ 1 & 1 & 1/2 \end{pmatrix}$ $B = \begin{pmatrix} 0 & -2 & 0 \\ -2 & 2 & 0 \\ 2 & 1 & 1 \end{pmatrix}$

$A^t = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 1 & 1/2 & 1/2 \end{pmatrix}$ $; \frac{B}{2} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 1 & 0 \\ 1 & 1/2 & 1/2 \end{pmatrix}$

$+$

$(-2)\ (-1)$

$R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$; $\det(R) = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$\Rightarrow -1$ d) CORRECTO

30.-

$$A=\begin{vmatrix} m & h & g \\ f & e & d \\ c & b & a \end{vmatrix}=10$$

$$B=\begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2m \end{vmatrix}$$

$$\begin{vmatrix} c & b & a \\ f & e & d \\ m & h & g \end{vmatrix}=-10 \Rightarrow \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & m \end{vmatrix}=10$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 2g & 2h & 2m \end{vmatrix}=20 \Rightarrow \begin{vmatrix} a & b & c \\ d-3a & e-3b & f-3c \\ 2g & 2h & 2m \end{vmatrix}=20$$

$$\frac{det(A)}{det(B)}=\frac{10}{20}=\frac{1}{2}$$ d) CORRECTO

31.-

$$det(A)=\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}=n$$

$$B=\begin{vmatrix} 6d & 4e & 2f \\ 3g & 2h & i \\ 9a & 6b & 3c \end{vmatrix}$$

$$\begin{vmatrix} g & h & i \\ d & e & f \\ a & b & c \end{vmatrix}=-n \Rightarrow \begin{array}{ccc} (3) & (2) & \\ \downarrow & \downarrow & \end{array} \begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}=n$$

$$\Rightarrow \begin{array}{c}(2)\\ \\ (3)\end{array}\begin{vmatrix} 3d & 2e & f \\ 3g & 2h & i \\ 3a & 2b & c \end{vmatrix} = 6n \Rightarrow \begin{vmatrix} 6d & 4e & 2f \\ 3g & 2h & i \\ 9a & 6b & 3c \end{vmatrix} = 36n$$

b)

$$\begin{vmatrix} d+f & e & f+e \\ a+c & b & c+b \\ g+i & h & i+h \end{vmatrix}$$

$$\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix} = -n \Rightarrow \begin{vmatrix} d+f & e & f+e \\ a+c & b & c+b \\ g+i & h & i+h \end{vmatrix} = -n$$

32 –

P(X):

$$\begin{array}{c} \\ \\ (-x)(-1) \\ \\ \end{array}\begin{vmatrix} x+2 & 1 & 1 & 1 \\ 1 & x+2 & 1 & 1 \\ 1 & 1 & x+2 & 1 \\ x & x & x & x \end{vmatrix} = 0 \quad (-x-2)$$

$$\begin{vmatrix} 0 & -x-1 & -(x+2)^2 & -x-1 \\ 0 & x+1 & -x-1 & 0 \\ 1 & 1 & x+2 & 1 \\ 0 & 0 & -x^2-x & 0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} -x-1 & -(x+2)^2 & -x-1 \\ x+1 & -x-1 & 0 \\ 0 & -x^2-x & 0 \end{vmatrix} = 0$$

$$(x^2+x)\begin{vmatrix} -x-1 & -x-1 \\ x+1 & 0 \end{vmatrix} = 0 \Rightarrow (x^2+x)(x+1)^2 = 0$$

$$x(x+1)(x+1)^2 = 0$$

$$x(x+1)^3 = 0$$

$x = 0$, $x = -1$

33.—

$$f(x)=\begin{vmatrix} a & b & -2a & 3b \\ -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix} \underset{(a)}{} \Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ a & b & -2a & 3b \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix}$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & b+ax & -2a & 3b \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \end{vmatrix} (b+ax) \Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & b+ax & -2a & 3b \\ 0 & 0 & -1 & x \end{vmatrix}$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & bx+ax^2-2a & 3b \\ 0 & 0 & -1 & x \end{vmatrix} \Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \\ 0 & 0 & ax^2+bx-2a & 3b \end{vmatrix} (ax^2+bx-2a)$$

$$\Rightarrow -\begin{vmatrix} -1 & x & 0 & 0 \\ 0 & -1 & x & 0 \\ 0 & 0 & -1 & x \\ 0 & 0 & 0 & \underbrace{\boxed{}}_{ax^3+bx^2-2ax+3b} \end{vmatrix} \Rightarrow \det = ax^3+bx^2-2ax+3b = f(x)$$

$\Rightarrow f(0) = 0+0-0+3b = -3$

$3b = -3$

$b=-1$

$f(1)=f(-1)$

$a+\cancel{b}-2a+\cancel{3b} = -a+\cancel{b}+2a+\cancel{3b}$

$-a = a$

$2a=0$

$a=0$

34.- a) CORRECTO, por propiedades

b) CORRECTO

$$-\begin{vmatrix} c & d \\ a & b \end{vmatrix} \Rightarrow \begin{vmatrix} d & c \\ b & a \end{vmatrix}$$

c) CORRECTO

$$K^2A = \begin{vmatrix} K^2a & b \\ K^2c & d \end{vmatrix} \Rightarrow K^3A = \begin{vmatrix} K^2a & Kb \\ K^2c & Kd \end{vmatrix}$$

d) CORRECTO

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \Rightarrow ad - bc$$

$$\begin{vmatrix} a & c \\ b & d \end{vmatrix} \Rightarrow ad - bc$$

e) FALSO

$$\begin{vmatrix} Ka & Kb \\ Kc & Kd \end{vmatrix} = K^2 \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

35.- a) $det = \begin{vmatrix} 5 & 0 \\ 1 & 3 \end{vmatrix} = \underline{\underline{15}}$

b) $det = \begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} = \underline{\underline{8}}$

c) $\begin{pmatrix} 5 & 0 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 2 & 13 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 1 \\ 0 & 4 \end{pmatrix} \times \begin{pmatrix} 5 & 0 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 11 & 3 \\ 4 & 12 \end{pmatrix}$

e) $\det(AB) = (130-10) = 120$

$\det(A) \times \det(B) = 15 \times 8 = 120$

e.1) $\det(B \times A) = 132 - 12 = 120$

$\det(A) \times \det(B) = 15 \times 8 = 120$

36.— $A = \begin{pmatrix} \cos\alpha & \operatorname{sen}\alpha \\ 1 & \cos\alpha \end{pmatrix}$

$\det(A) = \cos^2\alpha - \operatorname{sen}\alpha = 1$

$\Rightarrow \cancel{1} - \operatorname{sen}^2\alpha - \operatorname{sen}\alpha - \cancel{1} = 0$

$-\operatorname{sen}\alpha(\operatorname{sen}\alpha + 1) = 0$

$\operatorname{sen}\alpha = 0$

$\alpha = 0°, 180°, 360° \ldots$ $\quad \operatorname{sen}\alpha = -1$

$\alpha = 270°$

Sol: $\{ n\pi, -\frac{3\pi}{2} \} \cdot n \in \mathbb{Z}$.

37.— $(-1) \begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = 0$

$\begin{vmatrix} 1 & a & b+c \\ 0 & b-a & a-b \\ 0 & c-a & a-c \end{vmatrix} = 0$ $(b-a)(a-c) - (a-b)(c-a)$

$\Rightarrow ab - bc - a^2 + ac - (ac - a^2 - bc + ab)$

$\Rightarrow \cancel{ab} - \cancel{bc} - \cancel{a^2} + \cancel{ac} - \cancel{ac} + \cancel{a^2} + \cancel{bc} - \cancel{ab}$

$\det(A) = 0$

demostrado

38.-

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$$

a)

$$2*3 = \begin{vmatrix} a & b & c \\ d & e & f \\ 2g & 2h & 2i \end{vmatrix}$$

$\Rightarrow \underline{\underline{6}}$

b)

$$\begin{vmatrix} c & b & a \\ f & e & d \\ i & h & g \end{vmatrix} = -3 \Rightarrow \begin{vmatrix} c & b & -a \\ f & e & -d \\ i & h & -g \end{vmatrix} = -1(-3) \Rightarrow 3$$

$*-1$

c)

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3 \Rightarrow \begin{vmatrix} a-4c & b & c \\ d-4f & e & f \\ g-4i & h & i \end{vmatrix}$$

$*4$

39.-

$$\begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ \cos x - \operatorname{sen} x & -\operatorname{sen} x & \cos x \\ 1+\cos x+\operatorname{sen} x & \cos x & \operatorname{sen} x \end{vmatrix}$$

$(-1)\,(-\cos x)$

$\Rightarrow \frac{\operatorname{sen} x)}{}$

$$\begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ \cos x & -\operatorname{sen} x & \cos x-\operatorname{sen} x \\ \operatorname{sen} x & \cos x & 1+\cos x+\operatorname{sen} x \end{vmatrix} \Rightarrow \frac{1}{\operatorname{sen} x} \begin{vmatrix} \operatorname{sen} x & \cos x & \operatorname{sen} x \\ 0 & -1 & -\operatorname{sen}^2 x \\ 0 & 0 & 1+\cos x \end{vmatrix}$$

$$\Rightarrow -\frac{\cancel{\operatorname{sen} x}}{\operatorname{sen} x} \begin{vmatrix} -1 & -\operatorname{sen}^2 x \\ 0 & 1+\cos x \end{vmatrix} \Rightarrow -(-1)(1+\cos x)$$

$$\Rightarrow \boxed{1+\cos x}$$

40.- $(-1)\begin{vmatrix} 1 & \text{sen}\,a & \cos a \\ 1 & \text{sen}\,b & \cos b \\ 1 & \text{sen}\,c & \cos c \end{vmatrix} = \text{sen}(b-c) + \text{sen}(a-b) + \text{sen}(a-c)$

$$\Rightarrow \begin{vmatrix} 1 & \text{sen}\,a & \cos a \\ 0 & \text{sen}\,b-\text{sen}\,a & \cos b-\cos a \\ 0 & \text{sen}\,c-\text{sen}\,a & \cos c-\cos a \end{vmatrix} \Rightarrow (\text{sen}\,b-\text{sen}\,a)(\cos c-\cos a)-(\cos b-\cos a)(\text{sen}\,c-\text{sen}\,a)$$

$$\Rightarrow \text{sen}\,b\cos c - \text{sen}\,b\cos a - \text{sen}\,a\cos c + \text{sen}\,a\cos a - (\text{sen}\,c\cos b - \text{sen}\,a\cos b - \text{sen}\,c\cos a + \text{sen}\,a\cos a)$$

$$\Rightarrow \text{sen}\,b\cos c - \text{sen}\,b\cos a - \text{sen}\,a\cos c + \cancel{\text{sen}\,a\cos a} - \text{sen}\,c\cos b + \text{sen}\,a\cos b + \text{sen}\,c\cos a - \cancel{\text{sen}\,a\cos a}$$

$$\Rightarrow \underbrace{(\text{sen}\,b\cos c - \text{sen}\,c\cos b)}_{\text{sen}(b-c)} + \underbrace{(\text{sen}\,a\cos b - \text{sen}\,b\cos a)}_{\text{sen}(a-b)} + \underbrace{(\text{sen}\,c\cos a - \text{sen}\,a\cos c)}_{\text{sen}(c-a)}$$

$$\text{sen}(b-c) + \text{sen}(a-b) + \text{sen}(c-a)$$

41.-

a) $\begin{vmatrix} x+4 & x+14 \\ x & 2x+1 \end{vmatrix} \Rightarrow (x+4)(2x+1) - x(x+14)$

$\Rightarrow 2x^2 + x + 8x + 4 - x^2 - 14x$

$\Rightarrow x^2 - 5x + 4$

$\Rightarrow (x-4)(x-1)$

b) $\begin{matrix} -(x+1) \\ -(x+2) \end{matrix}$ $\begin{vmatrix} 1 & -1 & 9 \\ x+1 & x & 2 \\ x+2 & x+1 & x-1 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -1 & 9 \\ 0 & 2x+1 & -9x-7 \\ 0 & 2x+3 & -8x-19 \end{vmatrix}$

$\Rightarrow (2x+1)(-8x-19) - (-9x-7)(2x+3)$

$\Rightarrow -16x^2 - 38x - 8x - 19 - (-18x^2 - 27x - 14x - 21)$

$\Rightarrow -16x^2 - 46x - 19 + 18x^2 + 41x + 21$

$\Rightarrow 2x^2-5x+2$

$\Rightarrow \frac{1}{\cancel{2}}(\cancel{2}x-\cancel{4})(2x-1)$

$\Rightarrow (x-2)(2x-1)$

-3) C) $\Rightarrow \begin{vmatrix} 1 & -2 & 2 \\ x & -4 & 4 \\ 3 & x & 6 \end{vmatrix} \Rightarrow \begin{vmatrix} 1 & -2 & 2 \\ 0 & 2x-4 & -2x+4 \\ 0 & x+6 & 0 \end{vmatrix} \Rightarrow -(x+6)(-2x+4)$

$\Rightarrow (2x-4)(x+6)$

42.-

$p(x): \begin{vmatrix} 2\operatorname{sen}^2(2x) & 2\cos(x) \\ 3\operatorname{sen}(x) & 1 \end{vmatrix} = -1$

$2\operatorname{sen}^2(2x) - 6\operatorname{sen}x\cos x = -1$

$4\operatorname{sen}x\cos x - 6\operatorname{sen}x\cos x = -1$

$-2\operatorname{sen}x\cos x = -1$

$2\operatorname{sen}x\cos x = 1$

$\operatorname{sen}(2x) = 1$

$2x = 90°$

$x = 45°$

S.P: $\frac{\pi}{4}$

43.- $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} (-1) \Rightarrow \begin{vmatrix} x^2 & x & 1 \\ y^2-x^2 & y-x & 0 \\ z^2-x^2 & z-x & 0 \end{vmatrix} \Rightarrow (y^2-x^2)(z-x)-(y-x)(z^2-x^2)$

$\Rightarrow (y+x)(y-x)(z-x)-(y-x)(z-x)(z+x)$

$\Rightarrow (y-x)(z-x)(y+\cancel{x}-z-\cancel{x})$

$\Rightarrow (y-x)(z-x)(y-z)$ Demostrado

44.- (2)+ $\begin{vmatrix} x & y & z \\ 5 & 0 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1$

$\begin{vmatrix} x & y & z \\ 2x+5 & 2y & 2z+3 \\ x+1 & y+1 & z+1 \end{vmatrix} = 1$

45.- $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 5$

$\begin{vmatrix} a_{31} & a_{32} & a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = -5 \Rightarrow \begin{vmatrix} -a_{31} & -a_{32} & -a_{33} \\ a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \end{vmatrix} = 5$

$$\Rightarrow \begin{vmatrix} -a_{31} & -2a_{32} & a_{33} \\ a_{21} & 2a_{22} & a_{23} \\ a_{11} & 2a_{12} & a_{13} \end{vmatrix} = 10.$$

$$\Rightarrow \begin{vmatrix} -a_{31} & -2a_{32} & -a_{33} \\ 3a_{21} & -6a_{22} & -3a_{23} \\ a_{11} & 2a_{12} & a_{13} \end{vmatrix} = 30.$$

A 6.- $\begin{vmatrix} a_1 & b_1+b_2 \\ c_1 & d_1+d_2 \end{vmatrix} \Rightarrow \begin{vmatrix} a_1 & b_1 \\ c_1 & d_1 \end{vmatrix} + \begin{vmatrix} a_1 & b_2 \\ c_1 & d_2 \end{vmatrix}$

$a_1(d_1+d_2) - c_1(b_1+b_2)$

$a_1d_1 - b_1c_1 + a_1d_2 - b_2c_1$

$a_1(d_1+d_2) - c_1(b_1+b_2)$

demostrado.

47.-
$$\begin{cases} x + 2y + z = k \\ 2x + y + 4z = 6 \\ x - 4y + 5z = 9 \end{cases}$$

$$(-1)(-2)\left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 2 & 1 & 4 & 6 \\ 1 & -4 & 5 & 9 \end{array}\right) \Rightarrow (-2)\left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 0 & -3 & 2 & 6-2k \\ 0 & -6 & 4 & 9-k \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & k \\ 0 & -3 & 2 & 6-2k \\ 0 & 0 & 0 & -3+3k \end{array}\right)$$

Para que el sistema de ecuaciones sea consistente $-3+3k=0$

$k=1$

$k=1$; soluciones infinitas

$-3y+2z = 4$

$-3y = 4-2z$

$y = \dfrac{2z-4}{3}$

$x+2y+z=1$

$x = 1-z-2y$

$x = 1-z-\dfrac{2(2z-4)}{3}$

$x = \dfrac{3-3z-4z+8}{3}$

$x = \dfrac{11-7z}{3}$

Sol: $\left\{\dfrac{11-7a}{3}, \dfrac{2a-4}{3}, a\right\}$

48.

$$A=\begin{pmatrix}1&0&2\\2&1&0\\3&1&-1\end{pmatrix}$$

$$\begin{array}{c}(-3)(-2)\\ \\ \\\end{array}\Rightarrow\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\2&1&0&0&1&0\\3&1&-1&0&0&1\end{array}\right)\Rightarrow(-1)\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\0&1&-4&-2&1&0\\0&1&-7&-3&0&1\end{array}\right)$$

$$\begin{array}{c}(3)\\(-3)\\(2)(2)\end{array}\left(\begin{array}{ccc|ccc}1&0&2&1&0&0\\0&1&-4&-2&1&0\\0&0&-3&-1&-1&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}3&0&0&1&-2&2\\0&-3&0&2&-7&4\\0&0&-3&-1&-1&1\end{array}\right)$$

a)

$$\Rightarrow\left(\begin{array}{ccc|ccc}1&0&0&1/3&-2/3&2/3\\0&1&0&-2/3&7/3&-4/3\\0&0&1&1/3&1/3&-1/3\end{array}\right)$$

$$A^t=\begin{pmatrix}1&2&3\\0&1&1\\2&0&-1\end{pmatrix}$$

$$(-2)\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&1&0&1&0\\2&0&-1&0&0&1\end{array}\right)\xrightarrow{(4)(-2)}\left(\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&1&0&1&0\\0&-4&-7&-2&0&1\end{array}\right)$$

$$(3)\left(\begin{array}{ccc|ccc}1&0&1&1&-2&0\\0&1&1&0&1&0\\0&0&-3&-2&4&1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}3&0&0&1&-2&1\\0&3&0&-2&7&1\\0&0&-3&-2&4&1\end{array}\right)$$

b)

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & 7/3 & 1/3 \\ 0 & 0 & 1 & 2/3 & -4/3 & -1/3 \end{array}\right)$$

c) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$

$$x + 2y + 3z = 4$$
$$y + z = 0$$
$$2x + 0 - z = 1$$

$$\begin{array}{c} (-2) \\ \\ \\ \end{array} \Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 0 \\ 2 & 0 & -1 & 1 \end{array}\right)$$

$$\begin{array}{c} \\ (4)(-2) \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & -7 & -7 \end{array}\right) \Rightarrow \begin{array}{c} (3) \\ (3) \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -3 & -7 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 3 & 0 & 0 & 5 \\ 0 & 3 & 0 & -7 \\ 0 & 0 & -3 & -7 \end{array}\right)$$

$3x = 5$ $\quad$ $x = 5/3$

$3y = -7$ $\quad$ $y = -7/3$

$-3z = -7$ $\quad$ $z = 7/3$

e.1) $\begin{pmatrix} 1/3 & -2/3 & 2/3 \\ -2/3 & 7/3 & -4/3 \\ 1/3 & 1/3 & -1/3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

$$\frac{1}{3}x - \frac{2}{3}y + \frac{2}{3}z = 2$$
$$-\frac{2x}{3} + \frac{7}{3}y - \frac{4}{3}z = 3$$
$$\frac{x}{3} + \frac{y}{3} - \frac{z}{3} = -4$$

$$\begin{cases} x - 2y + 2z = 6 \\ -2x + 7y - 4z = 9 \\ x + y - z = -12 \end{cases}$$

$$(-1)(2)\left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ -2 & 7 & -4 & 9 \\ 1 & 1 & -1 & -12 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ 0 & 3 & 0 & 21 \\ 0 & 3 & -3 & -18 \end{array}\right)$$

$$(-1)(2)\left(\begin{array}{ccc|c} 1 & -2 & 2 & 6 \\ 0 & 1 & 0 & 7 \\ 0 & 1 & -1 & -6 \end{array}\right) \Rightarrow (2)\left(\begin{array}{ccc|c} 1 & 0 & 2 & 20 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & -13 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & -1 & -13 \end{array}\right) \quad x=-6 \quad y=7 \quad z=13$$

49.-

$$A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \qquad B=\begin{pmatrix} m & n & o \\ p & q & r \\ s & t & u \end{pmatrix}$$

$$3A+2B=\begin{pmatrix} 3 & 8 & -3 \\ -2 & 2 & -3 \\ 7 & 2 & 4 \end{pmatrix} \qquad A-B=\begin{pmatrix} 0 & 1 & 0 \\ -1 & -2 & -3 \\ 2 & 1 & 2 \end{pmatrix}$$

$$\begin{cases} 3a+2m=3 \\ a-m=0\,(2) \end{cases} \qquad \begin{array}{r} 3a+2m=3 \\ 2a-2m=0 \\ \hline 5a=3 \end{array} \qquad \begin{array}{r} 3b+2n=8 \\ 2b-2n=2 \\ \hline 5b=10 \end{array}$$

$a=3/5$ $\qquad$ $b=2$

$$\begin{cases} 3b+2n=8 \\ b-n=1\,(2) \end{cases}$$

$a-m=0$ $\qquad$ $b-n=1$

$m=a$ $\qquad$ $b-1=n$

$m=3/5$

$$\begin{cases} 3c+2o=-3 \\ c-o=0\,(2) \end{cases}$$

$n=2-1$

$n=1$

$$\begin{array}{l} 3c+2o=-3 \\ 2c-2o=0 \\ \hline 5c=-3 \end{array}$$

$c=-3/5$

$c-o=0$

$o=c$

$o=-3/5$

$$\begin{array}{l} 3f+2r=-3 \\ 2f-2r=-6 \\ \hline 5f=-9 \end{array}$$

$f=-9/5$

$r=f+3$

$r=6/5$

$$\begin{cases} 3d+2p=-2 \\ d-p=-1 \ (2) \end{cases}$$

$$\begin{cases} 3e+2q=2 \\ e-q=-2 \ (2) \end{cases}$$

$$\begin{cases} 3f+2r=-3 \\ f-r=-3 \ (2) \end{cases}$$

$$\begin{array}{l} 3d+2p=-2 \\ 2d-2p=-2 \\ \hline 5d=-4 \end{array}$$

$d=-4/5$

$d-p=-2$

$p=d+2$

$p=6/5$

$$\begin{array}{l} 3e+2q=2 \\ 2e-2q=-4 \\ \hline 5e=-2 \end{array}$$

$e=-2/5$

$e-q=-2$

$q=e+2$

$q=8/5$

$$(2)\begin{cases} 3g+2s=7 \\ g-s=2 \end{cases}$$

$$(2)\begin{cases} 3h+2t=2 \\ h-t=1 \end{cases}$$

$$(2)\begin{cases} 3i+2u=4 \\ i-u=2 \end{cases}$$

$$\begin{array}{l} 3g+2s=7 \\ 2g-2s=4 \\ \hline 5g=11 \end{array}$$

$g=11/5$

$g-s=2$

$s=g-2$

$s=1/5$

$$\begin{array}{l} 3h+2t=2 \\ 2h-2t=2 \\ \hline 5h=4 \end{array}$$

$h=4/5$

$h-t=1$

$t=h-1$

$t=-1/5$

$$\begin{array}{l} 3i+2u=4 \\ 2i-2u=4 \\ \hline 5i=8 \end{array}$$

$i=8/5$

$i-u=2$

$u=i-2$

$u=-2/5$

$$A=\begin{pmatrix} 3/5 & 2 & -3/5 \\ -4/5 & -2/5 & -9/5 \\ 11/5 & 4/5 & 8/5 \end{pmatrix}$$

$$B=\begin{pmatrix} 3/5 & 1 & -3/5 \\ 6/5 & 8/5 & 6/5 \\ 1/5 & -1/5 & -2/5 \end{pmatrix}$$

50 $\begin{pmatrix} 1 & \alpha \\ \beta & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1-\beta \\ \alpha \end{pmatrix}$

$$\begin{cases} x_1 + \alpha x_2 = 1 - \beta & (1) \\ \beta x_1 + x_2 = \alpha & (2) \end{cases}$$

El punto $P(2,-1)$ satisface a (1)
El punto $Q(2,0)$ satisface a (2)

Reemplazando $\underline{\underline{P}}$ en (1) y $\underline{\underline{Q}}$ en (2):

a) $$\begin{cases} 2 - \alpha = 1 - \beta & (1) \\ 2\beta = \alpha & (2) \end{cases}$$

Reemplazo (2) en (1)

$2 - 2\beta = 1 - \beta$

$-2\beta + \beta = 1 - 2$

$-\beta = -1$

$\boxed{\beta = 1}$

Reemplazo en (2)

$\alpha = 2\beta \qquad \alpha = 2(1) \qquad \boxed{\alpha = 2}$

b) $\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\begin{cases} x_1 + 2x_2 = 0 \\ x_1 + x_2 = 2 \end{cases}$$

El sistema tiene solución única ya que la determinante $\underline{\underline{\text{no}}}$ es cero.

$= 1(1) - 2(1) \qquad = 1 - 2 \qquad = -1$

51. $\begin{pmatrix} 1 & 2 & 1 \\ 0 & t & t \\ -1 & t & -1 \end{pmatrix}$

a) Por el método del pivoteo:

$$1\begin{vmatrix} t & t \\ t & -1 \end{vmatrix} - 2\begin{vmatrix} 0 & t \\ -1 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & t \\ -1 & t \end{vmatrix}$$

$$1(-t-t^2) - 2(0+t) + 1(0+t) = 0$$

$$\cancel{-t} - t^2 - 2t + \cancel{t} = 0$$

$$-t^2 - 2t = 0 \quad *$$

$$t^2 + 2t = 0$$

$$t(t+2) = 0$$

$$\boxed{t = 0} \qquad t+2=0 \qquad \boxed{t=-2}$$

b) Igualando $*$ a 1

$$-t^2 - 2t = 1$$

$$-t^2 - 2t - 1 = 0$$

$$t^2 + 2t + 1 = 0$$

$$(t+1)^2 = 0 \qquad t+1=0 \qquad \boxed{t=-1}$$

Reemplazo en la matriz inicial el valor de $t=-1$

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

Ahora, se halla la inversa de esta matriz, utilizando estos 5 pasos siguientes:

(1) det = 1

(2) Menores

$$\begin{pmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

(3) Cofactores

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & -1 \end{pmatrix}$$

(4) Transpuesta

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

(5) $\frac{1}{det}$ (Transpuesta)

$$= \frac{1}{1}\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix} \boxed{= \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}}$$

(52)

$$A = \begin{pmatrix} 1 & 2 & 1 \\ \lambda & 1 & 0 \\ -1 & 1 & \lambda \end{pmatrix}$$

a) Por el método del pivoteo

$$1\begin{vmatrix} 1 & 0 \\ 1 & \lambda \end{vmatrix} - 2\begin{vmatrix} \lambda & 0 \\ -1 & \lambda \end{vmatrix} + 1\begin{vmatrix} \lambda & 1 \\ -1 & 1 \end{vmatrix} = 0$$

$$1(\lambda - 0) - 2(\lambda^2 + 0) + 1(\lambda + 1) = 0$$

$$\lambda - 2\lambda^2 + \lambda + 1 = 0$$

$$-2\lambda^2 + 2\lambda + 1 = 0$$

$$2\lambda^2 - 2\lambda - 1 = 0$$

$a = 2 \qquad b = -2 \qquad c = -1$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$\lambda = \frac{2 \pm \sqrt{4 + 8}}{4}$$

$$\lambda = \frac{2 \pm \sqrt{12}}{4} \qquad \lambda = \frac{2 \pm 2\sqrt{3}}{4}$$

$$\lambda = \frac{\cancel{2}^{1}(1 \pm \sqrt{3})}{\cancel{4}_{2}} \qquad \boxed{\lambda = \frac{1 \pm \sqrt{3}}{2}}$$

b) $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ x_1 + x_2 = 0 \\ -x_1 + x_2 + x_3 = 0 \end{cases}$$

53 $3X-2Y=\begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$ (a') $\quad X+3Y=\begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$ (b')

Designando incógnitas cualesquiera para las matrices X i Y

$$X=\begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad Y=\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

a') $3X-2Y=\begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$

$$3\begin{pmatrix} a & b \\ c & d \end{pmatrix}-2\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 3a & 3b \\ 3c & 3d \end{pmatrix}-\begin{pmatrix} 2e & 2f \\ 2g & 2h \end{pmatrix}=\begin{pmatrix} 7 & 3 \\ 16 & 4 \end{pmatrix}$$

$3a-2e=7$ (1)
$3b-2f=3$ (2)
$3c-2g=16$ (3)
$3d-2h=4$ (4)

b') $X+3Y=\begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}+3\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}+\begin{pmatrix} 3e & 3f \\ 3g & 3h \end{pmatrix}=\begin{pmatrix} 6 & 12 \\ -2 & 27 \end{pmatrix}$$

Resolviendo este sistema homogéneo

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right| \begin{array}{l} F_1 - F_2 \\ \\ F_1 + F_3 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 2 & 0 \end{array}\right| -3F_2 + F_3$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right| -2F_2 + F_1$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right| \begin{array}{l} \\ \\ -F_3 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right| \begin{array}{l} \\ F_3 + F_1 \\ -F_3 + F_2 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right|$$

Luego, este sistema tiene la solución trivial $X_1 = 0 \quad X_2 = 0 \quad X_3 = 0$

$$S: \{X_1 = 0 \quad X_2 = 0 \quad X_3 = 0$$

Reemplazo en 2 ó 6

En 2: $3b - 2f = 3$

$3b - 2(3) = 3$ $\quad$ $3b - 6 = 3$

$3b = 3 + 6$ $\quad$ $3b = 9$

$b = \frac{\cancel{9}^{3}}{\cancel{3}_{1}}$ $\quad$ $\boxed{b = 3}$

*** Reuniendo 3 y 7

$$(-3)\begin{cases} 3c - 2g = 16 & (3) \\ c + 3g = -2 & (7) \end{cases}$$

$$\begin{array}{r} 3c - 2g = 16 \\ -3c - 9g = 6 \\ \hline // \quad -11g = 22 \end{array}$$

$11g = -22$

$g = -\frac{\cancel{22}^{2}}{\cancel{11}_{1}}$ $\quad$ $\boxed{g = -2}$

Reemplazo en 3 ó 7

En 7: $c + 3g = -2$

$c + 3(-2) = -2$ $\quad$ $c - 6 = -2$

$c = -2 + 6$ $\quad$ $\boxed{c = 4}$

**** Reuniendo 4 y 8

$$(-3)\begin{cases} 3d - 2h = 4 & (4) \\ d + 3h = 27 & (8) \end{cases}$$

$a + 3e = 6$ (5)
$b + 3f = 12$ (6)
$c + 3g = -2$ (7)
$d + 3h = 27$ (8)

*Reuniendo (1) y (5)

$$(-3)\begin{cases} 3a - 2e = 7 & (1) \\ a + 3e = 6 & (5) \end{cases}$$

$$\begin{array}{r} 3a - 2e = 7 \\ -3a - 9e = -18 \\ \hline // \quad -11e = -11 \end{array} \qquad 11e = 11$$

$$e = \frac{\cancel{11}^{1}}{\cancel{11}_{1}} \qquad \boxed{e = 1}$$

Reemplazo en (1) ó (5)

En (5): $a + 3e = 6$

$$a + 3(1) = 6 \qquad a + 3 = 6$$

$$a = 6 - 3 \qquad \boxed{a = 3}$$

** Reuniendo (2) y (6)

$$(-3)\begin{cases} 3b - 2f = 3 & (2) \\ b + 3f = 12 & (6) \end{cases}$$

$$\begin{array}{r} 3b - 2f = 3 \\ -3b - 9f = -36 \\ \hline // \quad -11f = -33 \end{array} \qquad 11f = 33$$

$$f = \frac{\cancel{33}^{3}}{\cancel{11}_{1}} \qquad \boxed{f = 3}$$

$$3d - 2h = 4$$
$$-3d - 9h = -81$$

$$// \quad -11h = -77 \qquad\qquad 11h = 77$$

$$h = \frac{\cancel{77}^{7}}{\cancel{11}_{1}} \qquad\qquad \boxed{h = 7}$$

Reemplazo en (4) ó (8)

En (4) : $3d - 2h = 4$

$$3d - 2(7) = 4 \qquad\qquad 3d - 14 = 4$$
$$3d = 4 + 14 \qquad\qquad 3d = 18$$
$$d = \frac{\cancel{18}^{6}}{\cancel{3}_{1}} \qquad\qquad \boxed{d = 6}$$

$$\boxed{X = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}} \qquad\qquad \boxed{Y = \begin{pmatrix} 1 & 3 \\ -2 & 7 \end{pmatrix}}$$

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$$\begin{cases} x \quad -3z = 1 \\ 2x \quad -6z = 3 \\ \quad y + z = 2 \end{cases}$$

$$\begin{pmatrix} 1 & 0 & -3 \\ 2 & 0 & -6 \\ 0 & 1 & 1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} 0 & -6 \\ 1 & 1 \end{vmatrix} - 0\cancel{\begin{vmatrix} 2 & -6 \\ 0 & 1 \end{vmatrix}}^{0} - 3\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}$$

$$1(0+6) - 3(2-0)$$
$$1(6) - 3(2) \qquad 6 - 6 \qquad \boxed{\det = 0}$$

(55) $*\begin{cases} x + ay + z = -1 \\ y + 2z = 1 \\ x + y - z = -a \end{cases}$

$$\begin{pmatrix} 1 & a & 1 \\ 0 & 1 & 2 \\ 1 & 1 & -1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} - a\begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} \neq 0$$

$$1(-1-2) - a(0-2) + 1(0-1) \neq 0$$

$$1(-3) - a(-2) + 1(-1) \neq 0$$

$$-3 + 2a - 1 \neq 0$$

$$2a - 4 \neq 0 \qquad 2a \neq 4$$

$$a \neq \frac{\cancel{4}^{2}}{\cancel{2}_{1}} \qquad \boxed{a \neq 2}$$

$**\begin{cases} x - ay + bz = 0 \\ 2x - y + z = 0 \\ ax - by - z = 0 \end{cases}$

$$\begin{pmatrix} 1 & -a & b \\ 2 & -1 & 1 \\ a & -b & -1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} -1 & 1 \\ -b & -1 \end{vmatrix} + a\begin{vmatrix} 2 & 1 \\ a & -1 \end{vmatrix} + b\begin{vmatrix} 2 & -1 \\ a & -b \end{vmatrix} = 0$$

$$1(1+b) + a(-2-a) + b(-2b+a) = 0$$

$$1 + b - 2a - a^2 - 2b^2 + ab = 0$$

Tomando $a=2$

$$1+b-2(2)-(2)^2-2b^2+(2)b=0$$
$$1+b-4-4-2b^2+2b=0$$
$$-2b^2+3b-7=0$$
$$2b^2-3b+7=0$$

$a=2$ $b=-3$ $c=7$

Como b^2-4ac es una cantidad negativa, las raices son complejas

$$b^2-4ac=(-3)^2-4(2)(7)$$
$$=9-56$$
$$=-47$$

56) $\begin{cases}(m+2)x+y+2z=0\\ x+my+z=0\\ 2x+2y+2z=0\end{cases}$

$$\begin{pmatrix} m+2 & 1 & 2 \\ 1 & m & 1 \\ 2 & 2 & 2 \end{pmatrix}$$

Por el método del pivoteo:

$$(m+2)\begin{vmatrix} m & 1 \\ 2 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & m \\ 2 & 2 \end{vmatrix}$$

$$(m+2)(2m-2)-1(2-2)+2(2-2m)$$
$$2m^2-2m+\cancel{4m}\cancel{-4}\cancel{-2}\cancel{+2}\cancel{+4}\cancel{-4m}$$
$$2m^2-2m=0$$
$$2m(m-1)=0$$
$$m(m-1)=0$$

$\boxed{m=0}$ $m-1=0$ $\boxed{m=1}$

57 $\begin{cases} x - my = m^2 n \\ (m) \quad \dfrac{x}{m} + y = 2mn \end{cases}$ $\quad m, n \in \mathbb{R}$, $m \neq 0$

$$x - my = m^2 n$$
$$x + my = 2m^2 n$$
$$2x \quad // \quad = 3m^2 n$$

$$\boxed{x = \frac{3m^2 n}{2}}$$

$$(-m)\begin{cases} x - my = m^2 n \\ \dfrac{x}{m} + y = 2mn \end{cases}$$

$$x - my = m^2 n$$
$$-x - my = -2m^2 n$$
$$\cancel{//} \quad -2my = -m^2 n$$
$$2my = m^2 n$$
$$y = \frac{m^{\cancel{2}} n}{2\cancel{m}} \qquad \boxed{y = \frac{mn}{2}}$$

El sistema tiene solución única

58 $\begin{pmatrix} 0 & -2 & \alpha \\ 1 & -\alpha & 1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$$\begin{cases} -2y + \alpha z = 1 \\ x - \alpha y + z = 2 \\ x + y = 3 \end{cases}$$

Por el método del pivoteo:

$$0\begin{vmatrix}-\alpha & 1\\ 1 & 0\end{vmatrix}^{0} + 2\begin{vmatrix}1 & 1\\ 1 & 0\end{vmatrix} + \alpha\begin{vmatrix}1 & -\alpha\\ 1 & 1\end{vmatrix} = 0$$

$$2(0-1) + \alpha(1+\alpha) = 0$$

$$2(-1) + \alpha + \alpha^2 = 0$$

$$\alpha^2 + \alpha - 2 = 0$$

$$(\alpha+2)(\alpha-1) = 0$$

$\alpha + 2 = 0$ $\qquad$ $\alpha - 1 = 0$

$\boxed{\alpha = -2}$ $\qquad$ $\boxed{\alpha = 1}$

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$$A\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}1\\ 1\\ -1\end{pmatrix}$$

$$\begin{pmatrix}1 & 2 & a\\ 0 & 1 & b\\ 3 & 2 & c\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}1\\ 1\\ -1\end{pmatrix}$$

$$\begin{cases} x_1 + 2x_2 + ax_3 = 1 & (1)\\ \quad\; x_2 + bx_3 = 1 & (2)\\ 3x_1 + 2x_2 + cx_3 = -1 & (3)\end{cases}$$

Multiplicando (1) por -3 y sumando con (3)

$$-3x_1 - 6x_2 - 3ax_3 = -3$$
$$3x_1 + 2x_2 + cx_3 = -1$$

$$// \quad -4x_2 - 3ax_3 + cx_3 = -4 \qquad (4)$$

$$-4x_2 + x_3(c-3a) = -4$$

Multiplicando 2 por 4 y sumando con 2

$$\begin{array}{l} 4X_2 + 4bX_3 = 4 \\ -4X_2 + X_3(c-3a) = -4 \\ \hline // \quad 4bX_3 + cX_3 - 3aX_3 = 0 \end{array}$$

$X_3(4b + c - 3a) = 0$

$\boxed{X_3 = 0}$

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$A = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$B = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$A = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 3\lambda \\ -2\lambda \end{pmatrix}$ $\qquad B = \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 4t \\ t \end{pmatrix}$

$A = \begin{pmatrix} 5 + 3\lambda \\ 1 - 2\lambda \end{pmatrix}$ $\qquad B = \begin{pmatrix} -2 + 4t \\ 2 + t \end{pmatrix}$

Como $A = B$, se debe cumplir:

$$\begin{cases} 5 + 3\lambda = -2 + 4t & (1) \\ 1 - 2\lambda = 2 + t & (2) \end{cases}$$

$$\begin{cases} 3\lambda - 4t = -7 & (1) \\ -2\lambda - t = 1 & (2) \end{cases} \quad (-4)$$

$$\begin{array}{l} 3\lambda - 4t = -7 \\ 8\lambda + 4t = -4 \\ \hline 11\lambda \quad // \quad = -11 \end{array}$$

$\lambda = -\frac{\cancel{11}^{1}}{\cancel{11}_{1}}$ $\qquad \boxed{\lambda = }$

Reemplazo en 1 ó 2:

En 2: $-2\lambda - t = 1$

$-2(-1) - t = 1$

$2 - t = 1$

$-t = 1 - 2$

$-t = -1$

$\boxed{t = 1}$

$S: \begin{cases} \lambda = -1 \\ t = 1 \end{cases}$

1) a) $\begin{cases} X_1 + 2X_2 + X_3 = 8 & (1) \\ X_2 + 3X_3 + X_4 = 15 & (2) \\ 4X_1 + X_3 + X_4 = 11 & (3) \\ X_1 + X_2 + 5X_4 = 23 & (4) \end{cases}$

$\left|\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 8 \\ 0 & 1 & 3 & 1 & 15 \\ 4 & 0 & 1 & 1 & 11 \\ 1 & 1 & 0 & 5 & 23 \end{array}\right|$ $\begin{array}{l} -4F_1 + F_3 \\ -F_1 + F_4 \end{array}$

$\left|\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 8 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & -8 & -3 & 1 & -21 \\ 0 & -1 & -1 & 5 & 15 \end{array}\right|$ $\begin{array}{l} -2F_2 + F_1 \\ 8F_2 + F_3 \\ F_2 + F_4 \end{array}$

$\left|\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 21 & 9 & 99 \\ 0 & 0 & 2 & 6 & 30 \end{array}\right|$ $\begin{array}{l} \\ \\ F_3/3 \\ F_4/2 \end{array}$

$$\left(\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 7 & 3 & 33 \\ 0 & 0 & 1 & 3 & 15 \end{array}\right) \quad F_3/7$$

$$\left(\begin{array}{cccc|c} 1 & 0 & -5 & -2 & -22 \\ 0 & 1 & 3 & 1 & 15 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 1 & 3 & 15 \end{array}\right) \quad \begin{array}{l} 5F_3+F_1 \\ -3F_3+F_2 \\ -F_3+F_4 \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1/7 & 11/7 \\ 0 & 1 & 0 & -2/7 & 6/7 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 0 & 18/7 & 72/7 \end{array}\right) \quad \begin{array}{l} \\ \\ \\ 7F_4/18 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1/7 & 11/7 \\ 0 & 1 & 0 & -2/7 & 6/7 \\ 0 & 0 & 1 & 3/7 & 33/7 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right) \quad \begin{array}{l} -\frac{F_4}{7}+F_1 \\ \frac{2F_4}{7}+F_2 \\ -\frac{3F_4}{7}+F_3 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right)$$

$$S: \left\{ X_1 = 1 \quad X_2 = 2 \quad X_3 = 3 \quad X_4 = 4 \right.$$

b) $$\begin{cases} X_1 - X_2 + X_3 - X_4 = -2 & (1) \\ X_1 + 2X_2 - 2X_3 - X_4 = -5 & (2) \\ 2X_1 + X_2 - 3X_3 + 2X_4 = -1 & (3) \\ X_1 + 2X_2 + 3X_3 - 6X_4 = -10 & (4) \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 1 & 2 & -2 & -1 & -5 \\ 2 & 1 & -3 & 2 & -1 \\ 1 & 2 & 3 & -6 & -10 \end{array}\right) \begin{array}{l} \\ -F_1 + F_2 \\ -2F_1 + F_3 \\ -F_1 + F_4 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 3 & -3 & 0 & -3 \\ 0 & 3 & -5 & 4 & 3 \\ 0 & 3 & 2 & -5 & -8 \end{array}\right) \begin{array}{l} \\ F_2/3 \\ \\ \\ \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & -1 & -2 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 3 & -5 & 4 & 3 \\ 0 & 3 & 2 & -5 & -8 \end{array}\right) \begin{array}{l} F_2 + F_1 \\ \\ -3F_2 + F_3 \\ -3F_2 + F_4 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & -2 & 4 & 6 \\ 0 & 0 & 5 & -5 & -5 \end{array}\right) \begin{array}{l} \\ \\ -F_3/2 \\ F_4/5 \end{array}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & -1 & -1 \end{array}\right) \begin{array}{l} \\ F_3 + F_2 \\ -F_3 + F_4 \\ \\ \end{array}$$

$$\left|\begin{array}{cccc:c} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & -2 & -4 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right| \begin{array}{l} F_4 + F_1 \\ 2F_4 + F_2 \\ 2F_4 + F_3 \end{array}$$

$$\left|\begin{array}{cccc:c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right|$$

$S: \{ X_1 = -1 \quad X_2 = 0 \quad X_3 = 1 \quad X_4 = 2$

c) $\begin{cases} 6X_1 - 5X_2 + 7X_3 + 8X_4 = 3 \\ 3X_1 + 11X_2 + 2X_3 + 4X_4 = 6 \\ 3X_1 + 2X_2 + 3X_3 + 4X_4 = 1 \\ X_1 + X_2 + X_3 = 0 \end{cases}$

$$\left|\begin{array}{cccc:c} 6 & -5 & 7 & 8 & 3 \\ 3 & 11 & 2 & 4 & 6 \\ 3 & 2 & 3 & 4 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{array}\right|$$

$$\left|\begin{array}{cccc:c} 1 & 1 & 1 & 0 & 0 \\ 3 & 11 & 2 & 4 & 6 \\ 3 & 2 & 3 & 4 & 1 \\ 6 & -5 & 7 & 8 & 3 \end{array}\right| \begin{array}{l} -3F_1 + F_2 \\ -3F_1 + F_3 \\ -6F_1 + F_4 \end{array}$$

d) $\begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} z = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$

$$\begin{pmatrix} x+2y \\ 3x-y \\ 2x-y \end{pmatrix} + \begin{pmatrix} -z \\ 2z \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} x+2y-z \\ 3x-y+2z \\ 2x-y+z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ -2 \end{pmatrix}$$

$$\begin{cases} x+2y-z = 3 \\ 3x-y+2z = -3 \\ 2x-y+z = -2 \end{cases}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & -3 \\ 2 & -1 & 1 & -2 \end{array}\right| \begin{array}{l} -3F_1+F_2 \\ -2F_1+F_3 \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -12 \\ 0 & -5 & 3 & -8 \end{array}\right| \; -F_2/7$$

$$\left|\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & -5 & 3 & -8 \end{array}\right| \begin{array}{l} -2F_2+F_1 \\ 5F_2+F_3 \\ \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 3/7 & -3/7 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & 0 & -4/7 & 4/7 \end{array}\right| \; -7F_3/4$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -1 & 0 & 4 & 1 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right|$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 4 & 1 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right| \begin{array}{l} \\ -F_2 \\ \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 8 & -1 & 4 & 6 \\ 0 & -11 & 1 & 8 & 3 \end{array}\right| \begin{array}{l} -F_2+F_1 \\ -8F_2+F_3 \\ 11F_2+F_4 \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 1 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & -1 & 36 & 14 \\ 0 & 0 & 1 & -36 & -8 \end{array}\right| \begin{array}{l} \\ \\ -F_3 \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 1 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -36 & -14 \\ 0 & 0 & 1 & -36 & -8 \end{array}\right| \begin{array}{l} -F_3+F_1 \\ -F_3+F_4 \\ \\ \\ \end{array}$$

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 40 & 15 \\ 0 & 1 & 0 & -4 & -1 \\ 0 & 0 & 1 & -36 & -14 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right|$$

El sistema es inconsistente (no tiene solución

$$\left|\begin{array}{ccc|c} 1 & 0 & 3/7 & -3/7 \\ 0 & 1 & -5/7 & 12/7 \\ 0 & 0 & 1 & -1 \end{array}\right| \quad \begin{array}{l} \frac{5F_3}{7} + F_2 \\ -\frac{3F_3}{7} + F_1 \end{array}$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right|$$

$S: \{ x = 0 \quad y = 1 \quad z = -1$

62.

$$\begin{cases} x + y + z = 2 \\ x + 3y + 2z = 5 \\ 2x + 3y + (a^2-1)z = a+1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 2 & 3 & a^2-1 \end{pmatrix}$$

Por el método del pivoteo:

$$1\begin{vmatrix} 3 & 2 \\ 3 & a^2-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 2 & a^2-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} = 0$$

$$1(3a^2-3-6) - 1(a^2-1-4) + 1(3-6) = 0$$

$$3a^2 - 9 - a^2 + 5 + 3 - 6 = 0$$

$$2a^2 - 7 = 0$$

$$2a^2 = 7$$

$$a^2 = \frac{7}{2} \qquad \boxed{a = \pm\sqrt{\frac{7}{2}}}$$

(63) a) Verdadero *

b) Verdadero **

c) Verdadero ***

d) Falso ****

e) Falso *****

$$\begin{cases} X_1 + 2X_2 - 3X_3 + X_4 - X_5 = 0 \\ 3X_1 - X_2 - 2X_3 - X_4 + X_5 = 0 \\ -2X_1 + X_2 - X_3 + X_4 + X_5 = 0 \\ X_1 + X_2 + X_3 - 2X_4 - X_5 = 0 \\ -3X_1 - 4X_2 + 4X_3 - X_4 + 4X_5 = 0 \end{cases}$$

* Como se observa, el sistema es homogéneo.

** $(2, 2, 2, 2, 2)$ es una solución

*** $m = 5$

**** La información es suficiente

***** El sistema propuesto satisface las condiciones iniciales.

El sistema tiene infinitas soluciones

(64)
$$\begin{cases} aX_1 - X_2 + X_3 = 0 \\ -X_1 + X_2 + 2X_3 = 0 \\ 3X_1 + 2X_2 - X_3 = 0 \end{cases}$$

$$\begin{pmatrix} a & -1 & 1 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix}$$

Veamos cuando la determinante es igual a cero y hallemos <u>a</u>.
Por el método del pivoteo:

$$a\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & 2 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} -1 & 1 \\ 3 & 2 \end{vmatrix} = 0$$

$$a(-1-4) + 1(1-6) + 1(-2-3) = 0$$

$$a(-5) + 1(-5) + 1(-5) = 0$$

$$-5a - 5 - 5 = 0$$

$$-5a - 10 = 0$$

$$-5a = 10$$

$$a = \frac{\cancel{10}^{\,2}}{\cancel{-5}_{\,1}} \qquad \boxed{a = -2}$$

La alternativa e) es falsa, ya que los sistemas homogéneos <u>no</u> pueden ser inconsistentes

65 a) $\begin{cases} (m-2)x + y + 2z = 0 \\ x + my + z = 0 \\ 2x + 2y + 2z = 0 \end{cases}$

El sistema tiene siempre la solución trivial

m puede tomar cualquier valor.

b) $\begin{cases} -mx + y - z = 1 & (1) \\ 2mx + y - z = 0 & (2) \\ -my + z = 0 & (3) \\ mx = 1 & (4) \end{cases}$

De la ecuación (4) se deduce que $x = \frac{1}{m}$, $\boxed{m \neq 0}$

Tomando solo hasta (3)

$$\begin{pmatrix} -m & 1 & -1 \\ 2m & 1 & -1 \\ 0 & -m & 1 \end{pmatrix}$$

Por el método del pivoteo:

$$-m \begin{vmatrix} 1 & -1 \\ -m & 1 \end{vmatrix} - 2m \begin{vmatrix} 1 & -1 \\ -m & 1 \end{vmatrix} \neq 0$$

$$-m(1-m) - 2m(1+m) \neq 0$$

$$-m + m^2 - 2m + 2m^2 \neq 0$$

$$3m^2 - 3m \neq 0$$

$$3m(m-1) \neq 0$$

$$m(m-1) \neq 0$$

$\boxed{m \neq 0}$ $\qquad m - 1 \neq 0 \quad \boxed{m \neq 1}$

$S: \{ m \neq 0 \wedge m \neq 1$

c) $\begin{cases} (m-1)x - 2y = 2 \\ -x + y + mz = 0 \\ -y + 2z = m \end{cases}$

$$\begin{pmatrix} m-1 & -2 & 0 \\ -1 & 1 & m \\ 0 & -1 & 2 \end{pmatrix}$$

Por el método del pivoteo

$$(m-1)\begin{vmatrix} 1 & m \\ -1 & 2 \end{vmatrix} + 1\begin{vmatrix} -2 & 0 \\ -1 & 2 \end{vmatrix} \neq 0$$

$$(m-1)(2+m) + 1(-4+0) \neq 0$$

$$2m + m^2 - 2 - m - 4 \neq 0$$

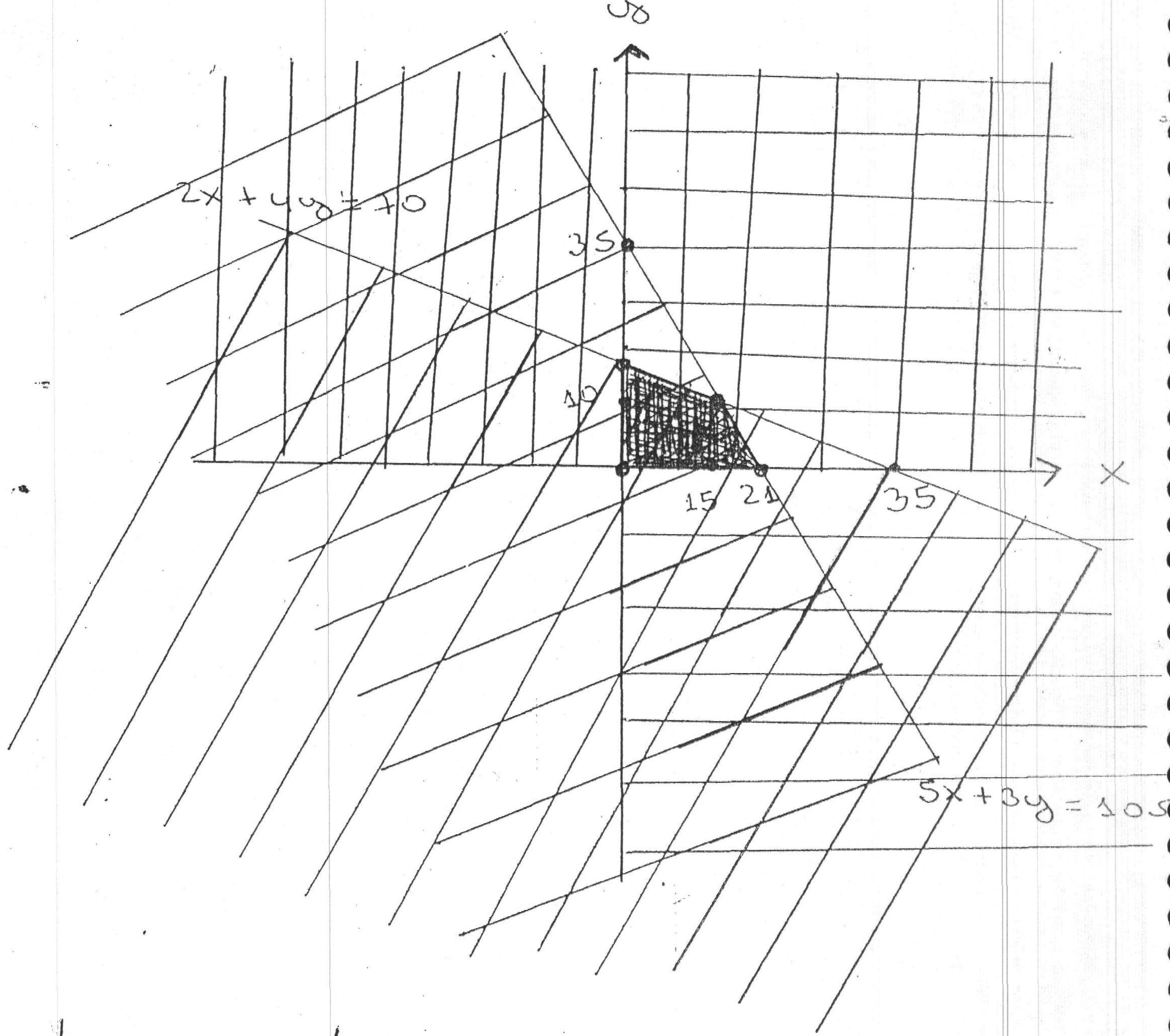
$$m^2 + m - 6 \neq 0$$

$$(m+3)(m-2) \neq 0$$

$m+3 \neq 0$ $\quad$ $m-2 \neq 0$

$\boxed{m \neq -3}$ $\quad$ $\boxed{m \neq 2}$

$S: \{ m \neq -3 \wedge m \neq 2$



La región más pintada es la solución del sistema y está representada por el trapecio formado.

Los vértices de dicho trapecio son: $P_1(0,0)$ $P_3(21,0)$

$P_2(0,10)$ $P_4(15,10)$

y también son soluciones del sistema de desigualdades.

66 a) $\begin{cases} 3^{x+y} = 81 & (1) \\ 3^{y-x} = 9 & (2) \end{cases}$

1) $3^{x+y} = 81$ $\qquad 3^{x+y} = 3^4$

$x + y = 4$ *

2) $3^{y-x} = 9$ $\qquad 3^{y-x} = 3^2$

$-x + y = 2$ **

$$\begin{array}{r} x + y = 4 \\ -x + y = 2 \\ \hline 2y = 6 \end{array}$$

$y = \frac{\cancel{6}^{3}}{\cancel{2}_{1}}$ $\qquad \boxed{y = 3}$

Reemplazo en (1) ó (2)

En (1): $x + y = 4$

$x + 3 = 4$

$x = 4 - 3$ $\qquad \boxed{x = 1}$

b) $\begin{cases} 3^x + 3^y = 36 & (1) \\ 3^{y-x} = 3 & (2) \end{cases}$

Igualando y en función de x en (2)

$3^{y-x} = 3^1$ $\qquad y - x = 1$

$\boxed{y = 1 + x}$ (3)

Reemplazando 3 en 1, y resolviendo.

$$3^x + 3^y = 36$$

$$3^x + 3^{x+1} = 36$$

$$3^x + 3(3^x) = 36$$

$$4(3^x) = 36$$

$$3^x = \frac{\cancel{36}^{\,9}}{\cancel{4}_{\,1}} \qquad 3^x = 9$$

$$3^x = 3^2 \qquad \boxed{x = 2}$$

Reemplazando en 3:

$$y = x + 1$$

$$y = 2 + 1 \qquad \boxed{y = 3}$$

c) $$\begin{cases} 2^x + 2^y = 20 & (1) \\ 2^{x+y} = 64 & (2) \end{cases}$$

Resolviendo en 2:

$$2^{x+y} = 64$$

$$2^{x+y} = 2^6$$

$$x + y = 6 \qquad \boxed{x = 6 - y} \; (3)$$

Reemplazo 3 en 1

$$2^x + 2^y = 20$$

$$2^{6-y} + 2^y = 20$$

$$\frac{2^6}{2^y} + 2^y = 20.$$

$$2^6 + 2^y(2^y) = 20(2^y)$$

$$2^6 + 2^{2y} = 20(2^y)$$

$$(2)^{2y} + 20(2^y) + 64 = 0$$

$$(2^y)^2 - 20(2^y) + 64 = 0$$

$$(2^y - 16)(2^y - 4) = 0$$

$2^y - 16 = 0$

$2^y = 16$

$2^y = 2^4$

$\boxed{y_1 = 4}$

$2^y - 4 = 0$

$2^y = 4$

$2^y = 2^2$

$\boxed{y_2 = 2}$

Reemplazando en 3

$x = 6 - y$

Con $y_1 = 4$

$x_1 = 6 - 4$

$\boxed{x_1 = 2}$

Con $y_2 = 2$

$x_2 = 6 - 2$

$\boxed{x_1 = 4}$

Las soluciones para el problema serian:

$$\begin{cases} x = 2 \\ y = 4 \end{cases} \qquad \begin{matrix} x = 4 \\ y = 2 \end{matrix}$$

d) $\begin{cases} 2^x + 3^y = 7 & (1) \\ 2^{2x+1} - 3^{2y} = 23 & (2) \end{cases}$

Tomando (1)

$2^x + 3^y = 7$

$\boxed{2^x = 7 - 3^y}$ (3)

Reemplazo (3) en (2):

$2^{2x+1} - 3^{2y} = 23$

$2(2^{2x}) - 3^{2y} = 23$

$2(2^x)^2 - 3^{2y} = 23$

$2(7-3^y)^2 - 3^{2y} = 23$

$2\left[49 - 14(3^y) + 3^{2y}\right] - 3^{2y} = 23$

$98 - 28(3^y) + 2(3^{2y}) - 3^{2y} = 23$

$3^{2y} - 28(3^y) + 98 - 23 = 0$

$(3^y)^2 - 28(3^y) + 75 = 0$

$(3^y - 25)(3^y - 3) = 0$

$3^y - 25 = 0$

$3^y = 25$

$\log_3 (3)^y = \log_3 25$

$y \log_3 3 = \log_3 25$

$\boxed{y_1 = \log_3 25}$

$3^y - 3 = 0$

$3^y = 3$

$\boxed{y_2 = 1}$

Reemplazando en 3

$2^x = 7 - 3^y$

Con $y_1 = \log_3 25$

$2^x = 7 - 3^{\log_3 25}$

$2^x = 7 - 25$

$2^x = -18$

Imposible

Con $y_2 = 1$

$2^x = 7 - 3^1$

$2^x = 7 - 3$

$2^x = 4$

$2^x = 2^2$

$\boxed{x = 2}$

$S: \begin{cases} x = 2 \\ y = 1 \end{cases}$

e) $\begin{cases} 2^{2x-y} = 32 & (1) \\ 3^{x-2y} = 3 & (2) \end{cases}$

Tomando 1

$2^{2x-y} = 32$

$2^{2x-y} = 2^5$

$2x - y = 5$

$-y = 5 - 2x$

$\boxed{y = 2x - 5}$ (3)

Tomando 2

$3^{x-2y} = 3$

$x - 2y = 1$

$-2y = 1 - x$

$2y = x - 1$

$\boxed{y = \frac{x-1}{2}}$ (4)

Igualando 3 y 4:

$y = 2x - 5$ $\qquad$ $y = \dfrac{x-1}{2}$

$2x - 5 = \dfrac{x-1}{2}$

$2(2x-5) = x - 1$

$4x - 10 = x - 1$ $\qquad$ $4x - x = -1 + 10$

$3x = 9$ $\qquad$ $x = \dfrac{\cancel{9}^{3}}{\cancel{3}_{1}}$ $\qquad$ $\boxed{x = 3}$

Reemplazando en 3 ó 4:

En 3: $y = 2x - 5$

$y = 2(3) - 5$

$y = 6 - 5$ $\qquad$ $\boxed{y = 1}$

$S: \begin{cases} x = 3 \\ y = 1 \end{cases}$

f) $\begin{cases} 3^x \cdot 4^y = 3^8 & (1) \\ 2^{x-1} \cdot 2^{y+1} = 2^6 & (2) \end{cases}$

Tomando 2

$2^{x-1} \cdot 2^{y+1} = 2^6$

$\dfrac{2^x}{2} \cdot 2(2^y) = 64$

$2^x \cdot \cancel{2}(2^y) = 128$

$2^y = \dfrac{128}{2(2^x)}$ $\qquad$ $2^y = \dfrac{64}{2^x}$

$2^y = \dfrac{2^6}{2^x}$ $\qquad$ $2^y = (2)^{6-x}$ $\qquad$ $\boxed{y = 6 - x}$

Reemplazando (3) en (1):

$$3^x \cdot 4^y = 3^8$$

$$3^x \cdot 4^{6-x} = 3^8$$

$$3^x \cdot \frac{4^6}{4^x} = 3^8$$

$$\frac{3^x}{4^x} = \frac{3^8}{4^6}$$

$*$ y $**$: Soluciones definitivas

$$\left(\frac{3}{4}\right)^x = \frac{6561}{4096}$$

$$\log_{\frac{3}{4}} \left(\frac{3}{4}\right)^x = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$x \log_{\frac{3}{4}} \frac{3}{4} = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$* \quad \boxed{x = \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)} \quad *$$

Reemplazo en (3)

$$y = 6 - x$$

$$\boxed{y = 6 - \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)}$$

De aquí se deduce

$$y = \log_{\frac{3}{4}} \left(\frac{3}{4}\right)^6 - \log_{\frac{3}{4}} \frac{6561}{4096}$$

$$y = \log_{\frac{3}{4}} \left(\frac{729}{4096}\right) - \log_{\frac{3}{4}} \left(\frac{6561}{4096}\right)$$

$$y = \log_{\frac{3}{4}} \left[\left(\frac{729}{4096}\right) \div \left(\frac{6561}{4096}\right)\right]$$

$$y = \log_{\frac{3}{4}} \left(\frac{\cancel{729}^{\,1}}{\cancel{4096}} \cdot \frac{\cancel{4096}}{\cancel{6561}_{\,9}}\right) \quad ** \quad \boxed{y = \log_{\frac{3}{4}} \left(\frac{1}{9}\right)} \; **$$

g) $\begin{cases} x + y = 110 & ① \\ \log x + \log y = 3 & ② \end{cases}$

Tomando ②:

$\log x + \log y = 3$

$\log(x y) = 3$ $\qquad x y = 10^3$

$x y = 1000$ $\qquad \boxed{x = \dfrac{1000}{y}}$ ③

Reemplazo ③ en ①

$x + y = 110$

$\dfrac{1000}{y} + y = 110$

$1000 + y^2 = 110 y$ $\qquad y^2 - 110y + 1000 = 0$

$(y - 100)(y - 10) = 0$

$y - 100 = 0$ $\qquad y - 10 = 0$

$\boxed{y_1 = 100}$ $\qquad \boxed{y_2 = 10}$

Reemplazando en ③:

$x = \dfrac{1000}{y}$

$x_1 = \dfrac{10\cancel{00}}{1\cancel{00}}$ $\qquad x_2 = \dfrac{100\cancel{0}}{1\cancel{0}}$

$\boxed{x_1 = 10}$ $\qquad \boxed{x_2 = 100}$

$S: \begin{cases} x = 10 \\ y = 100 \end{cases}$ $\qquad \begin{cases} x = 100 \\ y = 10 \end{cases}$

h) $\begin{cases} \log_2(x-y) = 2 & (1) \\ \log_2 x - \log_2 y = 1 & (2) \end{cases}$

De la ecuación (2):

$$\log_2 x - \log_2 y = 1$$

$$\log_2 \frac{x}{y} = 1$$

$$\frac{x}{y} = 2^1 \qquad \frac{x}{y} = 2 \qquad \boxed{x = 2y} \ (3)$$

Reemplazo (3) en (1):

$$\log_2(x-y) = 2$$

$$\log_2(2y-y) = 2$$

$$\log_2(y) = 2$$

$$\cancel{2^{\log_2}} y = 2^2 \qquad y = 2^2 \qquad \boxed{y = 4}$$

Reemplazo en (3):

$$x = 2y \qquad x = 2(4) \qquad \boxed{x = 8}$$

$$S: \begin{cases} x = 8 \\ y = 4 \end{cases}$$

i) $\begin{cases} \log x + \log y = 4 & (1) \\ y - 4x = 0 & (2) \end{cases}$

De (2) tenemos:

$$y - 4x = 0$$

$$\boxed{y = 4x} \ (3)$$

Reemplazo 3 en 1:

$$\log x + \log y = 4$$

$$\log xy = 4$$

$$\cancel{10^{\log}} xy = 10^4 \qquad xy = 10000$$

* Como $y = 4x$

$$x(4x) = 10000$$

$$4x^2 = 10000 \qquad x^2 = \frac{10000}{4}$$

$$x^2 = 2500 \qquad \cancel{\sqrt{x^2}} = \sqrt{2500}$$

$$\boxed{x = 50}$$

Reemplazando en 3:

$$y = 4x$$

$$y = 4(50) \qquad \boxed{y = 200}$$

$$S: \begin{cases} x = 50 \\ y = 200 \end{cases}$$

67 a) $\begin{cases} \operatorname{Sen} x + \operatorname{Sen} y = 1 & (1) \\ \operatorname{Cos}(x - y) = 1 & (2) \end{cases}$

$$x, y \in (0, 2\pi)$$

De 2 tenemos:

$$\operatorname{Cos}(x - y) = 1$$

$$x - y = \operatorname{Cos}^{-1}(1)$$

$$x - y = 0 \qquad \boxed{x = y} \quad (3)$$

Reemplazando 3 en 1

$$\operatorname{Sen} x + \operatorname{Sen} y = 1$$

$$\operatorname{Sen} x + \operatorname{Sen} x = 1$$

$$2 \operatorname{Sen} x = 1$$

$$\operatorname{Sen} x = \frac{1}{2}$$

$$x = \operatorname{Sen}^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x_2 = \frac{5\pi}{6}}$$

Reemplazo en 3

$$y = x$$

$$\boxed{y_1 = \frac{\pi}{6}} \qquad \boxed{y_2 = \frac{5\pi}{6}}$$

$$S: \begin{cases} x = \frac{\pi}{6} & x = \frac{5\pi}{6} \\ y = \frac{\pi}{6} & y = \frac{5\pi}{6} \end{cases}$$

b) $\begin{cases} \operatorname{Cos} x \operatorname{Tan} x = \frac{\sqrt{3}}{2} & (1) \\ \operatorname{Sen}(x+y) = 1 & (2) \end{cases}$

De 1, tenemos:

$$\operatorname{Cos} x \operatorname{Tan} x = \frac{\sqrt{3}}{2}$$

$$\cancel{\operatorname{Cos} x} \cdot \frac{\operatorname{Sen} x}{\cancel{\operatorname{Cos} x}} = \frac{\sqrt{3}}{2} \qquad x \neq 0 \quad x \neq 2\pi \quad x \neq \frac{\pi}{2}$$

$$\operatorname{Sen} x = \frac{\sqrt{3}}{2}$$

$$x = \operatorname{Sen}^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{x_2 = \frac{2\pi}{3}} \qquad \boxed{x_1 = \frac{\pi}{3}}$$

Reemplazando en (2):

$$\operatorname{Sen}(x+y) = 1$$

$$x+y = \operatorname{Sen}^{-1}(1)$$

$$x+y = \frac{\pi}{2}$$

Con $x_1 = \frac{\pi}{3}$

$$\frac{\pi}{3} + y = \frac{\pi}{2} \qquad y = \frac{\pi}{2} - \frac{\pi}{3} \qquad \boxed{y_1 = \frac{\pi}{6}}$$

Con $x_2 = \frac{2\pi}{3}$

$$\frac{2\pi}{3} + y = \frac{\pi}{2} \qquad y = \frac{\pi}{2} - \frac{2\pi}{3}$$

$$y = -\frac{\pi}{6} \qquad y = \frac{-\pi}{6} + 2\pi \qquad \boxed{y_2 = \frac{11\pi}{12}}$$

$$S: \begin{cases} x = \frac{\pi}{3} \\ y = \frac{\pi}{6} \end{cases} \qquad \begin{aligned} x &= \frac{2\pi}{3} \\ y &= \frac{11\pi}{12} \end{aligned}$$

c) $\begin{cases} \operatorname{Sen} x \operatorname{Sen} y = \frac{1}{4} & (1) \\ \operatorname{Cos} x \operatorname{Cos} y = \frac{3}{4} & (2) \end{cases}$ $\qquad x \neq 0$, $x \neq 2\pi$

Sumando (1) y (2), obtenemos:

$$\operatorname{Sen} x \operatorname{Sen} y + \operatorname{Cos} x \operatorname{Cos} y = 1$$

$$\operatorname{Cos}(x-y) = 1$$

$$x - y = \operatorname{Cos}^{-1}(1)$$

$$x - y = 0 \qquad\qquad x - y = 2\pi$$

$$\boxed{x = y} \ (3) \qquad\qquad \boxed{x = 2\pi + y}$$

Descarto por *

Reemplazo (3) en (1)

$$\text{Sen}\, x \ \text{Sen}\, y = \frac{1}{4}$$

$$\text{Sen}\, x \ \text{Sen}\, x = \frac{1}{4}$$

$$\text{Sen}^2 x = \frac{1}{4} \qquad \cancel{\sqrt{}}\,\text{Sen}^{\cancel{2}} x = \pm\sqrt{\frac{1}{4}}$$

$$\text{Sen}\, x = \pm\frac{1}{2} \qquad x = \text{Sen}^{-1}\left(\pm\frac{1}{2}\right)$$

$$x = \text{Sen}^{-1}\left(\frac{1}{2}\right)$$

$$\boxed{x_1 = \frac{\pi}{6}} \qquad \boxed{x_2 = \frac{5\pi}{6}}$$

$$x = \text{Sen}^{-1}\left(-\frac{1}{2}\right)$$

$$\boxed{x_3 = \frac{7\pi}{6}} \qquad \boxed{x_4 = \frac{11\pi}{6}}$$

Como $x = y$, obtenemos

$$S: \begin{cases} x = \frac{\pi}{6} \\ y = \frac{\pi}{6} \end{cases} \qquad \begin{matrix} x = \frac{5\pi}{6} \\ y = \frac{5\pi}{6} \end{matrix} \qquad \begin{matrix} x = \frac{7\pi}{6} \\ x = \frac{7\pi}{6} \end{matrix} \qquad \begin{matrix} x = \frac{11\pi}{6} \\ y = \frac{11\pi}{6} \end{matrix}$$

d) $\begin{cases} x + y = \dfrac{2\pi}{3} & (1) \\ \text{Sen}\, x + \text{Sen}\, y = \dfrac{3}{2} & (2) \end{cases}$

De (1): $x + y = \dfrac{2\pi}{3}$

$$\boxed{x = \frac{2\pi}{3} - y} \quad (3)$$

Reemplazo en (2)

$$\text{Sen}\, x + \text{Sen}\, y = \frac{3}{2}$$

$$\text{Sen}\left(\frac{2\pi}{3} - y\right) + \text{Sen}\, y = \frac{3}{2}$$

$$\text{Sen}\,\frac{2\pi}{3}\,\text{Cos}\, y - \text{Sen}\, y\,\text{Cos}\,\frac{2\pi}{3} + \text{Sen}\, y = \frac{3}{2}$$

$$\frac{\sqrt{3}}{2}\,\text{Cos}\, y + \frac{1}{2}\,\text{Sen}\, y + \text{Sen}\, y = \frac{3}{2}$$

$$\sqrt{3}\,\text{Cos}\, y + 3\,\text{Sen}\, y = 3$$

$$\sqrt{3}\,\sqrt{1 - \text{Sen}^2 y} = 3 - 3\,\text{Sen}\, y$$

$$3(1 - \text{Sen}^2 y) = 9 - 18\,\text{Sen}\, y + 9\,\text{Sen}^2 y$$

$$3 - 3\,\text{Sen}^2 y - 9 + 18\,\text{Sen}\, y - 9\,\text{Sen}^2 y = 0$$

$$-3\,\text{Sen}^2 y - 9\,\text{Sen}^2 y + 18\,\text{Sen}\, y - 6 = 0$$

$$-12\,\text{Sen}^2 y + 18\,\text{Sen}\, y - 6 = 0$$

$$2\,\text{Sen}^2 y - 3\,\text{Sen}\, y + 1 = 0$$

$$(2\,\text{Sen}\, y - 1)(\text{Sen}\, y + 1) = 0$$

$2\,\text{Sen}\,y - 1 = 0$

$2\,\text{Sen}\,y = 1$

$\text{Sen}\,y = \frac{1}{2}$

$y = \text{Sen}^{-1}\left(\frac{1}{2}\right)$

$$\boxed{y_1 = \frac{\pi}{6}}$$

$\text{Sen}\,y - 1 = 0$

$\text{Sen}\,y = 1$

$y = \text{Sen}^{-1}(1)$

$$\boxed{y_2 = \frac{\pi}{2}}$$

reemplazo en (3):

$x = \frac{2\pi}{3} - y$

Con $y_1 = \frac{\pi}{6}$

$x = \frac{2\pi}{3} - \frac{\pi}{6}$

$$\boxed{x_1 = \frac{\pi}{2}}$$

Con $y_2 = \frac{\pi}{2}$

$x_2 = \frac{2\pi}{3} - \frac{\pi}{2}$

$$\boxed{x_2 = \frac{\pi}{6}}$$

S: $\begin{cases} x = \frac{\pi}{2} \\ y = \frac{\pi}{6} \end{cases}$ $\quad x = \frac{\pi}{6}$, $\quad y = \frac{\pi}{2}$

e) $\begin{cases} \operatorname{Sen} x \operatorname{Sen} y = \operatorname{Cos} x \operatorname{Cos} y & (1) \\ x - y = \dfrac{\pi}{6} & (2) \end{cases}$

$x \neq 0$ $\quad x \neq 2\pi$

De (1):

$\operatorname{Sen} x \operatorname{Sen} y - \operatorname{Cos} x \operatorname{Cos} y = 0$

$\operatorname{Cos} x \operatorname{Cos} y - \operatorname{Sen} x \operatorname{Sen} y = 0$

$\operatorname{Cos}(x+y) = 0$

$x + y = \operatorname{Cos}^{-1}(0)$

$x + y = 0$ $\qquad$ $x + y = 2\pi$

$\boxed{x = -y}$ (3) $\qquad$ $\boxed{x = 2\pi - y}$ (4)

De (2):

$x - y = \dfrac{\pi}{6}$ $\qquad$ $\boxed{x = \dfrac{\pi}{6} + y}$ (5)

Igualando (3) y (5):

$\dfrac{180}{12}$

$-y = \dfrac{\pi}{6} + y$ $\qquad$ $-2y = \dfrac{\pi}{6}$

$y = \dfrac{\frac{\pi}{6}}{-2}$ $\qquad$ $y = -\dfrac{\pi}{12} + 2\pi$

$\boxed{y = \dfrac{23\pi}{12}}$

Igualando (4) y (5):

$2\pi - y = \dfrac{\pi}{6} + y$

$2\pi - \dfrac{\pi}{6} = y + y$ $\qquad$ $\dfrac{11\pi}{6} = 2y$

$2y = \dfrac{11\pi}{6}$ $\qquad$ $\boxed{y = \dfrac{11\pi}{12}}$

S: $\begin{cases} x = \dfrac{13\pi}{12} \\ y = \dfrac{11\pi}{12} \end{cases}$ $\qquad$ $x = \dfrac{\pi}{12}$ $\quad y = \dfrac{23\pi}{12}$

$$\frac{\sqrt{3}}{2}\cos y + \frac{1}{2}\operatorname{Sen} y + \operatorname{Sen} y = \frac{2}{3}$$

$$\frac{\sqrt{3}}{2}\cos y + \frac{3}{2}\operatorname{Sen} y = \frac{2}{3}$$

$$3\sqrt{3}\cos y + 9\operatorname{Sen} y = 4$$

$$9\operatorname{Sen} y = 4 - 3\sqrt{3}\cos y$$

$$9\sqrt{1-\cos^2 y} = 4 - 3\sqrt{3}\cos y$$

$$\left(9\sqrt{1-\cos^2 y}\right)^2 = \left(4 - 3\sqrt{3}\cos y\right)^2$$

$$81(1-\cos^2 y) = 16 - 24\sqrt{3}\cos y + 27\cos^2 y$$

$$81 - 81\cos^2 y = 16 - 24\sqrt{3}\cos y + 27\cos^2 y$$

$$0 = 16 - 24\sqrt{3}\cos y + 27\cos^2 y - 81 + 81\cos^2 y = 0$$

$$81\cos^2 y - 24\sqrt{3}\cos y - 65 = 0$$

$a=-2$ $\qquad a = 81 \qquad b = -24\sqrt{3} \qquad c = -65.$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{(-24\sqrt{3})^2 - 4(81)(-65)}}{2(81)}$$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{1728 + 21060}}{162}$$

$$\cos y = \frac{24\sqrt{3} \pm \sqrt{22788}}{162}$$

$$\cos y = \frac{24\sqrt{3} \pm 3\sqrt{2532}}{162}$$

Las soluciones son inconmensurables.

f) $\begin{cases} \text{Sen}\,x + \text{Cos}\,y = \frac{1}{2} & (1) \\ \text{Sen}\,x + \text{Sen}\,y = \frac{3}{2} & (2) \end{cases}$

Restando (1) de (2)

$$\begin{array}{r} \text{Sen}\,x + \text{Sen}\,y = \frac{3}{2} \\ -\text{Sen}\,x - \text{Cos}\,y = -\frac{1}{2} \\ \hline \text{Sen}\,y - \text{Cos}\,y = 1 \end{array}$$

$$\text{Sen}\,y = 1 + \text{Cos}\,y$$

$$\text{Sen}^2 y = 1 + 2\,\text{Cos}\,y + \text{Cos}^2 y$$

$$1 - \text{Cos}^2 y = 1 + 2\,\text{Cos}\,y + \text{Cos}^2 y$$

$$\cancel{1} + 2\,\text{Cos}\,y + \text{Cos}^2 y - \cancel{1} + \text{Cos}^2 y = 0$$

$$2\,\text{Cos}^2 y + 2\,\text{Cos}\,y = 0$$

$$2\,\text{Cos}\,y\,(\text{Cos}\,y + 1) = 0$$

$$\text{Cos}\,y\,(\text{Cos}\,y + 1) = 0$$

* $\text{Cos}\,y = 0$

$y = \text{Cos}^{-1}(0)$

$\boxed{y_1 = \frac{\pi}{2}}$ $\boxed{y_2 = \frac{3\pi}{2}}$

** $\text{Cos}\,y + 1 = 0$

$\text{Cos}\,y = -1$ $y = \text{Cos}^{-1}(-1)$

$\boxed{y = \pi}$

Reemplazando en (1) ó (2):

En 1: $\text{Sen}\,x + \text{Cos}\,y = \frac{1}{2}$

$y_1 = \frac{\pi}{2}$ $\text{Sen}\,x + \text{Cos}\,\frac{\pi}{2} = \frac{1}{2}$

$\text{Sen}\,x = \frac{1}{2}$ $x = \text{Sen}^{-1}\left(\frac{1}{2}\right)$

$\boxed{x_1 = \frac{\pi}{6}}$ $\boxed{x_1 = \frac{5\pi}{6}}$

Con $y_2 = \frac{3\pi}{2}$

$\operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2}$

$\operatorname{Sen} x = \frac{1}{2}$

$\boxed{x_1 = \frac{\pi}{6}}$ $\boxed{x = \frac{5\pi}{6}}$

Con $y_3 = \pi$

$\operatorname{Sen} x + \operatorname{Cos} y = \frac{1}{2}$

$\operatorname{Sen} x + \operatorname{Cos} \pi = \frac{1}{2}$

$\operatorname{Sen} x = \frac{1}{2} - \operatorname{Cos} \pi$

$\operatorname{Sen} x = \frac{1}{2} - (-1)$

$\operatorname{Sen} x = \frac{3}{2}$ $x = \operatorname{Sen}^{-1}\left(\frac{3}{2}\right)$

Esto es imposible ya que el seno está entre $[-1, 1]$

Por lo tanto, la solución del problema, nos queda:

$S: \begin{cases} x = \frac{\pi}{6} \\ y = \frac{3\pi}{2} \end{cases}$ $x = \frac{5\pi}{6}$ $y = \frac{3\pi}{2}$

$\begin{cases} x = \frac{\pi}{6} \\ y = \frac{\pi}{2} \end{cases}$ $x = \frac{5\pi}{6}$ $y = \frac{\pi}{2}$

(68) $\begin{cases} p^2 + x^2 = 169 & (1) \\ p = x + 7 & (2) \end{cases}$

De (2), reemplazo en (1)

$$p^2 + x^2 = 169$$
$$(x+7)^2 + x^2 = 169$$
$$x^2 + 14x + 49 + x^2 = 169$$
$$2x^2 + 14x + 49 - 169 = 0$$
$$2x^2 + 14x - 120 = 0$$
$$x^2 + 7x - 60 = 0$$
$$(x+12)(x-5) = 0$$

$x + 12 = 0$ $\qquad$ $x - 5 = 0$

$\boxed{x = -12}$ $\qquad$ $\boxed{x = 5}$

Imposible

(69) x = tiempo de trabajo del padre
y = tiempo de trabajo del hijo

$$\begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{12} & (1) \\ x = y - 7 & (2) \end{cases}$$

Reemplazo (2) en (1)

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12}$$
$$\frac{1}{y-7} + \frac{1}{y} = \frac{1}{12}$$
$$12y + 12(y-7) = y(y-7)$$
$$12y + 12y - 84 = y^2 - 7y$$
$$0 = y^2 - 7y - 24y + 84 \qquad y^2 - 31y + 84 = 0$$

$(y-28)(y-3)=0$

$y-28=0 \qquad y-3=0$

$\boxed{y_1=28} \qquad \boxed{y_2=3}$

cero descarto por naturaleza $y_2=3$

Reemplazo en (2):

$x=y-7$

$x=28-7$

$\boxed{x=21}$

(10)

$$\begin{cases} R_1+R_2=25 & (1) \\ \frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{6} & (2) \end{cases}$$

De (1), tenemos:

$R_1+R_2=25 \qquad \boxed{R_1=25-R_2}$ (3)

Reemplazo en (2), el valor (3):

$\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{6}$

$\frac{1}{25-R_2}+\frac{1}{R_2}=\frac{1}{6}$

$6R_2+6(25-R_2)=R_2(25-R_2)$

$(25=150R_2-6R_2^2)$

$(6R_2^2-150R_2+25=0)$

$6R_2+150-6R_2=25R_2-R_2^2$

$R_2^2-25R_2+\cancel{6R_2}+150-\cancel{6R_2}=0$

$R_2^2-25R_2+150=0$

$$(R_2 - 15)(R_2 - 10) = 0$$

$R_2 - 15 = 0$ $\qquad$ $R_2 - 10 = 0$

$\boxed{R_2 = 15}$ $\qquad$ $\boxed{R_2 = 10}$

Reemplazo en 3:

$$R_1 = 25 - R_2$$

Con $R_2 = 15$

$R_1 = 25 - 15$

$\boxed{R_1 = 10}$

Con $R_2 = 10$

$R_1 = 25 - 10$

$\boxed{R_2 = 15}$

$\begin{cases} R_1 = 10 \\ R_2 = 15 \end{cases}$ $\qquad$ $R_1 = 15$, $R_2 = 10$

71 $\begin{cases} 2x^2 - 3y^2 = 15 & (1) \\ x^2 + 2y^2 = 11 & (2) \end{cases}$

$$\begin{array}{r} 2x^2 - 3y^2 = +15 \\ -2x^2 - 4y^2 = -22 \\ \hline // \quad -7y^2 = -7 \end{array}$$

$7y^2 = 7$

$y^2 = \frac{\cancel{7}^{\,1}}{\cancel{7}_{\,1}}$ $\qquad$ $y^2 = 1$ $\qquad$ $y = \pm 1$

Reemplazo en 1 ó 2:

En 2: $x^2 + 2y^2 = 11$

$x^2 + 2(\pm 1)^2 = 11$

$x^2 + 2 = 11$ $\qquad$ $x^2 = 11 - 2$

$x^2 = 9$ $\qquad$ $x = \pm 3$

S: $\begin{cases} x = 3 \\ y = 1 \end{cases}$ $\quad$ $x = 3$, $y = -1$ $\quad$ $x = -3$, $y = 1$ $\quad$ $x = -3$, $y = -1$

b) $\begin{cases} x^2 + y^2 = 25 & (1) \\ x + y = 1 & (2) \end{cases}$

De (2):

$x + y = 1$ $\boxed{x = 1 - y}$ (3)

Reemplazo (3) en (1)

$x^2 + y^2 = 25$

$(1 - y)^2 + y^2 = 25$

$1 - 2y + y^2 + y^2 - 25 = 0$

$2y^2 - 2y + 24 = 0$

$y^2 - y + 12 = 0$

$(y - 4)(y + 3) = 0$

$y - 4 = 0$ $y + 3 = 0$

$\boxed{y_1 = 4}$ $\boxed{y_2 = -3}$

Reemplazo en (3)

$x = 1 - y$

Con $y_1 = 4$ Con $y_2 = -3$

$x = 1 - 4$ $x = 1 + (-3)$

$\boxed{x_1 = -3}$ $\boxed{x_2 = 4}$

S: $\begin{cases} x = -3 \\ y = 4 \end{cases}$ $\begin{matrix} x = 4 \\ y = -3 \end{matrix}$

c) $\begin{cases} x^2 + 3xy = -8 & (1) \\ xy + y^2 = -4 & (2) \end{cases}$

De (2):

$$xy + y^2 = -4$$

$$xy = -4 - y^2 \qquad \boxed{x = \frac{-4-y^2}{y}} \quad (3)$$

Reemplazo (3) en (1):

$$x^2 + 3xy = -8$$

$$\left(\frac{-4-y^2}{y}\right)^2 + 3xy = -8$$

$$\frac{16 + 8y^2 + y^4}{y^2} + 3\cancel{y}\left(\frac{-4-y^2}{\cancel{y}}\right) = -8$$

$$16 + 8y^2 + y^4 + y^2(-12-3y^2) = -8y^2$$

$$16 + 8y^2 + y^4 - 12y^2 - 3y^4 + 8y^2 = 0$$

$$-2y^4 + 4y^2 + 16 = 0$$

$$y^4 - 2y^2 - 8 = 0$$

$$(y^2 - 4)(y^2 + 2) = 0$$

$$y^2 - 4 = 0 \qquad\qquad y^2 + 2 = 0$$

$$y^2 = 4 \qquad\qquad y^2 = -2$$

$$\boxed{y = \pm 2} \qquad\qquad \text{Imposible}$$

Reemplazo en (3)

$$x = \frac{-4-y^2}{y}$$

Con $y_1 = 2$

$$x_1 = \frac{-4-(2)^2}{2} \qquad\qquad x_1 = \frac{-4-4}{2}$$

$$x_1 = -\frac{8}{2} \qquad\qquad \boxed{x_1 = -4}$$

on $y = -2$

$x_2 = \frac{-4 - (-2)^2}{-2}$ $\qquad x_2 = \frac{-4-4}{-2}$

$x_2 = \frac{-\cancel{8}^{\,4}}{\cancel{-2}_{\,1}}$ $\qquad \boxed{x_2 = 4}$

S: $\begin{cases} x = -4 \\ y = 2 \end{cases}$ $\qquad \begin{matrix} x = 4 \\ y = -2 \end{matrix}$

d) $\begin{cases} x^2 + y^2 = 29 & (1) \\ xy = 10 & (2) \end{cases}$

De (2):

$xy = 10$ $\qquad \boxed{x = \frac{10}{y}}$

Reemplazo en (1):

$x^2 + y^2 = 29$

$\left(\frac{10}{y}\right)^2 + y^2 = 29$

$\frac{100}{y^2} + y^2 = 29$ $\qquad 100 + y^4 = 29y^2$

$y^4 - 29y^2 + 100 = 0$

$(y^2 - 25)(y^2 - 4) = 0$

$y^2 - 25 = 0$ $\qquad y^2 - 4 = 0$

$y^2 = 25$ $\qquad y^2 = 4$

$\boxed{y = \pm 5}$ $\qquad \boxed{y = \pm 2}$

S: $\begin{cases} x = 2 \\ y = 5 \end{cases}$ $\qquad \begin{matrix} x = 5 \\ y = 2 \end{matrix}$

e) $\begin{cases} xy - 6 = \dfrac{y^3}{x} & (1) \\ xy + 24 = \dfrac{x^3}{y} & (2) \end{cases}$

Multiplicando miembro a miembro (1) y (2) nos queda

$$(xy-6)(xy+24) = x^2y^2$$

$$x^2y^2 - 6xy + 24xy - 144 = x^2y^2$$

$$\cancel{x^2y^2} - \cancel{x^2y^2} + 18xy = 144$$

$$18xy = 144$$

$$x = \frac{\overset{8}{\cancel{144}}}{\underset{1}{\cancel{18}}y} \qquad \boxed{x = \frac{8}{y}} \quad (3)$$

Reemplazo en (1) ó (2)

En (1): $xy - 6 = \dfrac{y^3}{x}$

$\frac{8}{y}$) $y - 6 = \dfrac{y^3}{\frac{8}{y}}$

$8 - 6 = \dfrac{y^4}{8}$ $\qquad 8(2) = y^4$

$y^4 = 16$ $\qquad \sqrt[4]{y^4} = \sqrt[4]{16}$

$$\boxed{y = \pm 2}$$

Reemplazando en (3):

$x = \dfrac{8}{y}$

$y_1 = 2$ $\qquad x_1 = \dfrac{\cancel{8}^{4}}{\cancel{2}_1}$ $\qquad \boxed{x_1 = 4}$

$y_2 = -2$ $\qquad x_2 = \dfrac{\cancel{8}^{4}}{\cancel{-2}_1}$ $\qquad \boxed{x_2 = -4}$

S: $\begin{cases} x = 4 \\ y = 2 \end{cases}$ $\qquad \begin{matrix} x = -4 \\ y = -2 \end{matrix}$

f) $\begin{cases} 3x^2 - 8xy + 4y^2 = 0 & (1) \\ 5x^2 - 7xy - 6y^2 = 0 & (2) \end{cases}$

Factorizando (1):

$(3x - 2y)(x - 2y) = 0$

$3x - 2y = 0$ $\qquad x - 2y = 0$

$3x = 2y$ $\qquad \boxed{x = 2y}$ (4)

$\boxed{x = \dfrac{2y}{3}}$ (3)

Reemplazo en (2), el valor de (3):

$5x^2 - 7xy - 6y^2 = 0$

$5\left(\dfrac{2y}{3}\right)^2 - 7\left(\dfrac{2y}{3}\right)y - 6y^2 = 0$

$$5\left(\frac{4y^2}{9}\right)-\frac{14y^2}{3}-6y^2=0$$

$$20y^2-42y^2-54y^2=0$$

$$-76y^2=0$$

$$y^2=0 \qquad \boxed{y=0}$$

Reemplazo en (3):

$$x=\frac{2y}{3} \qquad x=\frac{2(0)}{3} \qquad \boxed{x=0}$$

Lo mismo ocurre al reemplazar el valor de (4) en (2)

$$S:\begin{cases} x=0 \\ y=0 \end{cases}$$

* g)

$$\begin{cases} 3x^2-2xy=160 & (1) \\ x^2-3xy-2y^2=8 & (2) \end{cases}$$

Despejando y en función de x en (1):

$$\boxed{y=\frac{3x^2-160}{2x}} \quad (3)$$

Reemplazo (3) en (2):

$$x^2-3x\left(\frac{3x^2-160}{2x}\right)-2\left(\frac{3x^2-160}{2x}\right)^4=8$$

$$x^2-\frac{3(3x^2-160)}{2}-\frac{9x^4-960x^2+25600}{2x^2}=$$

$$2x^4-9x^4+480x^2-9x^4+960x^2+25600=16x^2$$

Reduciendo, nos queda:

$$-16x^4+1424x^2-25600=0$$

$$x^4-89x^2+1600=0$$

actorizando:

$$(x^2-64)(x^2-25)=0$$

$x^2-64=0$ $\qquad$ $x^2-25=0$

$x^2=64$ $\qquad$ $x^2=25$

$\boxed{x=\pm 8}$ $\qquad$ $\boxed{x=\pm 5}$

Reemplazando en 3

$$y=\frac{3x^2-160}{2x}$$

Con $x=8$ $\qquad$ $y=2$

Con $x=-8$ $\qquad$ $y=-2$

Con $x=5$ $\qquad$ $y=-\frac{17}{2}$

Con $x=-5$ $\qquad$ $y=\frac{17}{2}$

S: $\begin{cases} x=8 \\ y=2 \end{cases}$ $\quad$ $\begin{matrix} x=-8 \\ y=-2 \end{matrix}$ $\quad$ $\begin{matrix} x=5 \\ y=-\frac{17}{2} \end{matrix}$ $\quad$ $\begin{matrix} x=-5 \\ y=\frac{17}{2} \end{matrix}$

72

$$\begin{cases} 5x + 3y \leq 105 \\ 2x + 4y \leq 70 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

a)
$$\begin{matrix} (2) \\ (-5) \end{matrix} \begin{cases} 5x + 3y = 105 & (1) \\ 2x + 4y = 70 & (2) \end{cases}$$

$$10x + 6y = 210$$
$$-10x - 20y = -350$$
$$// \quad -14y = -140$$

$14y = 140 \qquad y = \frac{\cancel{140}^{10}}{\cancel{14}_{1}} \qquad \boxed{y = 10}$

Reemplazo en (1) ó (2)

En (2): $2x + 4y = 70$

$$2x + 4(10) = 70$$

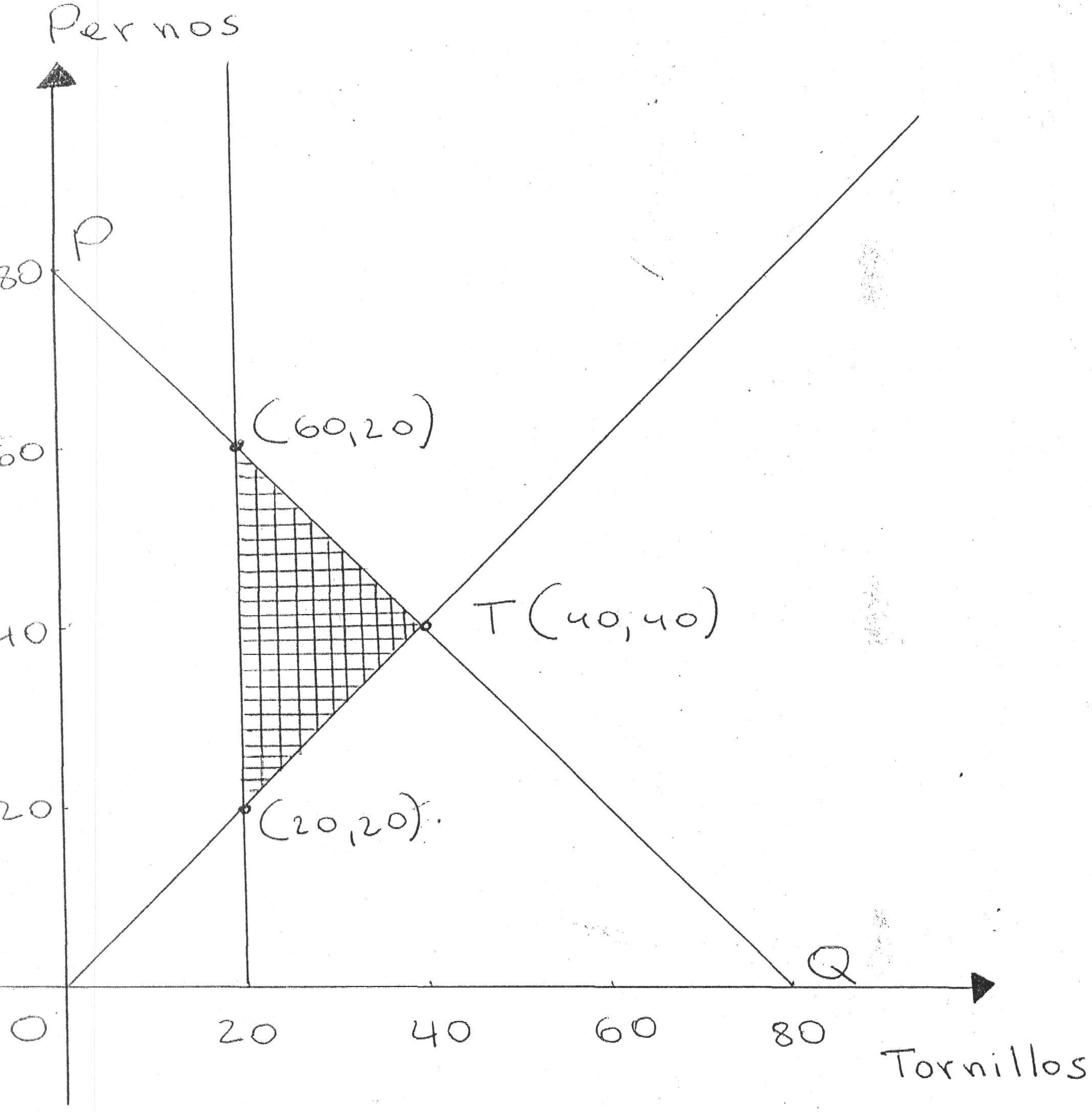
$2x + 40 = 70 \qquad 2x = 70 - 40$

$2x = 30 \qquad x = \frac{\cancel{30}^{15}}{\cancel{2}_{1}} \qquad \boxed{x = 15}$

$$P(15, 10)$$

S: $\{ X = 15 \quad Y = 10$

73

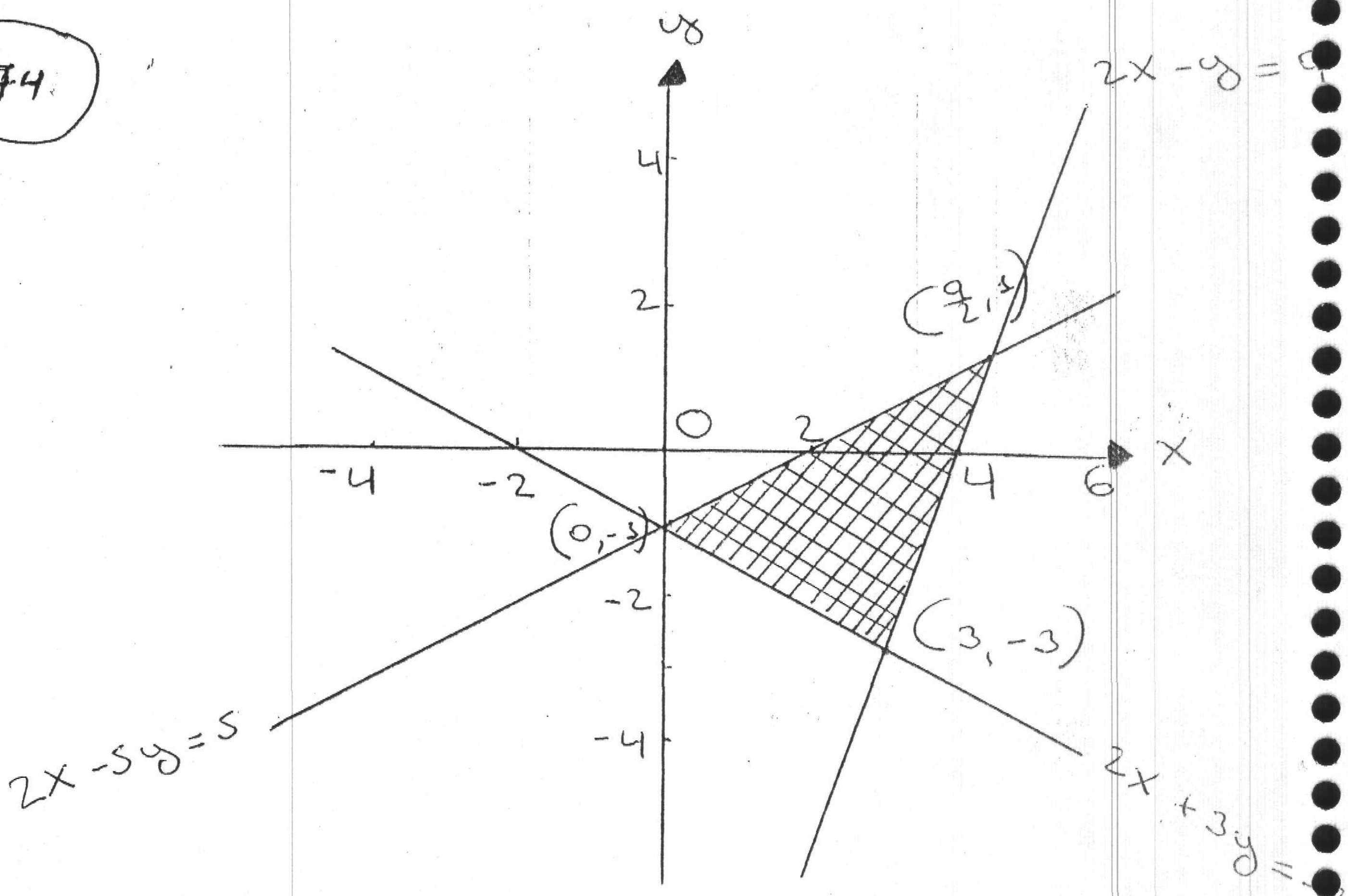


Los vértices del triángulo formado son: a) (20, 20), b) (60, 20), c) (40, 40)

Se observa claramente que $Q \geq 20$ y que $P \geq Q$ y además por b se puede establecer $P \leq 80 - Q$

$$\begin{cases} P \geq Q \\ P \leq 80 - Q \\ Q \geq 20 \end{cases}$$

(74)



Los vértices del triángulo son:

a) $(0, -1)$ b$(3, -3)$ c) $(\frac{9}{2}, 1)$

Se observa que : $2x + 3y + 3 \geq 0 \quad \wedge$

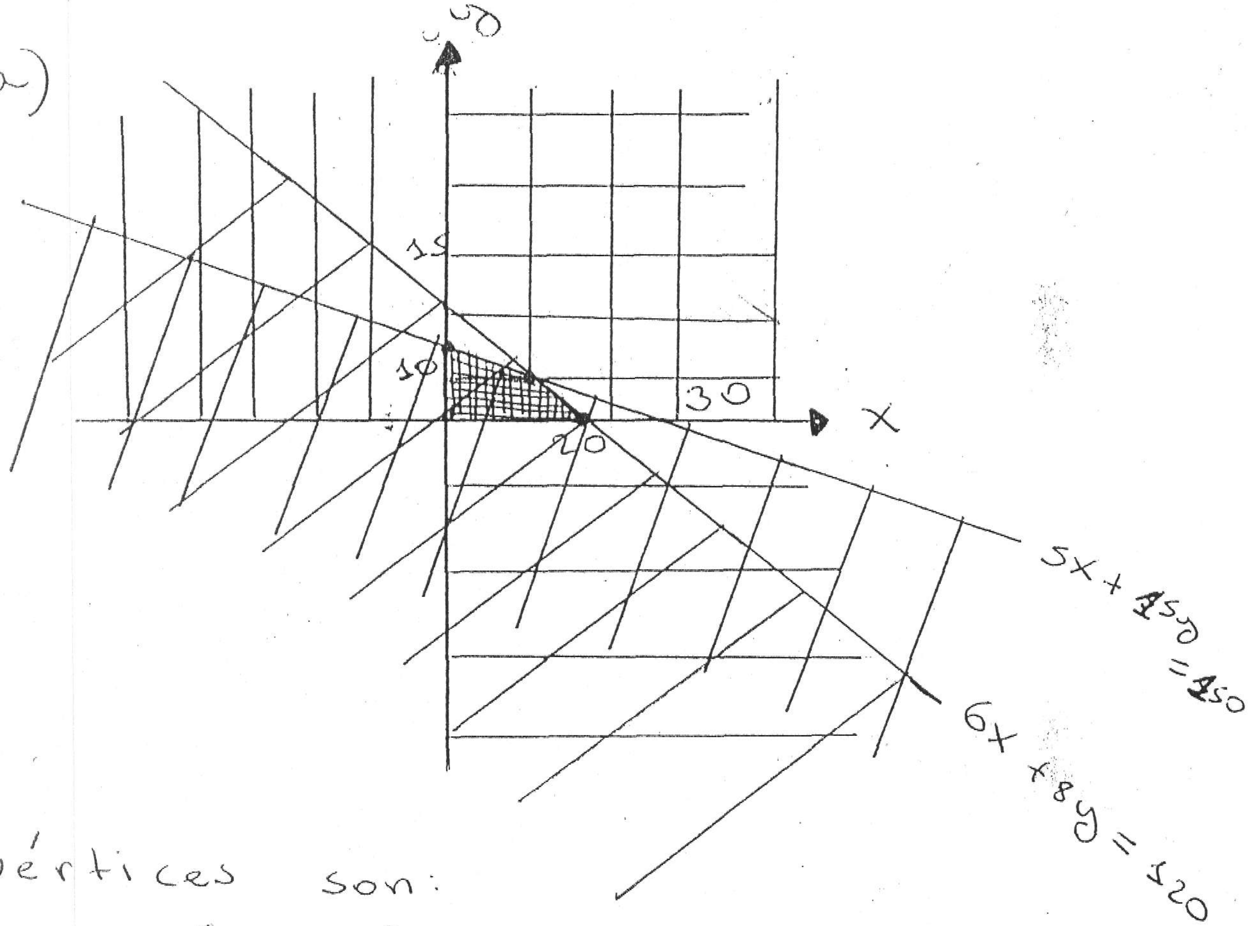
$2x - y - 9 \leq 0 \quad \wedge \quad 2x - 5y - 5 \geq 0$

Por tanto, el sistema corresponde a

$$\begin{cases} 2x + 3y \geq -3 \\ 2x - y - 9 \leq 0 \\ 2x - 5y - 5 \geq 0 \end{cases}$$

La región sombreada en el interior del triángulo corresponde al conjunto solución del sistema de desigualdades descrito.

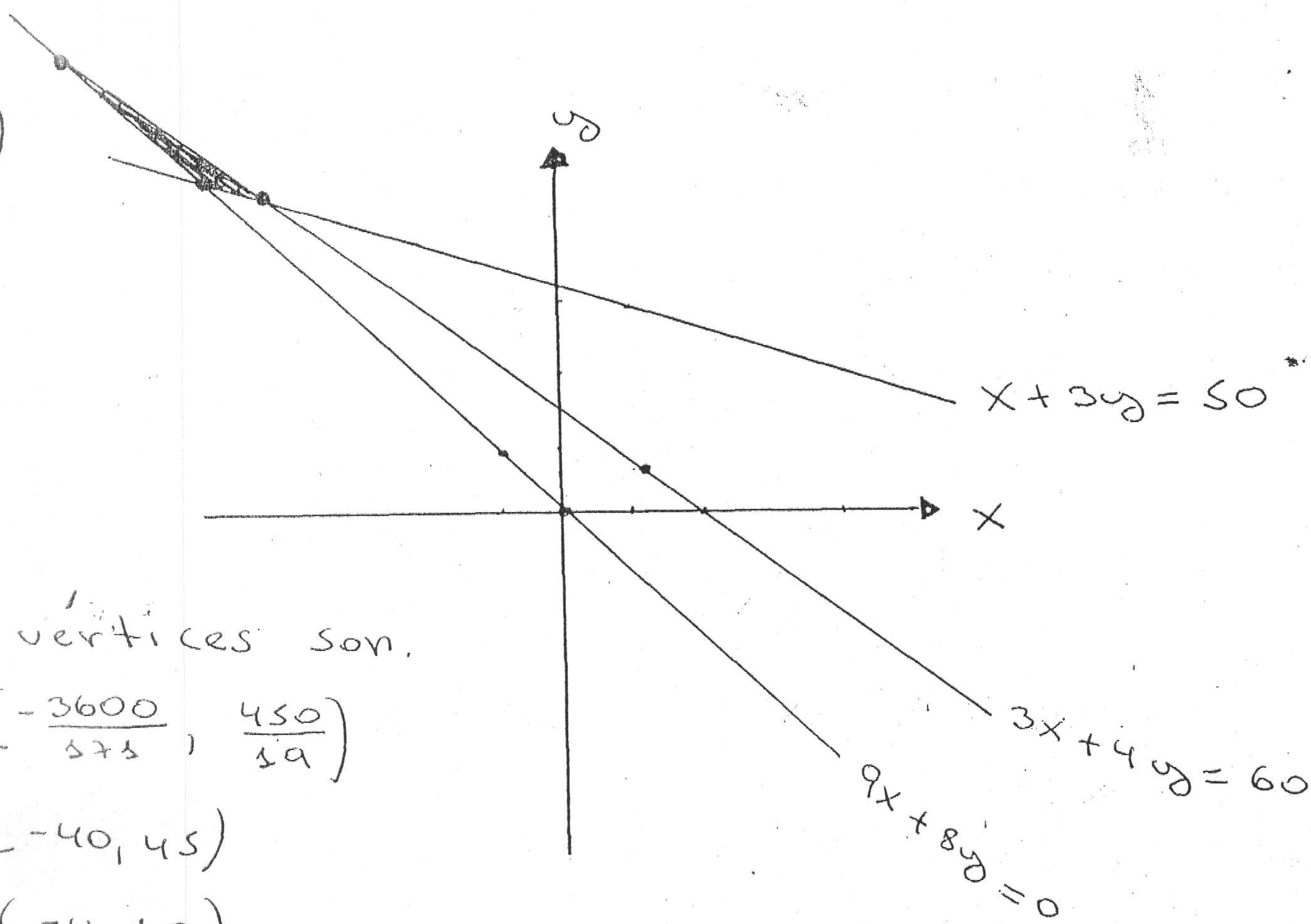
75) a)



Los vértices son:

$(0,0)$, $(0,10)$, $(20,0)$, $(12,6)$

b)



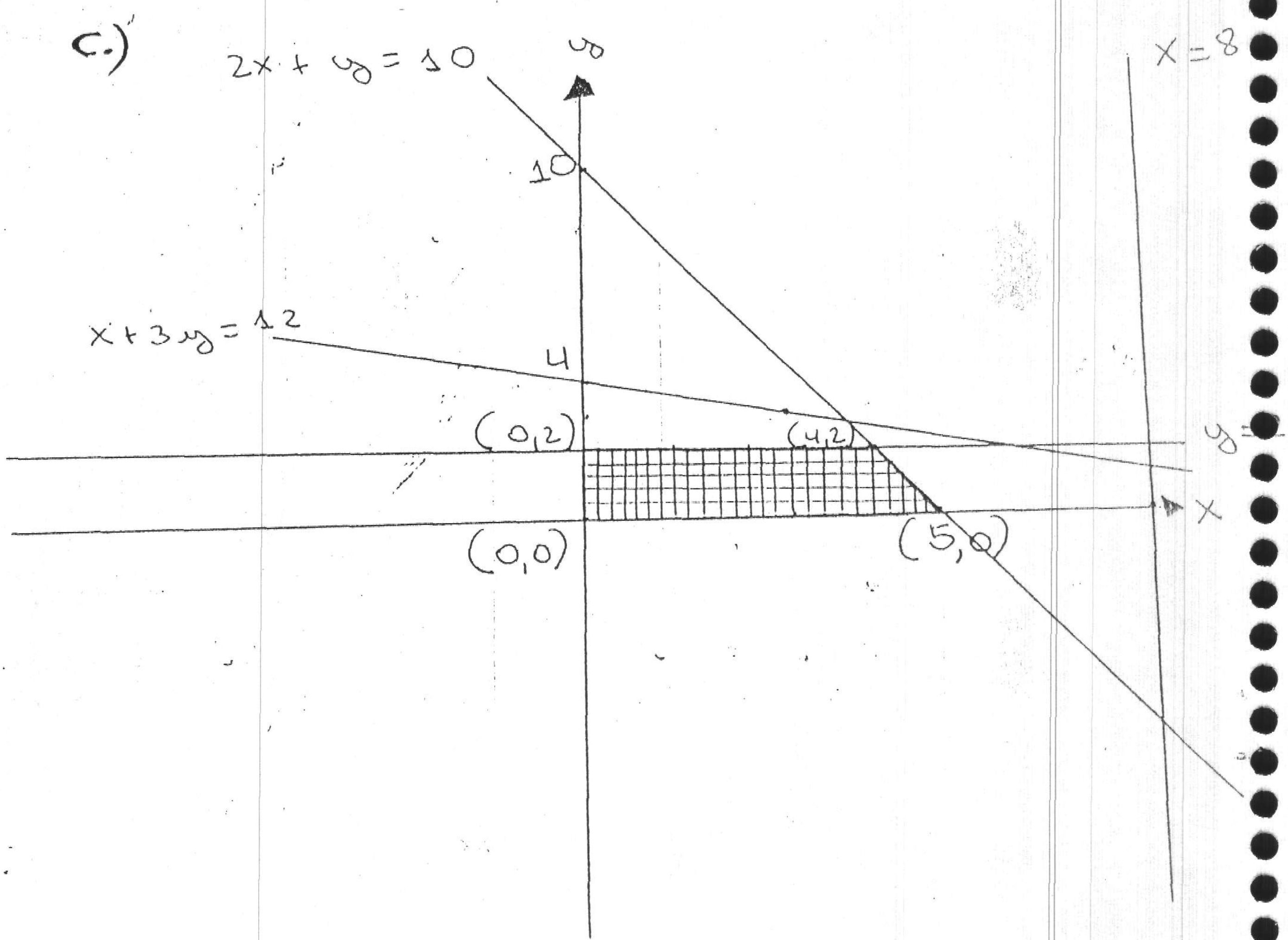
Los vértices son.

$P\left(-\frac{3600}{171}, \frac{450}{19}\right)$

$Q(-40, 45)$

$R(-4, 18)$

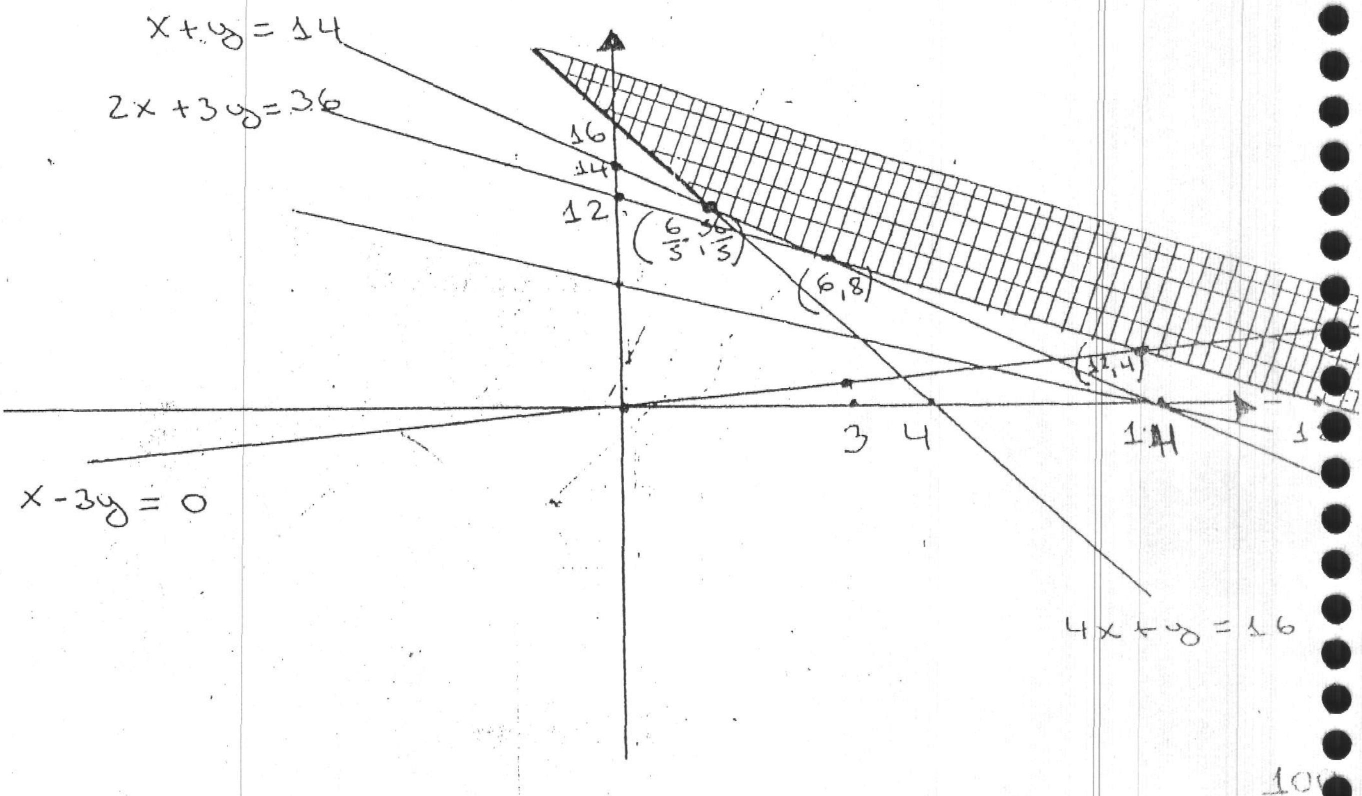
c.)



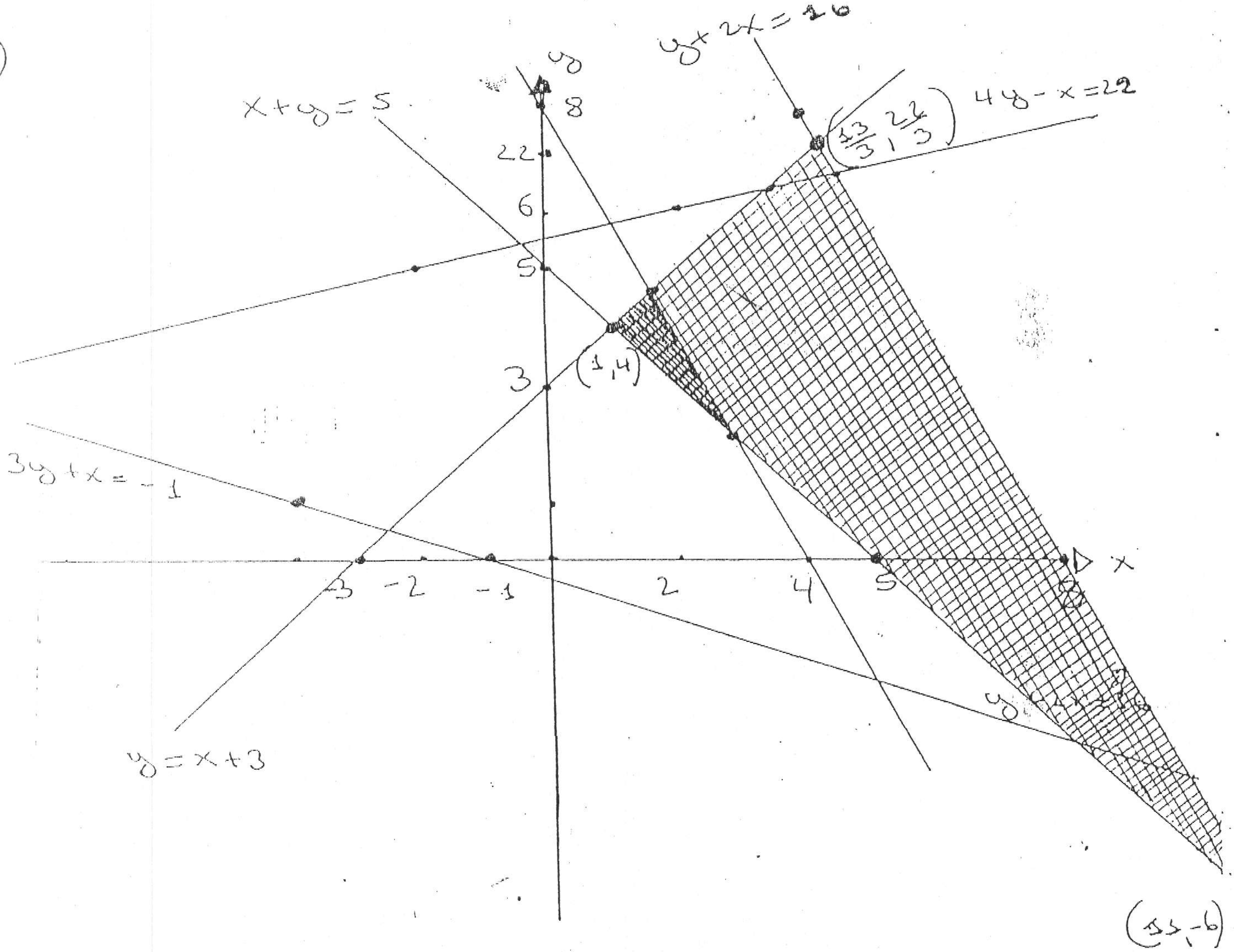
Los vértices son:

(0,2) , (0,0) , (4,2) , (5,0)

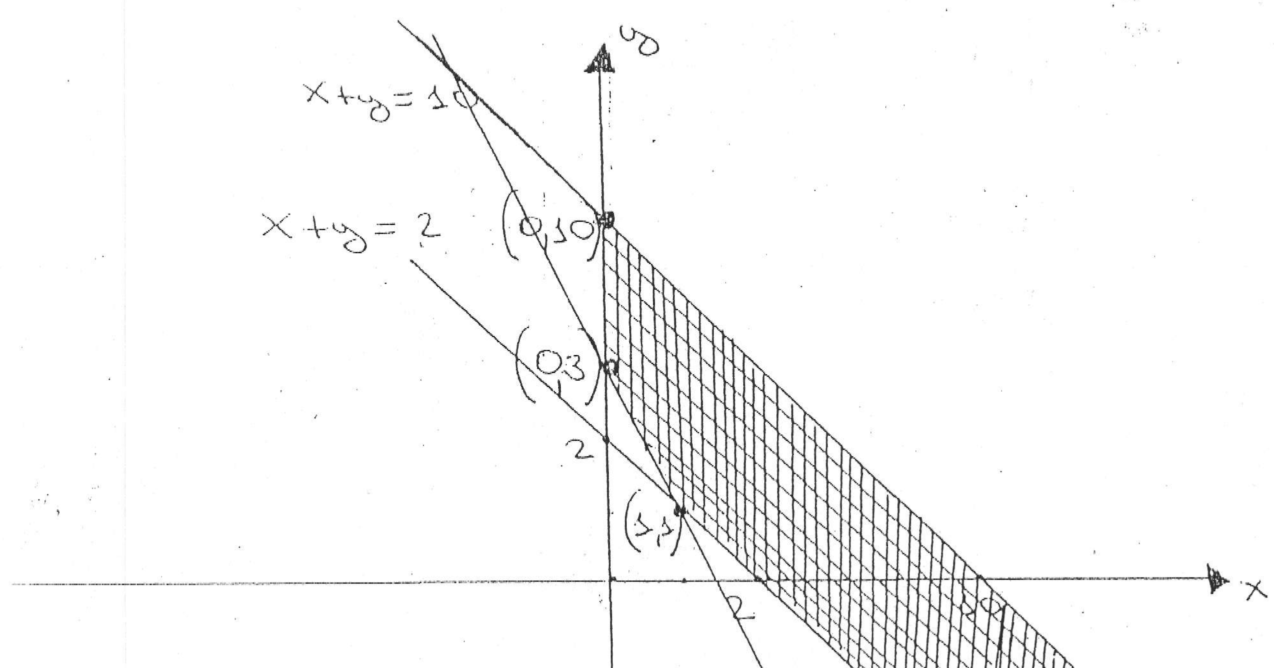
d)



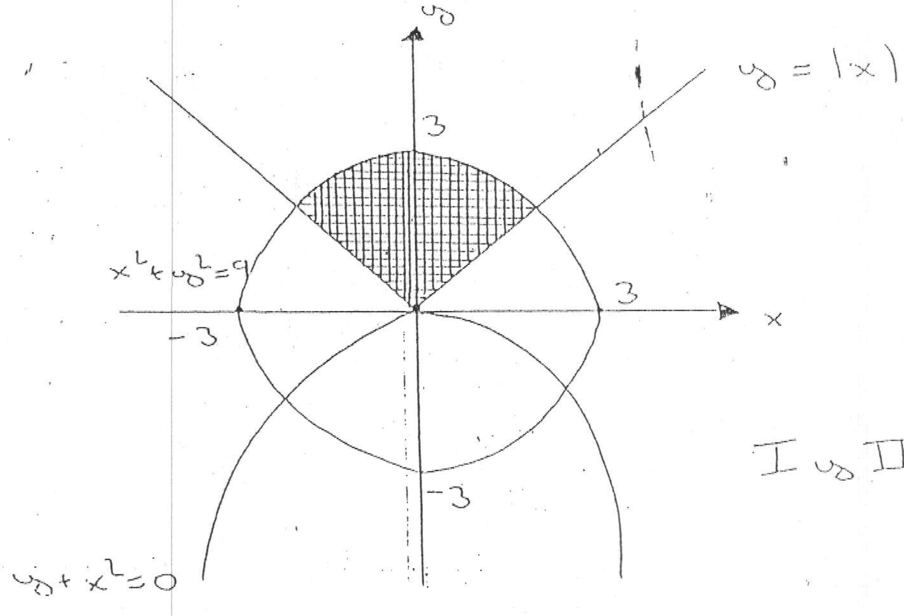
e)


y + 2x = 16
x + y = 5
4y - x = 22
y
8
6
5
3
(1,4)
3y + x = -1
y = x + 3
-3
-2
-1
2
4
5
x
(11,-6)

f)

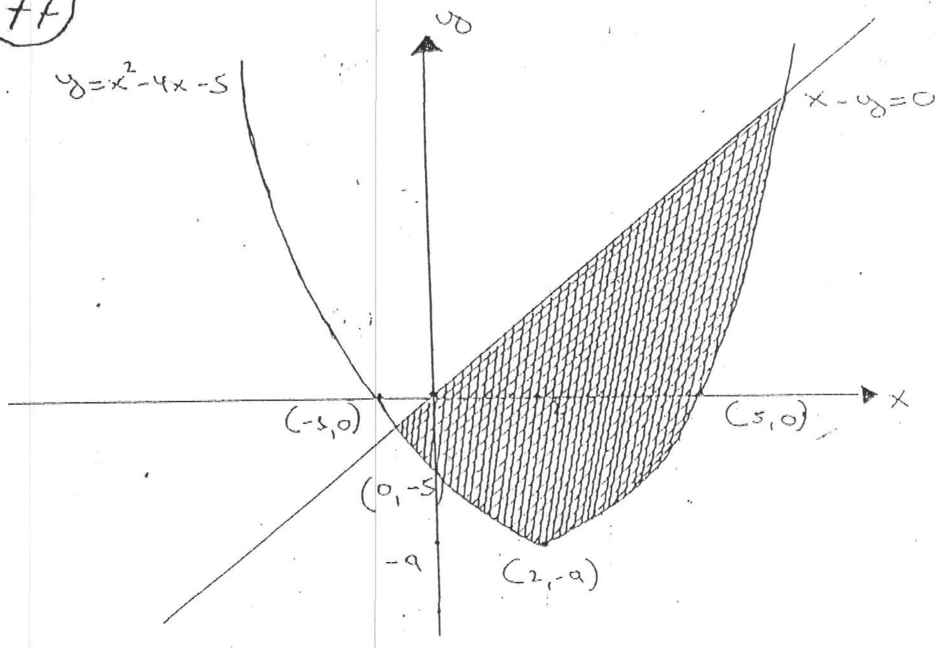

y
x + y = 10
x + y = 2
(0,10)
(0,3)
2
(1,1)
2
x
2x + y = 3

76.

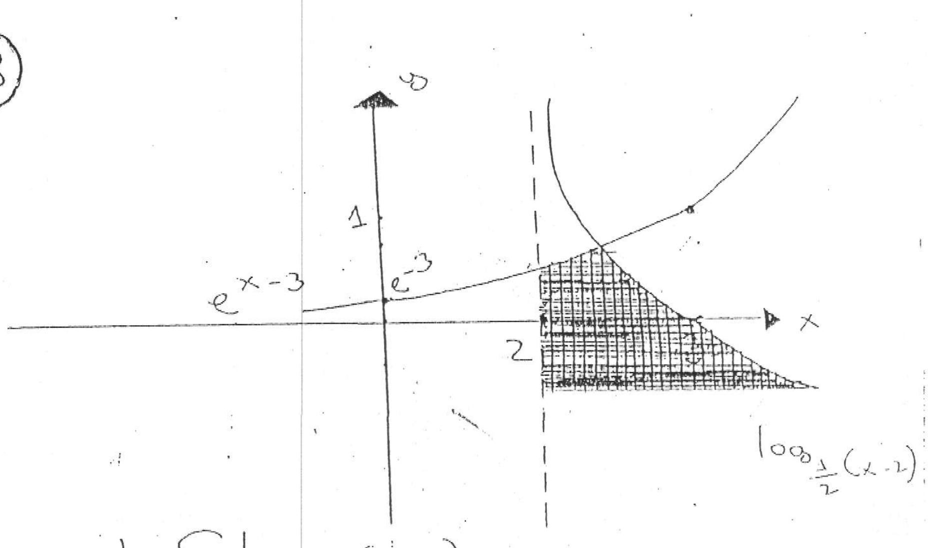


I y II

77

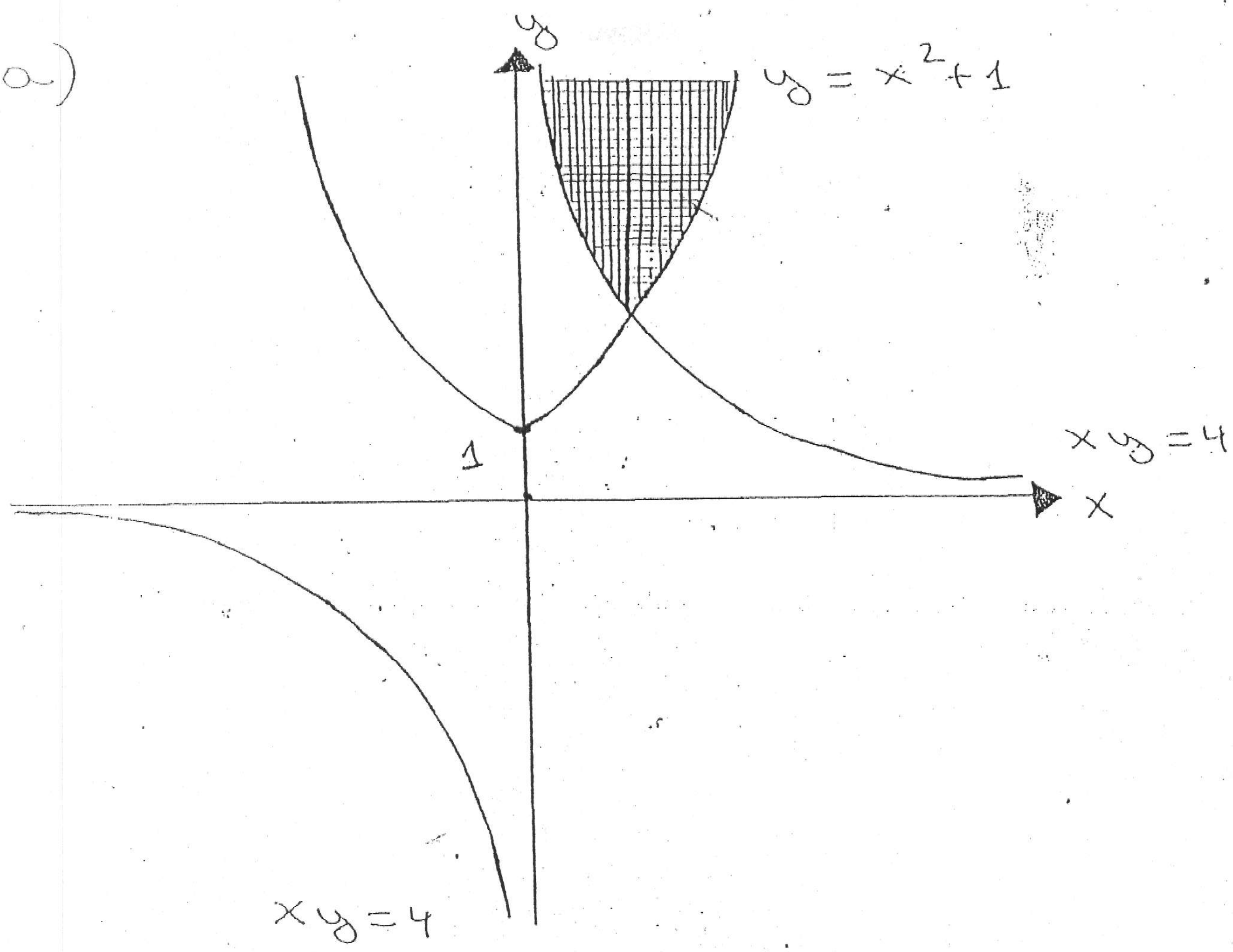


78

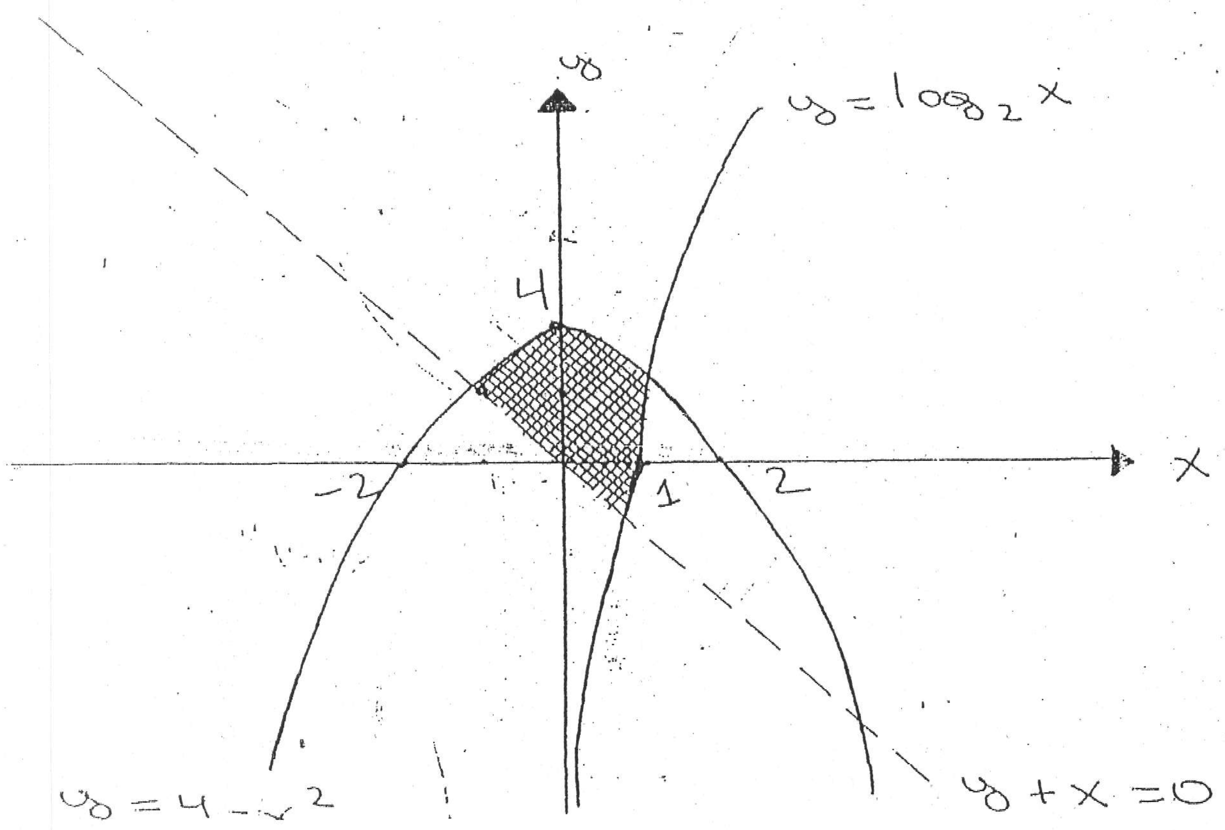


$$S: \begin{cases} \log_{\frac{1}{2}}(x-2) \geq y \\ e^{x-3} \geq y \end{cases}$$

79 a)

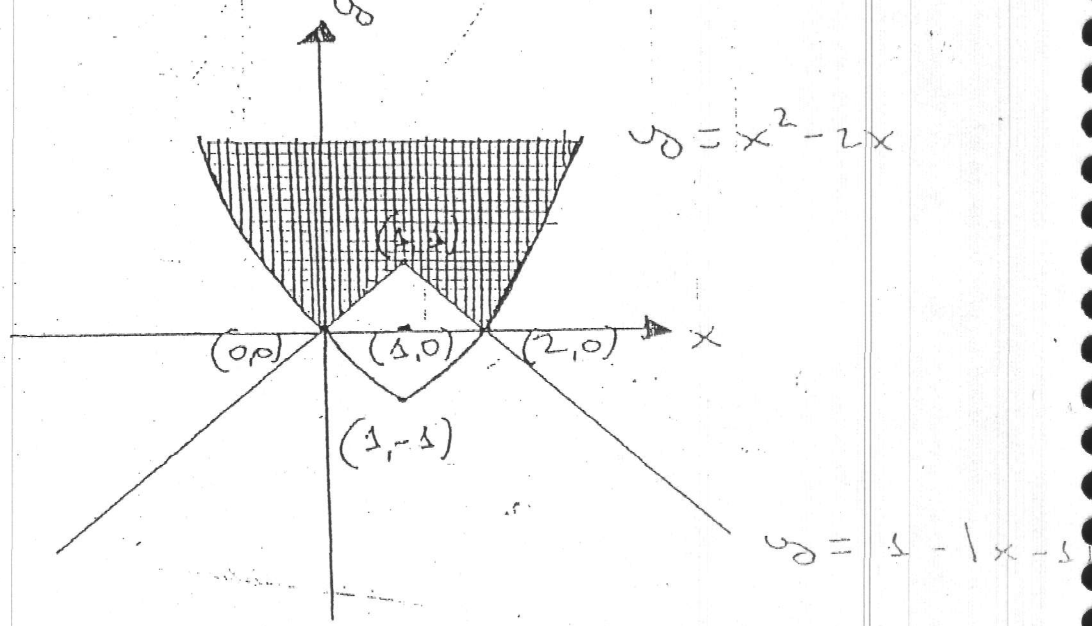


b)



(80)

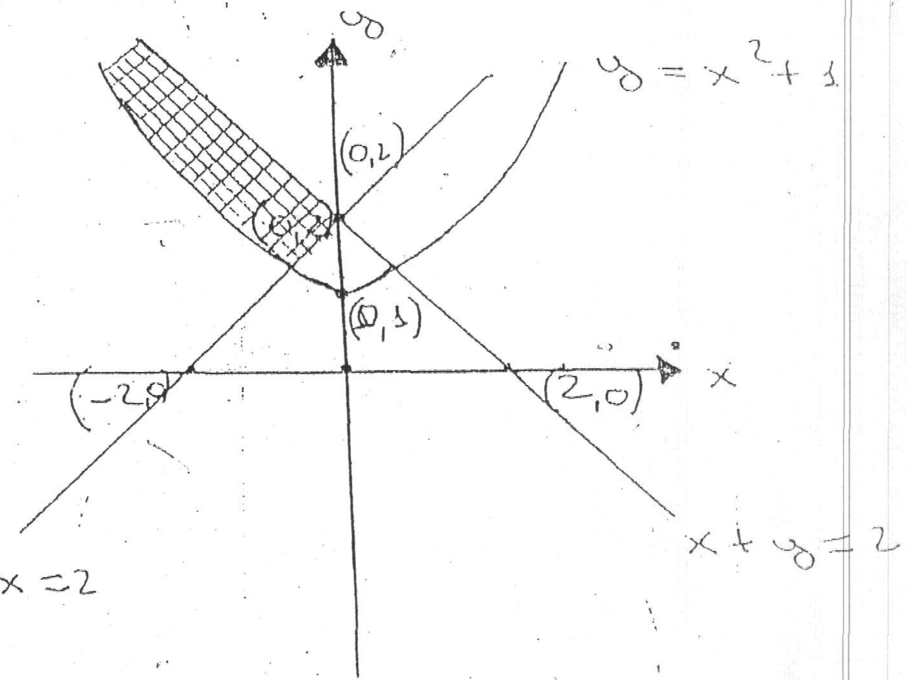
$$\begin{cases} y \geq x^2 - 2x \\ y > 1 - |x+1| \end{cases}$$



a) F b) F c) F d) F e) V

C es falso porque el punto (1, 1) no pertenece a la gráfica ya que $y > 1 - |x-1|$

(81)



c) Solo en el II cuadrante

40

Solución
EJERCICIOS PROPUESTOS
CAPÍTULO SEIS
NÚMEROS COMPLEJOS

(1) $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$

Por propiedades de los # complejos esto es verdadero

$z = 1 + i \qquad w = 2 - i$

$z \cdot w = 2 + i + 1 \qquad \boxed{\overline{z \cdot w} = 3 - i}$ *

$\overline{z} = 1 - i \qquad \overline{w} = 2 + i$

$\boxed{\overline{z} \cdot \overline{w} = 3 - i}$ **

Luego * y ** son iguales (a)

(2) $\left(\sqrt{1 - \sqrt{3}}\right)^2 = \sqrt{(1-\sqrt{3})^2}$

Si efectuamos $1 - \sqrt{3}$ nos da una cantidad negativa; no se puede sacar raiz cuadrada a esa cantidad negativa, mientras que en el otro lado se simplifican el exponente con el indice (b)

(3) $f(x) = x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

$f(i) = 1 - a_1(i) + a_2(-1) + a_3(i) + a_4 = 0$

$f(1+i) = -4 + a_1(-2+2i) + a_2(2i) + a_3(1+i) + a_4 = 0$

$f(1+i) = -4 + 2ia_1 - 2a_1 + 2ia_2 + a_3 + ia_3 + a_4 = 0$

$\therefore a_1 + a_2 + a_3 + a_4 = 4$ (d)

(4) $p(x) : \det\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} \cancel{p}\det\begin{vmatrix} 8x & 2 \\ -1 & x \end{vmatrix} - \det\begin{vmatrix} 4 & 1 \\ 2 & x \end{vmatrix} = 0$

$\det\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} = -2 \qquad \det = 8x^2 + 2 \qquad \det = 4x - 2$

$\cancel{-2} + 8x^2 \cancel{+2} - 4x + 2 = 0 \qquad 8x^2 - 4x + 2 = 0$

$4x^2 - 2x + 1 \qquad x_1 + x_2 = -\dfrac{-\cancel{2}^{1}}{\cancel{4}_{2}} \qquad \boxed{x_1 + x_2 = \frac{1}{2}}$ (a)

(5) $\frac{2i}{x} + iy - 3i = 3i - \frac{3}{x} + y$

$i\left(\frac{2}{x} + y - 3\right) = y - \frac{3}{x} + 2 = 0$

$y - \frac{3}{x} + 2 = 0$ $\boxed{y = \frac{3}{x} - 2}$ *

Reemplazo * en $\frac{2}{x} + y - 3 = 0$ $\boxed{x = 1}$

$\frac{2}{x} + \frac{3}{x} - 2 - 3 = 0$ $\frac{5}{x} = 5$

Reemplazo en *

$y = \frac{3}{x} - 2$

$y = \frac{3}{1} - 2$ $y = 3 - 2$ $\boxed{y = 1}$

S: $\{x = 1 \quad y = 1$

(6) i) $3 + 2ix + 3yi = 8i + x - 2y$

$2ix + 3iy - 8i = x - 2y - 3$

$i(2x + 3y - 8) = x - 2y - 3$

$\begin{cases} 2x + 3y = 8 \\ x - 2y = 3 \end{cases}$

$2x + 3y = 8$
$-2x + 4y = -6$

$// \; 7y = 14$ $\boxed{y = 2}$

$x - 2y = 3$
$x - 2(2) = 3$
$x = 3 + 4$ $\boxed{x = 7}$

ii) $(1 - i)x + (2 + i)y = 4 + 2i$

$x - ix + 2y + iy = 4 + 2i$

$x + 2y - 4 = ix - iy + 2i$

$x + 2y - 4 = i(x - y + 2)$

$\begin{cases} x + 2y = 4 \\ x - y = -2. \end{cases}$

$x + 2y = 4$
$-x + y = 2$

$// \; 3y = 6$
$y = \frac{6}{3}$ $\boxed{y = 2}$

$x - y = -2$
$x - 2 = -2$
$x = 2 - 2$ $\boxed{x = 0}$

iii) $\frac{8i}{x} + iy - 2 = 7i - \frac{10}{x} + y$

$\frac{8i}{x} + iy - 7i = y + 2 - \frac{10}{x}$

$i\left(\frac{8}{x} + y - 7\right) = y + 2 - \frac{10}{x}$

$\begin{cases} \frac{8}{x} + y = 7 \\ -\frac{10}{x} + y = -2 \end{cases}$ $\quad -\frac{8}{x} - y = -7$

$-\frac{10}{x} + y = -2$

$-\frac{18}{x} \quad // = -9$

$\frac{18}{x} = 9$

$9x = 18$

$x = \frac{18}{9}$ $\boxed{x = 2}$

$\frac{8}{x} + y = 7$

$\frac{8}{2} + y = 7$

$4 + y = 7 \qquad y = 7 - 4 \qquad \boxed{y = 3}$

*** Del ejercicio 3

Las raíces son: $i, -i, 1+i, 1-i$

$(x-i)(x+i)(x-1-i)(x-1+i) = x^4 - 2x^3 + 3x^2 - 2x + 2$

Por lo tanto: $a_1 + a_2 + a_3 + a_4 = 1$ ***

7) $A = \begin{pmatrix} 1 & 0 & 2i \\ 0 & 1-i & 1 \\ -i & 0 & 1 \end{pmatrix} \qquad \bar{A} = \begin{pmatrix} 1 & 0 & -2i \\ 0 & 1+i & 1 \\ i & 0 & 1 \end{pmatrix}$

$A - \bar{A} = \begin{pmatrix} 0 & 0 & 4i \\ 0 & -2i & 0 \\ -2i & 0 & 0 \end{pmatrix}$

$\det(A - \bar{A}) = 4i \begin{vmatrix} 0 & -2i \\ -2i & 0 \end{vmatrix}$

$4i(-4i^2) = -16i^3 \qquad -16(-i) \qquad \boxed{= +16i}$

8) $\frac{-4 + mi}{2 - 3i} = n - 2i$

$-4 + mi = (2-3i)(n-2i)$

$-4 + mi = 2n - 4i - 3in - 6$

$-4 + mi = (2n - 6) - i(4 + 3n)$

$-4 + mi = (2n-6) - i(4+3n)$

$-4 = 2n - 6 \qquad mi = -i(4+3n)$

$-4 + 6 = 2n \qquad m = -4 - 3n$

$2 = 2n \qquad m = -4 - 3(1)$

$\boxed{n = 1} \qquad \boxed{m = -7}$

(9) $z_1 = 1 + \sqrt{3}\,i \qquad z_2 = -1 + \sqrt{3}\,i$

$$\frac{z_2}{z_1} = \frac{-1+\sqrt{3}\,i}{1+\sqrt{3}\,i} \cdot \frac{1-\sqrt{3}\,i}{1-\sqrt{3}\,i} = \frac{2+2\sqrt{3}\,i}{4}$$

$$\frac{2+2\sqrt{3}\,i}{4} = \frac{\cancel{2}^{1}(1+\sqrt{3}\,i)}{\cancel{4}_{2}} = \frac{1+\sqrt{3}\,i}{2}$$

$$\left(\frac{z_2}{z_1}\right)^2 = \left(\frac{1+\sqrt{3}\,i}{2}\right)^2 = \frac{-2+2\sqrt{3}\,i}{4}$$

(C)

$$2\left(\frac{z_2}{z_1}\right)^2 \cancel{2}\left[\cancel{2}\left(\frac{-1+\sqrt{3}\,i}{\cancel{4}}\right)\right] \boxed{= -1+\sqrt{3}}$$

(10) $$z = \frac{4+Ki}{2+i}$$

$$z = \frac{4+Ki}{2+i} \cdot \frac{2-i}{2-i} = \frac{8+K-4i+2Ki}{5}$$

$$z = \frac{8+K+i(2K-4)}{5}$$

* Para que z sea real ~~es~~ puro necesario que
$2K-4 = 0 \qquad 2K = 4 \qquad K = \frac{\cancel{4}^{2}}{\cancel{2}_{1}} \qquad \boxed{K=2}$

* Para que z sea imaginario puro:
$8+K=0 \qquad \boxed{K=-8}$

(11) $x^3 - 5x^2 + 7x + 13 = 0$
Los divisores de 13 son: $\pm 1, \pm 13$
El polinomio se anula cuando $x = -1$

$$\frac{x^3-5x^2+7x+13}{x+1} = x^2 - 6x + 13$$

Ahora, hay que resolver $x^2-6x+13=0$

$a=1 \qquad b=-6 \qquad c=13$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \qquad x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(13)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-52}}{2} \qquad x=\frac{6\pm\sqrt{-16}}{2}$$

$$x=\frac{6\pm4i}{2} \qquad x=\cancel{2}\left(\frac{3\pm2i}{\cancel{2}}\right) \qquad \boxed{x=3\pm2i}$$

2)

$$\sqrt{a+ib}=u+iv$$

$$u=\sqrt{\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)} \qquad v=\sqrt{\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)}$$

$$\sqrt{a+ib}=u+iv$$

$$(\cancel{\sqrt{a+ib}})^{\cancel{2}}=(u+iv)^2$$

$$a+ib=u^2+2iuv+i^2v^2$$

$$* \quad a+ib=u^2+2iuv-v^2$$

Resolviendo $u^2+2iuv-v^2$

$$\left[\cancel{\sqrt{\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)}}\right]^{\cancel{2}}+2i\sqrt{\tfrac{1}{4}\left[(\cancel{\sqrt{a^2+b^2}})^{\cancel{2}}-a^2\right]}-\left[\cancel{\sqrt{\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)}}\right]^{\cancel{2}}$$

$$\tfrac{1}{2}\left(a+\sqrt{a^2+b^2}\right)+\cancel{2}i\left(\tfrac{1}{\cancel{2}}\right)\sqrt{\cancel{a^2}+b^2-\cancel{a^2}}-\tfrac{1}{2}\left(-a+\sqrt{a^2+b^2}\right)$$

$$\frac{a}{2}+\cancel{\frac{\sqrt{a^2+b^2}}{2}}+i\sqrt{b^2}+\frac{a}{2}-\cancel{\frac{\sqrt{a^2+b^2}}{2}}$$

$$\frac{a}{2}+bi+\frac{a}{2}=a+bi \quad *$$

Luego: $a+bi=a+bi$

13 a) $(1-4i)(3+11i)-(1+i)^{-1}$

$3+11i-12i+44 \quad - \frac{1}{1+i}$

$47-i-\frac{1}{1+i} \qquad = \frac{(1+i)(47-i)-1}{1+i}$

$\frac{47-i+47i+\cancel{1}-\cancel{1}}{1+i} = \frac{47+46i}{1+i}$

$\frac{47+46i}{1+i}\cdot\frac{1-i}{1-i} = \frac{47-47i+46i+46}{(1+i)(1-i)}$

$= \frac{93-i}{1+1} \qquad \boxed{=\frac{93-i}{2}}$

b) $(1-i)^3(1+i)$

$(1-i)(1-i)^2(1+i)$

$(1-i)^2(1-i^2)$

$-2i(2) \quad \boxed{=-4i}$

c) $\left[\frac{2i}{1+i}\right]^4$

$\frac{2i}{1+i}\cdot\frac{1-i}{1-i} = \frac{2+2i}{2} = 2\left(\frac{1+i}{2}\right) = 1+i$

$(1+i)^4 = 1+4i+6i^2+4i^3+i^4$

$= 1+\cancel{4i}-6-\cancel{4i}+1 \qquad \boxed{=-4}$

d) $\frac{i}{1+i}+\frac{1+i}{i}$

$= \frac{i^2+(1+i)^2}{i(1+i)} = \frac{-1+2i}{-1+i}$

$$\frac{-1+2i}{-1+i}\cdot\frac{-1-i}{-1-i}=\frac{1+i-2i+2}{1+1}\boxed{=\frac{3-i}{2}}$$

$$\left[1+i\sqrt{3}+(1-\sqrt{3})i\right]^3$$

$$\left[1+\cancel{\sqrt{3}i}+i-\cancel{\sqrt{3}i}\right]^3$$

$$(1+i)^3=1+3i+3i^2+i^3$$

$$=1+3i-3-i\qquad\boxed{=-2+2i}$$

$$\begin{array}{l} -(1-3i) \\ (2+3i) \end{array}\begin{cases}(2+3i)x-(1+i)y=3+4i & (1)\\ (1-3i)x-(1-2i)y=-2-6i & (2)\end{cases}$$

$$-(1-3i)(2+3i)x+(1+i)(1-3i)y=-(3+4i)(1-3i)$$

$$(1-3i)(2+3i)x-(1-2i)(2+3i)y=(-2-6i)(2+3i)$$

$$y\left[(1+i)(1-3i)-(1-2i)(2+3i)\right]=-15+5i+14-18i$$

$$y\left[4-2i-8+i\right]=-15+5i+14-18i$$

$$y(-4-1i)=-1-13i$$

$$y(4+1i)=1+13i\qquad y=\frac{1+13i}{4+1i}$$

$$y=\frac{1+13i}{4+1i}\cdot\frac{4-1i}{4-1i}=\frac{4-1i+52i-13i^2}{16+1}$$

$$y=\frac{17+51i}{17}\qquad y=\frac{\cancel{17}(1+3i)}{\cancel{17}}\qquad\boxed{y=1+3i}$$

Reemplazo en (1) ó (2)

En (1): $(2+3i)x-(1+i)y=3+4i$

$$(2+3i)x-(1+i)(1+3i)=3+4i$$

$$(2+3i)x-(-2+4i)=3+4i$$

(15) $A = \begin{pmatrix} 1+i & 2+i & 4+i \\ 0 & 1 & 2 \\ -i & 0 & i \end{pmatrix}$, hallar A^{-1}

1) $\det|A| = (1+i)\begin{vmatrix} 1 & 2 \\ 0 & i \end{vmatrix} - (2+i)\begin{vmatrix} 0 & 2 \\ -i & i \end{vmatrix} + (4+i)\begin{vmatrix} 0 & 1 \\ -i & 0 \end{vmatrix}$

$\det|A| = (1+i)(i) - (2+i)(2i) + (4+i)(i)$

$\det|A| = i + \cancel{i^2} - \cancel{4i} - \cancel{2i^2} + \cancel{4i} + \cancel{i^2}$

$\det|A| = i$

2) Menores

$\begin{pmatrix} i & 2i & i \\ -1+2i & -2+5i & -1+2i \\ i & 2+2i & 1+i \end{pmatrix}$

3) Cofactores

$\begin{pmatrix} i & -2i & i \\ 1-2i & -2+5i & 1-2i \\ i & -2-2i & 1+i \end{pmatrix}$

4) Transpuesta

$\begin{pmatrix} i & 1-2i & i \\ -2i & -2+5i & -2-2i \\ i & 1-2i & 1+i \end{pmatrix}$

5) Fórmula

$A^{-1} = \dfrac{1}{\det A}(A^T)$

5) $\dfrac{1}{i}\begin{pmatrix} i & 1-2i & i \\ -2i & -2+5i & -2-2i \\ i & 1-2i & 1+i \end{pmatrix}$

$A^{-1} = \begin{pmatrix} 1 & -2-i & 1 \\ -2 & 5+2i & -2+2i \\ 1 & -2-i & 1-i \end{pmatrix}$

(16) a) $\begin{vmatrix} 1+i & 2+i & 4+i \\ 0 & 1 & 2 \\ -i & 0 & i \end{vmatrix}$

$\det = (1+i)\begin{vmatrix} 1 & 2 \\ 0 & i \end{vmatrix} - (2+i)\begin{vmatrix} 0 & 2 \\ -i & i \end{vmatrix} + (4+i)\begin{vmatrix} 0 & 1 \\ -i & 0 \end{vmatrix}$

$\det = (1+i)(i) - (2+i)(2i) + (4+i)(i)$

$\det = i + \cancel{i^2} - \cancel{4i} - \cancel{2i^2} + \cancel{4i} + \cancel{i^2}$

$\boxed{\det = i}$

$$(2+3i)x = 3+4i-2+4i$$

$$(2+3i)x = 1+8i$$

$$x = \frac{1+8i}{2+3i}$$

$$x = \frac{1+8i}{2+3i} \cdot \frac{2-3i}{2-3i} = \frac{2-3i+16i+24}{4+9}$$

$$x = \frac{26+13i}{13} \qquad \frac{\cancel{13}(2+i)}{\cancel{13}} \qquad \boxed{x = 2+i}$$

b) $\begin{cases} (2+i)x + 2y = 1+7i & (1) \\ (1-i) + yi = 0 & (2) \end{cases}$

De la ecuación (2)

$$1-i+yi = 0$$

$$yi = -1+i \qquad y = \frac{-1+i}{i}$$

$$\frac{-1+i}{i} \cdot \frac{i}{i} = \frac{i(-1+i)}{i^2}$$

$$-1(-i+i^2) = i-i^2 = 1+i \qquad \boxed{y = 1+i}$$

Reemplazo en (1)

$$(2+i)x + 2y = 1+7i$$

$$(2+i)x + 2(1+i) = 1+7i$$

$$(2+i)x + 2+2i = 1+7i$$

$$(2+i)x = 1+7i-2-2i$$

$$(2+i)x = -1+5i \qquad x = \frac{-1+5i}{2+i}$$

$$x = \frac{-1+5i}{2+i} \cdot \frac{2-i}{2-i}$$

$$x = \frac{-2+i+10i+5}{4+1} \qquad \boxed{x = \frac{3+11i}{5}}$$

b) $\begin{vmatrix} 1-i & 0 & i \\ 2-i & 1 & 0 \\ 4-i & 2 & -i \end{vmatrix}$

$$det = (1-i)\begin{vmatrix} 1 & 0 \\ 2 & -i \end{vmatrix} + i\begin{vmatrix} 2-i & 1 \\ 4-i & 2 \end{vmatrix}$$

$$det = (1-i)(-i) + i(4-2i-4+i)$$

$$det = -i + i^2 + i(-i)$$

$$det = -i + \cancel{i^2} - \cancel{i^2} \qquad \boxed{det = -i}$$

17 a) $\dfrac{(3+5i)(2-i)^3}{-1+4i}$

$$\frac{(3+5i)(2-11i)}{-1+4i} = \frac{61-23i}{-1+4i}$$

$$\frac{61-23i}{-1+4i} \cdot \frac{-1-4i}{-1-4i} = \frac{-153-221}{17}$$

$$\cancel{17}\left(\frac{-9-13i}{\cancel{17}}\right) = -9-13i$$

b) $(2+3i)(3-4i)^2$

$$(2+3i)(-7-24i)$$

$$= -14-48i-21i+72 \qquad \boxed{= 58-69i}$$

c) $\frac{1}{2}(1+i)(1+i^{-8})$

$$\frac{1}{2}(1+i)\left(1+\frac{1}{i^8}\right) = \frac{1}{2}(1+i)(1+1)$$

$$= \frac{1}{\cancel{2}}(1+i)(\cancel{2}) \qquad \boxed{= 1+i}$$

d) $\dfrac{(1+i)(2+i)(3+i)}{1-i}$

$\dfrac{(1+i)^2(2+i)(3+i)}{(1+i)(1-i)} = \dfrac{2i(2+i)(3+i)}{1+1}$

$= \dfrac{\cancel{2}i(6+2i+3i-1)}{\cancel{2}} = i(5+5i)$

$= 5i+5i^2$ $\boxed{= -5+5i}$

e) $(2+5i)^3$

$= 8+3(2)^2(5i)+3(2)(5i)^2+125i^3$

$= 8+60i+150i^2+125i^3$

$= 8+60i-150-125i$

$\boxed{= -142-65i}$

18) $\left(\dfrac{z_1}{z_1+z_2}\right) + \left(\dfrac{z_2}{z_1+z_2}\right) = 1$

$\dfrac{z_1+z_2}{z_1+z_2} = 1$

19) $\sqrt{z} = \dfrac{2}{1-i} + 1-4i$

$\sqrt{z} = \dfrac{2+(1-i)(1-4i)}{1-i}$

$\sqrt{z} = \dfrac{2-3-5i}{1-i}$ $\sqrt{z} = \dfrac{-1-5i}{1-i}$

$\sqrt{z} = \dfrac{-1-5i}{1-i} \cdot \dfrac{1+i}{1+i}$ $\sqrt{z} = \dfrac{4-6i}{2}$,

$\sqrt{z} = \dfrac{\cancel{2}(2-3i)}{\cancel{2}}$ $\sqrt{z} = 2-3i$

$(\cancel{\sqrt{z}})^{\cancel{2}} = (2-3i)^2$ $z = 4 - 12i + 9i^2$

$z = 4 - 12i - 9$ $\boxed{z = -5 - 12i}$

(20) $(x+iy)(2+3i)$

$2x + 3ix + 2iy + 3i^2y$

$2x + i(3x+2y) - 3y = 0$

Para que esto sea un número real:

$3x + 2y = 0$

$3x = -2y$ $\boxed{\dfrac{x}{y} = -\dfrac{2}{3}}$

(21) a) $-\sqrt{3} + i = z$

Hay que hallar: módulo y argumento

módulo $= \sqrt{a^2+b^2}$ $|z|$

argumento $=$ $\Theta = \text{Tan}^{-1}\left(\dfrac{b}{a}\right)$

siendo $z = a + bi$

a: parte real b: parte imaginaria

$|z| = \sqrt{(-\cancel{\sqrt{3}})^{\cancel{2}} + (1)^2} = \sqrt{3+1} = \sqrt{4} = 2$

$\Theta = \text{Tan}^{-1}\left(\dfrac{1}{-\sqrt{3}}\right)$ $\Theta = \text{Tan}^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$

Θ está en el II cuadrante ya que: $x(-)$ $y(+)$

$\Theta = \text{Tan}^{-1}\left(-\dfrac{\sqrt{3}}{3}\right)$ $\Theta = \dfrac{5\pi}{6}$

Forma Polar: $|z|\,\text{Cis}\,\Theta$ $\boxed{= 2\,\text{Cis}\,\dfrac{5\pi}{6}}$

b) $z = 3 - 3i$

$|z| = \sqrt{(3)^2 + (-3)^2}$

$|z| = \sqrt{9+9}$ $\qquad |z| = \sqrt{18}$

$|z| = 3\sqrt{2}$

$\Theta = \operatorname{Tan}^{-1}\left(\frac{-3}{3}\right)$ $\qquad \Theta = \operatorname{Tan}^{-1}(-1)$ $\qquad x = +$, $y = -$

$\Theta = 7\frac{\pi}{4}$ $\qquad \boxed{3\sqrt{2}\ Cis\ \frac{7\pi}{4}}$

c) $z = 1 + \sqrt{3}\, i$

$|z| = \sqrt{(1)^2 + (\sqrt{3})^2}$

$|z| = \sqrt{1+3}$

$|z| = \sqrt{4}$

$|z| = 2$

$\Theta = \operatorname{Tan}^{-1}\left(\frac{\sqrt{3}}{1}\right)$ $\qquad \Theta = \operatorname{Tan}^{-1}(\sqrt{3})$

$\Theta = \frac{\pi}{3}$ $\qquad \boxed{= 2\ Cis\ \frac{\pi}{3}}$

Θ cae en el I cuadrante ya que:
$x\,(+)$, $y\,(+)$

(22) $\dfrac{1+\cos\alpha+i\,\text{Sen}\,\beta}{1+\cos\beta+i\,\text{Sen}\,\beta}$

$$\frac{1+\cos\alpha+i\,\text{Sen}\,\beta}{1+\cos\beta+i\,\text{Sen}\,\beta}\cdot\frac{1+\cos\beta-i\,\text{Sen}\,\beta}{1+\cos\beta-i\,\text{Sen}\,\beta}$$

Resolviendo y reduciendo

$$\frac{1+\cos\beta+\cos\alpha(1+\cos\beta)+\text{Sen}^2\beta+i\,\text{Sen}\,\beta(\cos\beta-\cos\alpha)}{2(1+\cos\beta)}$$

Parte real:

$$\frac{1+\cos\beta+\cos\alpha+\cos\alpha\cos\beta+\text{Sen}^2\beta}{2(1+\cos\beta)}$$

Parte imaginaria:

$$\frac{i\,\text{Sen}\,\beta(\cos\beta-\cos\alpha)}{2(1+\cos\beta)}$$

(23) $\left[\dfrac{1+\text{Sen}\,\alpha+i\cos\alpha}{1+\text{Sen}\,\alpha-i\cos\alpha}\right]^n$

$$\frac{(1+\text{Sen}\,\alpha+i\cos\alpha)(1+\text{Sen}\,\alpha+i\cos\alpha)}{(1+\text{Sen}\,\alpha-i\cos\alpha)(1+\text{Sen}\,\alpha+i\cos\alpha)}$$

$$=\frac{1+\text{Sen}^2\alpha-\cos^2\alpha+2\,\text{Sen}\,\alpha+2i\cos\alpha(1+\text{Sen}\,\alpha)}{2(1+\text{Sen}\,\alpha)}$$

$$=\frac{2\,\text{Sen}\,\alpha(\text{Sen}\,\alpha+1)+2i\cos\alpha(\text{Sen}\,\alpha+1)}{2(1+\text{Sen}\,\alpha)}$$

$$=\frac{\cancel{2(\text{Sen}\,\alpha+1)}(\text{Sen}\,\alpha+i\cos\alpha)}{\cancel{2(\text{Sen}\,\alpha+1)}}$$

$$=(\text{Sen}\,\alpha+i\cos\alpha)^n \;*$$

$$\cos\left[n\left(\frac{\pi}{2}-\alpha\right)\right] + i\,\text{Sen}\left[n\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$\left[\cos\left(\frac{\pi}{2}-\alpha\right) + i\,\text{Sen}\left(\frac{\pi}{2}-\alpha\right)\right]^{n}$$

$$\left[\left(\cancel{\cos\frac{\pi}{2}}\overset{0}{\cancel{\cos\alpha}} + \cancel{\text{Sen}}\overset{1}{\frac{\pi}{2}}\,\text{Sen}\,\alpha\right) + i\left(\cancel{\text{Sen}}\overset{1}{\frac{\pi}{2}}\cos\alpha - \text{Sen}\,\alpha\,\overset{0}{\cancel{\cos\frac{\pi}{2}}}\right)\right]^{n}$$

$$\left(\text{Sen}\,\alpha + i\cos\alpha\right)^{n} \;*\qquad \text{L.Q.Q.D.}$$

4) a) $i^{25} = i$

$i^{25} = i^{21} = i^{17} = i^{13} = i^{9} = i^{5} = i$ (V)

b) $|z_1 + z_2| \leq |z_1| + |z_2|$

Es propiedad de los números complejos (V)

c) $(x, y)(0, 1) = (x, y)$

$(x, y)(0, 1) = (0, y)$ (F)

d) $z \cdot z = z^2$

Es propiedad de los números complejos (V)

(25) $\left(\frac{1+\sqrt{3}i}{-1+\sqrt{3}i}\right)^{10}$

$$\frac{1+\sqrt{3}i}{-1+\sqrt{3}i}\cdot\frac{-1-\sqrt{3}i}{-1-\sqrt{3}i} = \frac{-1-\sqrt{3}i-\sqrt{3}i-3i^2}{1-3i^2}$$

$$= \frac{2-2\sqrt{3}i}{1+3} = \frac{\cancel{2}(1-\sqrt{3}i)}{\cancel{4}_2} = \frac{1-\sqrt{3}i}{2}$$

$$z=\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^{10}$$

$$|z| = \sqrt{\left(\frac{1}{2}\right)^2+\left(-\frac{\sqrt{3}}{2}\right)^2} \qquad |z| = \sqrt{\frac{1}{4}+\frac{3}{4}}$$

$$|z| = \sqrt{1} \qquad |z| = 1 \ *$$

$$\Theta = \text{Tan}^{-1}\left(\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) \qquad \Theta = \text{Tan}^{-1}(-\sqrt{3})$$

Θ está en el IV cuadrante ya que: $x(+)$, $y(-)$

$$\Theta = \frac{5\pi}{3} \ **$$

$$z^N = |z|^N e^{i\Theta n}$$

$$= (1)^{10}\left(\text{Cos}\,\frac{5\pi}{3} + i\,\text{Sen}\,\frac{5\pi}{3}\right)^{10}$$

$$= 1\,(\text{Cos}\,3000^\circ + i\,\text{Sen}\,3000^\circ)$$

$$= \text{Cos}\,120^\circ + i\,\text{Sen}\,120^\circ$$

$$\boxed{= -\frac{1}{2}+\frac{\sqrt{3}}{2}i} \quad (a)$$

(26) $\left(z + z^{-1}\right)^5$

$= \left(z + \frac{1}{z}\right)^5 \qquad = \left(\frac{z^2 + 1}{z}\right)^5$

$= \frac{z^{10} + 5z^8 + 10z^6 + 10z^4 + 5z^2 + 1}{z^5}$ *

$\cos 5\theta = \cos(3\theta + 2\theta)$

$= \cos 3\theta \cos 2\theta - \text{Sen } 3\theta \text{ Sen } 2\theta$

$= (4\cos^3\theta - 3\cos\theta)(2\cos^2\theta - 1) - 2\,\text{Sen}\theta\cos\theta(3\,\text{Sen}\theta - 4\,\text{Se}$

$= 8\cos^5\theta - 4\cos^3\theta - 6\cos^3\theta + 3\cos\theta - 2\,\text{Sen}^2\theta\cos\theta(3 - 4\,\text{Sen}^2\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(1 - \cos^2\theta)(3 - 4 + 4\cos$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(1 - \cos^2\theta)(4\cos^2\theta$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 2\cos\theta(5\cos^2\theta - 4\cos^4\theta -$

$= 8\cos^5\theta - 10\cos^3\theta + 3\cos\theta - 10\cos^3\theta + 8\cos^5\theta + 2\cos\theta$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

* $\cos^3\theta = (3\cos\theta + \cos 3\theta)/4$

$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

Reemplazando *

$\cos 5\theta = 16\cos^5\theta - 20\left(\frac{3\cos\theta + \cos 3\theta}{4}\right) + 5\cos\theta$

$\cos 5\theta = 16\cos^5\theta - 5(3\cos\theta + \cos 3\theta) + 5\cos\theta$

$\cos 5\theta = 16\cos^5\theta - 15\cos\theta - 5\cos 3\theta + 5\cos\theta$

$\cos 5\theta = 16\cos^5\theta - 5\cos 3\theta - 10\cos\theta$

$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = 16\cos^5\theta$

$$\boxed{\cos^5\theta = \frac{\cos 5\theta + 5\cos 3\theta + 10\cos\theta}{16}}$$

Lo que se quería demostrar.

$n = 1 \qquad m = 5 \qquad \lambda = 10$

(27) a) $\left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\,\mathrm{Sen}\,\frac{\pi}{4}\right)\right]^4$

$= (\sqrt{2})^4\left[\cos \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + i\,\mathrm{Sen}\,\cancel{4}\left(\frac{\pi}{\cancel{4}}\right)\right]$

$= (2)^2(\cos\pi + i\,\mathrm{Sen}\,\pi)$

$= 4\,\mathrm{Cis}\,\pi$

b) $\left[3\left(\cos\frac{\pi}{5} + i\,\mathrm{Sen}\,\frac{\pi}{5}\right)\right]^{-3}$

$= (3)^{-3}\left[\cos(-3)\frac{\pi}{5} + i\,\mathrm{Sen}\,(-3)\frac{\pi}{5}\right]$

$= \frac{1}{27}\left[\cos\left(-\frac{3\pi}{5}\right) + i\,\mathrm{Sen}\left(-\frac{3\pi}{5}\right)\right]$

$= \frac{1}{27}\,\mathrm{Cis}\left(-\frac{3\pi}{5}\right)$

(28) $z = 2 + 2i$

$zi = i(2+2i)$

$zi = 2i + 2i^2$

$zi = -2 + 2i$

Cambia el signo de la parte real

(29) $z - \frac{1}{z} = \frac{z^2 - 1}{z}$

En el caso de que z sea un número complejo, haciendo operaciones al final seguirá existiendo parte imaginaria. Por lo tanto, z debe ser real $z \neq 0$

(30) i) $\left[1 + \sqrt{3}i + (1 - \sqrt{3})i\right]^3$

$= \left[1 + \cancel{\sqrt{3}i} + i - \cancel{\sqrt{3}i}\right]^3$

$= (1+i)^3$

$= 1 + 3i + 3i^2 + i^3$

$= 1 + 3i - 3 - i$ $\boxed{= -2 + 2i}$

ii) $\left(\frac{1+i}{1-i}\right)^2 - \left(\frac{1-i}{1+i}\right)^3$

$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2i}{1+1} = \frac{\cancel{2}i}{\cancel{2}} = i$

$\frac{1-i}{1+i} \cdot \frac{1-i}{1-i} = \frac{-2i}{1+1} = -\frac{\cancel{2}i}{\cancel{2}} = -i$

$= (i)^2 - (-i)^3$

$= i^2 - (i^3)$ $= -1 - (+i)$

$\boxed{= -1 + i}$

31 a) $e^{-i\pi}$

$\cos\pi + i\,\mathrm{Sen}\,\pi = -1$

Parte real $= -1$ Módulo $= 1$

Parte imaginaria $= 0$

b) $e^{1-i\frac{\pi}{6}}$

$= e \cdot e^{-i\frac{\pi}{6}}$

$= e\left[\cos\frac{11\pi}{6} + i\,\mathrm{Sen}\,\frac{11\pi}{6}\right]$

$= e\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) = \frac{\sqrt{3}e}{2} - \frac{e}{2}i$

Módulo $= e$ Parte real $= \frac{\sqrt{3}e}{2}$

Parte imaginaria $= -\frac{e}{2}$

c) e^{1+i}

$= e \cdot e^{i}$

$= e\left[\cos 1 + i\,\mathrm{Sen}\,1\right]$

Módulo $= e$ Parte real $= e\cos 1$

Parte imaginaria $= e\,\mathrm{Sen}\,1$

32 $z_1 = 7 - i$ $z_2 = 3 + i$

$\frac{z_1}{z_2} = \frac{7-i}{3+i}$ $\frac{7-i}{3+i} \cdot \frac{3-i}{3-i} = \frac{21 - 7i - 3i + i^2}{3^2 + 1^2}$

$= \frac{21 - 10i - 1}{10} = \frac{20 - 10i}{10} = \frac{\cancel{10}(2-i)}{\cancel{10}} = 2 - i$

$e^{2-i} = e^2 \cdot e^{-i}$ Módulo $= e^2$

(33) $z = \frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4} + \dots$

$A = \frac{1}{1+i}$ $\qquad R = \frac{1}{(1+i)^2} \cdot \cancel{\frac{1+i}{1}} = \frac{1}{1+i}$

$S = \frac{A}{1-R}$ $\qquad S = \frac{\frac{1}{1+i}}{1 - \frac{1}{1+i}} = \frac{\frac{1}{1+i}}{\frac{\cancel{1}+i-\cancel{1}}{1+i}}$

$S = \frac{1}{\cancel{1+i}} \cdot \frac{\cancel{1+i}}{i}$ $\qquad S = \frac{1}{i}$ $\qquad S = i^{-1}$

$z^2 = (i^{-1})^2$ $\qquad z^2 = (i^2)^{-1}$ $\qquad \boxed{z^2 = -1}$

(34) a) $e^{i\pi}$

$= \cos\pi + i \cancel{\operatorname{sen}\pi}^{\,0}$ $\qquad \boxed{= -1}$

b) $-2e^{-i\pi}$

$= -2[\cos\pi + i \operatorname{sen}\pi]$

$= -2(-1)$ $\qquad \boxed{= 2}$

c) $i + e^{i\frac{3\pi}{2}}$

$i + \left(\cancel{\cos}\frac{3\pi}{2} + i \operatorname{Sen}\frac{3\pi}{2}\right)$

$= i + i(-1)$ $\quad i - i \quad \boxed{=0}$

d) $e^{i\frac{\pi}{4}} - e^{-i\frac{\pi}{4}}$

$\left(\cos\frac{\pi}{4} + i\operatorname{Sen}\frac{\pi}{4}\right) - \left(\cos\frac{7\pi}{8} + i\operatorname{Sen}\frac{7\pi}{8}\right)$

$\cancel{\frac{\sqrt{2}}{2}} + \frac{\sqrt{2}}{2}i - \cancel{\frac{\sqrt{2}}{2}} - \left(-\frac{\sqrt{2}}{2}i\right)$

$\frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}i \quad = \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right)i \quad \boxed{=\sqrt{2}\,i}$

e) $e^{i\frac{\pi}{3}} + e^{i\frac{5\pi}{6}}$

$\left(\cos\frac{\pi}{3} + i\operatorname{Sen}\frac{\pi}{3}\right) + \left(\cos\frac{7\pi}{6} + i\operatorname{Sen}\frac{7\pi}{6}\right)$

$\frac{1}{2} + \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$\frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2}i$

$$\boxed{= \frac{1}{2}\left(1 - \sqrt{3}\right) + i\,\frac{1}{2}\left(\sqrt{3} - 1\right)}$$

3.5 a) $2e^{\frac{\pi}{8}}$

$= 2\left(\cos\frac{\pi}{8} + i\operatorname{Sen}\frac{\pi}{8}\right) \quad \boxed{= 2\operatorname{Cis}\frac{\pi}{8}}$

b) e^{2+i}

$= e^2 \cdot e^i \quad = e^2\left(\cos 1 + i\operatorname{Sen} 1\right) \quad \boxed{= e^2 \operatorname{Cis} 1}$

c) $2e^{1+\frac{\pi}{4}i}$

$= 2e \cdot e^{i\frac{\pi}{4}} \quad = 2e\left(\cos\frac{\pi}{4} + i\operatorname{Sen}\frac{\pi}{4}\right) \quad \boxed{= 2e\operatorname{Cis}\frac{\pi}{4}}$

d) 2^{3i}

$= 8^{i} \quad = e^{i \ln 8} \quad \boxed{= Cis(\ln 8)}$

e) $3^{-1/2}$

$= \left(\frac{1}{3}\right)^{1/2} \quad = \sqrt{\frac{1}{3}} \quad \boxed{= \frac{\sqrt{3}}{3}}$

f) 5^{1+i}

$= 5 \cdot 5^{i} \quad = 5 \cdot e^{i \ln 5} \quad \boxed{= 5(Cis(\ln 5))}$

g) 10^{1-i}

$= 10 \cdot 10^{-i} \quad = \frac{10}{10^{i}}$

$= \frac{10}{e^{i \ln 10}} \quad \boxed{= 10/[Cos(\ln 10) - i\, Sen(\ln 10)]}$

36) a) h) $((Cos\ 30^\circ + i\, Sen\ 30^\circ))^2 = Cos\ \frac{\pi}{3} + i\, Sen\ \frac{\pi}{3}$

$((Cos\ 30^\circ + i\, Sen\ 30^\circ)^2 = Cos\ 2(30^\circ) + i\, Sen\ 2(30^\circ)$

$= Cos\ 60^\circ + i\, Sen\ 60^\circ$

$Cos\ 60^\circ + i\, Sen\ 60^\circ = Cos\ \frac{\pi}{3} + i\, Sen\ \frac{\pi}{3}$

b) i) $((Cos\ \frac{\pi}{6} + i\, Sen\ \frac{\pi}{6})^3 = i$

$= Cos\ 3\left(\frac{\pi}{6}\right) + i\, Sen\ 3\left(\frac{\pi}{6}\right)$

$= Cos\ \frac{\pi}{2} + i\, Sen\ \frac{\pi}{2}$

$= 0 + i(1) = i \qquad i = i$

(37) i) $2\ Cis\ \frac{\pi}{4}$

$= \left[2\left(Cos\ \frac{\pi}{4} + i\ Sen\ \frac{\pi}{4}\right)\right]^4$

$= 2^4\left(Cos\ \cancel{4}\left(\frac{\pi}{\cancel{4}}\right) + i\ Sen\ \cancel{4}\left(\frac{\pi}{\cancel{4}}\right)\right)$

$= 16\left(Cos\ \pi + i\ Sen\ \pi\right)$

$= 16(-1 + 0i)$

$= 16(-1)$ $\boxed{= -16}$

ii) $\sqrt{3}\ Cis\ 225^\circ$

$= \left[\sqrt{3}\left(Cos\ 225^\circ + i\ Sen\ 225^\circ\right)\right]^4$

$= \left(\sqrt{3}\right)^4\left(Cos\ 900^\circ + i\ Sen\ 900^\circ\right)$

$= \sqrt{81}\left(Cos\ 180^\circ + i\ Sen\ 180^\circ\right)$

$= 9(-1 + 0i)$

$= 9(-1)$ $\boxed{= -9}$

iii) $\frac{1}{2}\ e^{i\alpha}$

$= \left[\frac{1}{2}\left(Cos\ \alpha + i\ Sen\ \alpha\right)\right]^4$

$= \left(\frac{1}{2}\right)^4\left(Cos\ 4\alpha + i\ Sen\ 4\alpha\right)$

$\boxed{= \frac{1}{16}\left(Cos\ 4\alpha + i\ Sen\ 4\alpha\right)}$

(38) i) $27\ Cis\ \frac{3\pi}{2}$

$= \left[27\left(Cos\ \frac{3\pi}{2} + i\ Sen\ \frac{3\pi}{2}\right)\right]^{1/3}$

$= (27)^{1/3}\left[Cos\ \cancel{\tfrac{1}{3}}\left(\frac{\cancel{3}\pi}{2}\right) + i\ Sen\ \cancel{\tfrac{1}{3}}\left(\frac{\cancel{3}\pi}{2}\right)\right]$

$= 3(0 + i)$ $\boxed{= 3i}$

$|z| = 27$ $\quad \Theta = \frac{3\pi}{2}$ $\quad \frac{2\pi}{3} = 120^\circ$

** $z^{1/3} = (27)^{1/3} e^{i\theta/3}$

$= 3\left(\cos \frac{7\pi}{6} + i \operatorname{Sen} \frac{7\pi}{6}\right)$

$= 3\left(-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$$\boxed{z_2 = -\frac{3\sqrt{3}}{2} - \frac{3}{2} i}$$

** $z^{1/3} = 3\left(\cos \frac{11\pi}{6} + i \operatorname{Sen} \frac{11\pi}{6}\right)$

$= 3\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$

$$\boxed{z_3 = \frac{3\sqrt{3}}{2} - \frac{3}{2} i}$$

ix) $z = -1 + 0i$

* $|z| = 1$ $\quad \Theta = \operatorname{Tan}^{-1}\left(\frac{0}{-1}\right)$ $\quad \Theta = \pi$ *

$\frac{2\pi}{4} = \frac{\pi}{2}$ *

** $z^{1/4} = (1)^{1/4} e^{i\theta/4}$

$= \cos \frac{\pi}{4} + i \operatorname{Sen} \frac{\pi}{4}$

$$\boxed{z_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i}$$

** $z^{1/4} = \cos \frac{3\pi}{4} + i \operatorname{Sen} \frac{3\pi}{4}$

$$\boxed{z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i}$$

** $z^{1/4} = \cos \frac{5\pi}{4} + i \operatorname{Sen} \frac{5\pi}{4}$

$$\boxed{z_3 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}$$

$$** \quad z^{1/4} = \cos\frac{7\pi}{4} + i\,\mathrm{Sen}\,\frac{7\pi}{4}$$

$$\boxed{z_4 = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} i}$$

iii) $z = i$

$|z| = 1 \qquad \Theta = \mathrm{Tan}^{-1}\left(\frac{1}{0}\right) \qquad \Theta = \frac{\pi}{2}$

$z^{1/N} = |z|^{1/N} e^{i(\Theta + 2\pi K)/N} \qquad K = 0$

$= 1\left(\cos\frac{\pi}{8} + i\,\mathrm{Sen}\,\frac{\pi}{8}\right)$

$$\boxed{z_1 = \cos\frac{\pi}{8} + i\,\mathrm{Sen}\,\frac{\pi}{8}}$$

$\frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\boxed{z_2 = \cos\frac{5\pi}{8} + i\,\mathrm{Sen}\,\frac{5\pi}{8}}$$

$$\boxed{z_3 = \cos\frac{9\pi}{8} + i\,\mathrm{Sen}\,\frac{9\pi}{8}}$$

$$\boxed{z_4 = \cos\frac{13\pi}{8} + i\,\mathrm{Sen}\,\frac{13\pi}{8}}$$

iv) $z = 64\cos\pi$

$z = 64(-1) \qquad z = -64$

$|z| = 64 \quad \Theta = \mathrm{Tan}^{-1}\left(\frac{0}{-64}\right) \qquad \Theta = \pi$

$** \quad z^{1/3} = (64)^{1/3} e^{i\Theta/3}$

$= 4\left(\cos\frac{\pi}{3} + i\,\mathrm{Sen}\,\frac{\pi}{3}\right)$

$= 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) \qquad \boxed{z_1 = 2 + 2\sqrt{3}i}$

$\frac{2\pi}{1} = \frac{2\pi}{3}$

$^{*} z^{1/3} = 4((\cos\pi + i\,\text{Sen}\,\pi)$

$= 4(-1 + 0i)$

$\boxed{z_2 = -4}$

$** \quad z_3^{1/3} = 4((\cos\frac{5\pi}{3} + i\,\text{Sen}\,\frac{5\pi}{3})$

$= 4(\frac{1}{2} + i(-\frac{\sqrt{3}}{2}))$

$\boxed{z_3 = 2 - 2\sqrt{3}\,i}$

i) $z = 27\,\text{Cis}\,\frac{3\pi}{2}$

$z = 27(\cos\frac{3\pi}{2} + i\,\text{Sen}\,\frac{3\pi}{2})$

$z = 27(0 - i)$

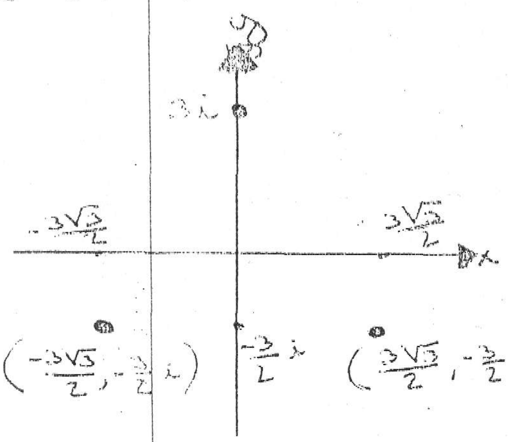
$z = -27i$

$|z| = 27 \qquad \Theta = \text{Tan}^{-1}(-\frac{27}{0}) \qquad \Theta = \frac{3\pi}{2}$

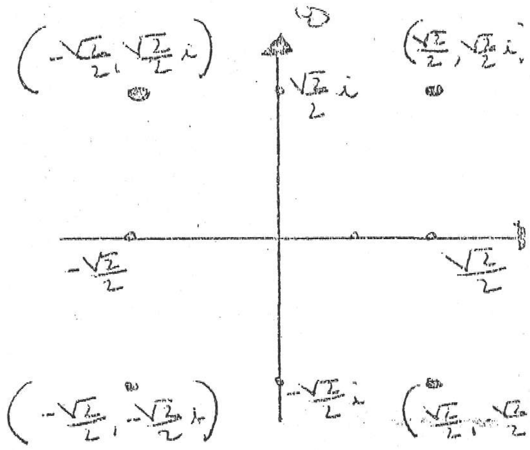
Las soluciones de i) están correctas.

Gráficos

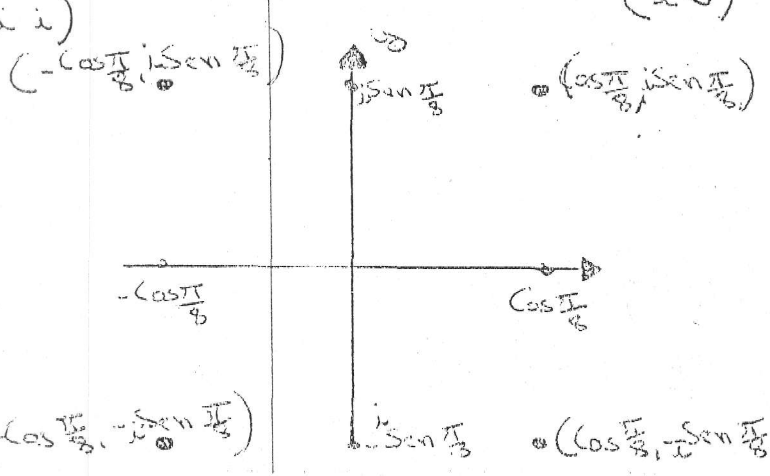
i)



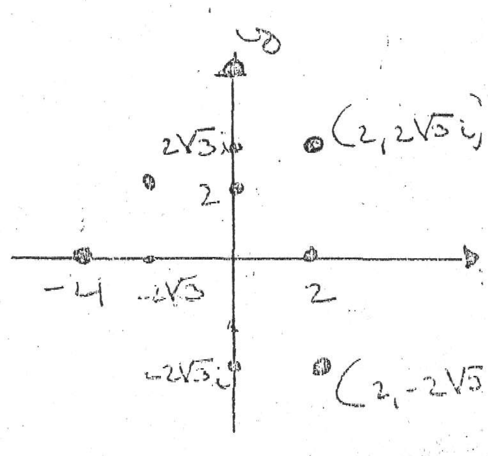
ii)



iii)



(iv)



(39) Como uno de los vértices del hexágono regular es $(0, 2i)$, entonces:

$$(2i)^6 = 2^6 i^6 = 64(-1) = -64$$

Por lo tanto $z = -64 + 0i$

Hay que hallar $z^{1/6} = (-64 + 0i)^{1/6}$

$|z| = \sqrt{(-64)^2 + (0)^2}$ $\quad |z| = \sqrt{4096}$ $\quad * |z| = 64$

$\theta = \tan^{-1}\left(\frac{0}{-64}\right)$ $\quad ** \theta = \pi$ ya que $x = -$ $y = 0$

Módulo $= 64$ Argumento $= \pi$

$$z^{1/N} = |z|^{1/N} e^{i(\theta + 2\pi K)/N} \quad K = 0, 1, 2, 3, 4, \ldots$$

$$* z^{1/6} = (64)^{1/6} e^{i\theta/N} \quad \text{con } K = 0$$

$$= 2\left(\cos\frac{\pi}{6} + i \operatorname{sen}\frac{\pi}{6}\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= 2\left(\frac{1}{2}\right)(\sqrt{3} + i) \qquad \boxed{z_1 = \sqrt{3} + i}$$

Luego, utilizo la fórmula $\frac{2\pi}{N}$ para sumarlo a $\frac{\pi}{6}$ y seguir hallando raíces.

$$\frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$* z^{1/6} = 2\left(\cos\frac{\pi}{2} + i \operatorname{sen}\frac{\pi}{2}\right)$$

$$= 2(0 + i) \qquad \boxed{z_2 = 2i}$$

$$* z^{1/6} = 2\left(\cos\frac{5\pi}{6} + i \operatorname{sen}\frac{5\pi}{6}\right)$$

$$= 2\left(-\frac{\sqrt{3}}{2} + i\left(\frac{1}{2}\right)\right)$$

$$= 2\left(\frac{1}{2}\right)(-\sqrt{3} + i) \qquad \boxed{z_3 = -\sqrt{3} + i}$$

$$z^{1/6} = 2\left(\cos\frac{7\pi}{6} + i\,\text{Sen}\,\frac{7\pi}{6}\right)$$

$$= 2\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}(i)\right)$$

$$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)(-\sqrt{3} - i)$$

$$= -\sqrt{3} - i \qquad \boxed{z_4 = -\sqrt{3} - i}$$

$$z^{1/6} = 2\left(\cos\frac{3\pi}{2} + i\,\text{Sen}\,\frac{3\pi}{2}\right)$$

$$= 2(0 - i)$$

$$= -2i \qquad \boxed{z_5 = -2i}$$

$$z^{1/6} = 2\left(\cos\frac{11\pi}{6} + i\,\text{Sen}\,\frac{11\pi}{6}\right)$$

$$= 2\left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right)$$

$$= \cancel{2}\left(\frac{1}{\cancel{2}}\right)(\sqrt{3} - i)$$

$$= \sqrt{3} - i \qquad \boxed{z_6 = \sqrt{3} - i}$$

En la práctica, bastaba hallar z_1, z_2 y z_3, para después utilizando sus conjugadas hallar z_4, z_5 y z_6.

(40) $z = \sqrt[4]{8 + 8\sqrt{3}\,i}$

$$z = (8 + 8\sqrt{3}\,i)^{1/4}$$

$$|z| = \sqrt{(8)^2 + (8\sqrt{3})^2} \qquad |z| = \sqrt{64 + 192}$$

$$|z| = \sqrt{256} \qquad |z| = 16$$

$$\theta = \text{Tan}^{-1}\left(\frac{8\sqrt{3}}{8}\right) \qquad \theta = \text{Tan}^{-1}(\sqrt{3})$$

$$\theta = \frac{\pi}{3}$$

Módulo $= 16$ Argumento $= \frac{\pi}{3}$

$$z^{1/N} = |z|^{1/N} e^{i(\theta + 2\pi K)/N}$$ $\quad * \frac{2\pi}{4} = \frac{\pi}{2}$

$$z^{1/4} = (16)^{1/4} e^{i\theta/N}$$ con $K=0$

$$= 2\left(\cos\frac{\pi}{12} + i\,\text{Sen}\,\frac{\pi}{12}\right)$$

$$= 2\left[\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$\boxed{z_1 = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right) + \left(\frac{\sqrt{6}-\sqrt{2}}{2}\right)i}$$

$$z^{1/4} = (16)^{1/4}\left(\cos\frac{7\pi}{12} + i\,\text{Sen}\,\frac{7\pi}{12}\right)$$

$$= 2\left[\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)\right]$$

$$\boxed{z_2 = \left(\frac{-\sqrt{6}-\sqrt{2}}{2}\right) + \left(\frac{\sqrt{6}-\sqrt{2}}{2}\right)i}$$

$$z^{1/4} = 2\left(\cos\frac{13\pi}{12} + i\,\text{Sen}\,\frac{13\pi}{12}\right)$$

$$= 2\left[\left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right]$$

$$\boxed{z_3 = \left(\frac{-\sqrt{6}-\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}-\sqrt{6}}{2}\right)i}$$

$$z^{1/4} = 2\left(\cos\frac{19\pi}{12} + i\,\text{Sen}\,\frac{19\pi}{12}\right)$$

$$= 2\left[\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) + i\left(\frac{\sqrt{2}-\sqrt{6}}{4}\right)\right]$$

$$\boxed{z_4 = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}-\sqrt{6}}{2}\right)i}$$

41 $\quad z = a + bi$

Se debe cumplir: $z = (\bar{z})^2$

$= (a-bi)^2 \qquad = a^2 - 2abi + b^2 i^2$

$$\boxed{=(a^2 - b^2) - 2abi}$$

a) $x^2 - 6x + 13 = 0$

$a = 1 \quad b = -6 \quad c = 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 52}}{2} \qquad x = \frac{6 \pm \sqrt{-16}}{2}$$

$$x = \frac{6 \pm 4i}{2} \qquad x = \frac{\cancel{2}(3 \pm 2i)}{\cancel{2}}$$

$$\boxed{x_1 = 3 + 2i} \qquad \boxed{x_2 = 3 - 2i}$$

b) $2x^2 + 5x + 6$

$a = 2 \quad b = 5 \quad c = 6$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(6)}}{2(2)}$$

$$x = \frac{-5 \pm \sqrt{25 - 48}}{4} \qquad \boxed{x = \frac{-5 \pm \sqrt{23}\, i}{4}}$$

c) $x^2 - 2(1+i)x + (2i - 1) = 0$

$a = 1 \quad b = -2(1+i) \quad c = 2i - 1$

$$x = \frac{-[-2(1+i)] \pm \sqrt{[-2(1+i)]^2 - 4(1)(2i-1)}}{2(1)}$$

$$x = \frac{2 + 2i \pm \sqrt{\cancel{8i} - \cancel{8i} + 4}}{2}$$

$$x = \frac{2 + 2i \pm \sqrt{4}}{2} \qquad x = \frac{2 + 2i \pm 2}{2}$$

$$x = \frac{\cancel{2}(1 + i \pm 1)}{\cancel{2}} \qquad x = 1 + i \pm 1$$

$$x_1 = 1 + i + 1 \qquad x_2 = \cancel{1} + i \cancel{- 1}$$

$$\boxed{x_1 = 2 + i} \qquad \boxed{x_2 = i}$$

d) $x^3 - 2x + 4 = 0$

$$(x + 2)(x^2 - 2x + 2) = 0$$

$$x + 2 = 0$$

$$\boxed{x_1 = -2}$$

$$x^2 - 2x + 2 = 0$$

$$a = 1 \qquad b = -2 \qquad c = 2$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 8}}{2} \qquad x = \frac{2 \pm \sqrt{-4}}{2}$$

$$x = \frac{2 \pm 2i}{2} \qquad x = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$\boxed{x_1 = 1 + i} \qquad \boxed{x_2 = 1 - i}$$

e) $x^2 - 3x + (3 - i) = 0$

$$a = 1 \qquad b = -3 \qquad c = 3 - i$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(3 - i)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{4 - 12 + 4i}}{2}$$

$$\boxed{x = \frac{3 \pm \sqrt{4i - 8}}{2}}$$

F) $x^3 + 3x^2 - 3x - 14 = 0$

$(x-2)(x^2 + 5x + 7) = 0$

$x - 2 = 0 \qquad \boxed{x_1 = 2}$

$x^2 + 5x + 7 = 0$

$a = 1 \qquad b = 5 \qquad c = 7$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 28}}{2} \quad \therefore \quad \boxed{x = \frac{-5 \pm \sqrt{3}\, i}{2}}$$

3) $z = 2 + 3i \qquad \bar{z} = 2 - 3i$

z y $\bar{z}$ son conjugados

$z^3 = (2 + 3i)^3 \qquad z^3 = 8 + 36i + 54i^2 + 27i^3$

$z^3 = -46 + 9i$

$(\bar{z})^3 = (2 - 3i)^3 \qquad (\bar{z})^3 = 8 - 36i + 54i^2 - 27i^3$

$(\bar{z})^3 = -46 - 9i$

Luego z^3 y $(\bar{z})^3$ también son conjugados

44) $\log_{\frac{1}{2}} |z| + \log_{\frac{1}{2}} (|z| + 1) < \log_{\frac{1}{2}} (2|z| + 5)$

$\log_2 |z| + \log_2 (|z| + 1) > \log_2 (2|z| + 5)$

$\log_2 [(|z|)(|z| + 1)] > \log_2 (2|z| + 5)$

$$|z|(|z|+1) > 2|z|+5$$

$$|z|^2+|z|-2|z|-5>0$$

$$|z|^2-|z|-5=0$$

$a=1 \quad b=-1 \quad c=-5$

$$|z|=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-5)}}{2(1)}$$

$$|z|=\frac{1\pm\sqrt{1+20}}{2} \qquad |z|=\frac{1\pm\sqrt{21}}{2}$$

$|z|=\frac{1+\sqrt{21}}{2}$ Descarto $\frac{1-\sqrt{21}}{2}$ ya que esto es un valor negativo y $|z|$ siempre es +.

Por lo tanto: $\boxed{|z|>\frac{1+\sqrt{21}}{2}}$

(45) $-2 < 2\log_{\frac{1}{3}}|x+1-i\sqrt{5}| < -\log_3 2$

$$2\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}2-\log_{\frac{1}{3}}\left(\frac{1}{3}\right)$$

$$2\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}6$$

$$\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \frac{1}{2}\log_{\frac{1}{3}}6$$

$$\log_{\frac{1}{3}}|x+1-\sqrt{5}i| < \log_{\frac{1}{3}}\sqrt{6}$$

$$|x+1-\sqrt{5}i| < \sqrt{6}$$

$|x+1-\sqrt{5}i| < \sqrt{6}$ $\qquad$ $|x+1-\sqrt{5}i| > -\sqrt{6}$

$x+1-\sqrt{5}i < \sqrt{6}$ $\qquad$ $x+1-\sqrt{5}i > -\sqrt{6}$

* $\boxed{x<\sqrt{6}-1+\sqrt{5}i}$ $\qquad$ ** $\boxed{x>\sqrt{5}i-1-\sqrt{6}}$

$$2\log_{\frac{1}{3}}|x+1-i\sqrt{5}| > -2$$

$$-2\log_{3}|x+1-i\sqrt{5}| > -2$$

$$\log_{3}|x+1-i\sqrt{5}| < 1$$

$$\log_{3}|x+1-i\sqrt{5}| < \log_{3} 3$$

$$|x+1-i\sqrt{5}| < 3$$

$x+1-i\sqrt{5} < 3$ ———— $x+1-i\sqrt{5} > -3$

$x < 3-1+i\sqrt{5}$ $\qquad$ $x > -3-1+i\sqrt{5}$

*** $\boxed{x < 2+i\sqrt{5}}$ $\qquad$ **** $\boxed{x > -4+i\sqrt{5}}$

Intersectando las soluciones:

$$\boxed{\left(-4+i\sqrt{5},\ -\sqrt{6}-1+i\sqrt{5}\right) \cup \left(\sqrt{6}-1+\sqrt{5}i,\ 2+\sqrt{5}i\right)}$$

(46) $a^2+ab+b^2=0$

$a=1 \quad b=b \quad c=b^2$

$$a=\frac{-b\pm\sqrt{(b)^2-4(1)(b^2)}}{2}$$

$$a=\frac{-b\pm\sqrt{b^2-4b^2}}{2} \qquad a=\frac{-b\pm\sqrt{-3b^2}}{2}$$

$$a=\frac{-b\pm\sqrt{3}ib}{2}$$

$$\left[a-\left(\frac{-b+\sqrt{3}ib}{2}\right)\right]\left[a-\left(\frac{-b-\sqrt{3}ib}{2}\right)\right]$$

$$\left(a+\frac{b-\sqrt{3}ib}{2}\right)\left(a+\frac{b+\sqrt{3}ib}{2}\right)$$

$$\left[a+\left(\frac{1-\sqrt{3}i}{2}\right)b\right]\left[a+\left(\frac{1+\sqrt{3}i}{2}\right)b\right]$$

$$\boxed{\left[a+\left(\frac{1-i\sqrt{3}}{2}\right)b\right]\left[a+\left(\frac{1+i\sqrt{3}}{2}\right)b\right]}$$

A-25

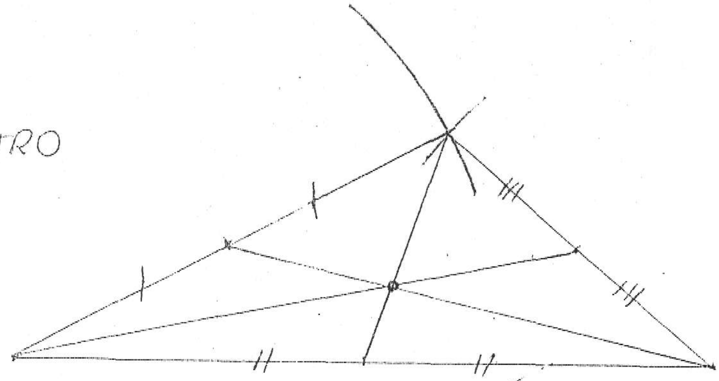
Solución
EJERCICIOS PROPUESTOS
CAPÍTULO SIETE
MATEMÁTICAS
GEOMETRÍA PLANA

TRIÁNGULOS

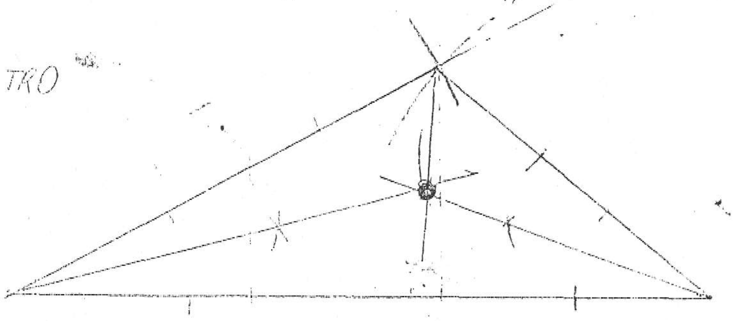
1.

a) $a = 10cm$
$b = 5cm$
$c = 7cm$

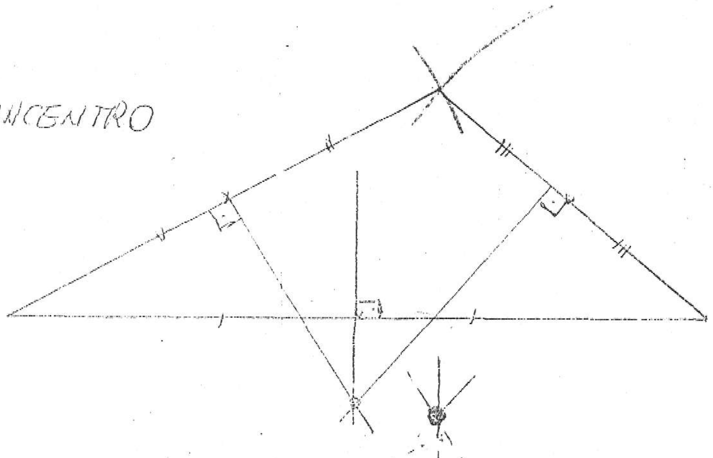
BARICENTRO



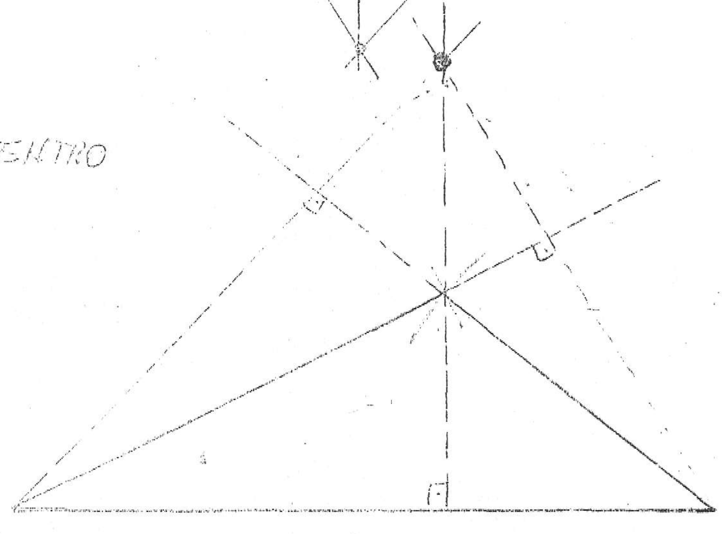
INCENTRO



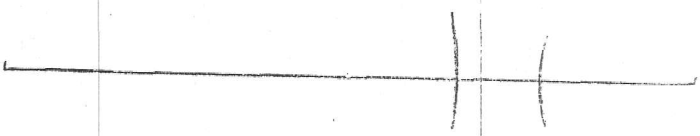
CIRCUNCENTRO



ORTOCENTRO



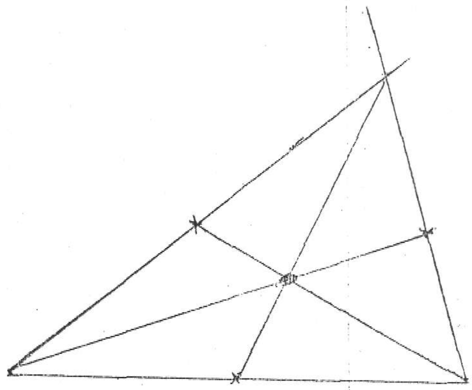
b)



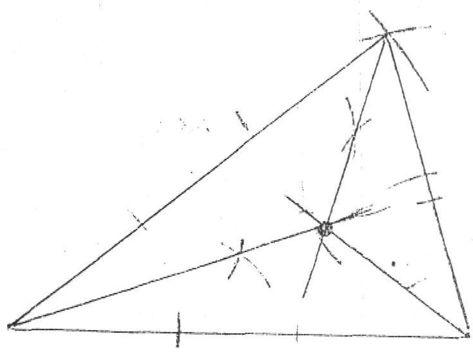
NO SE PRODUCE INTERSECCION.
NO HAY TRIANGULO

c) BARICENTRO

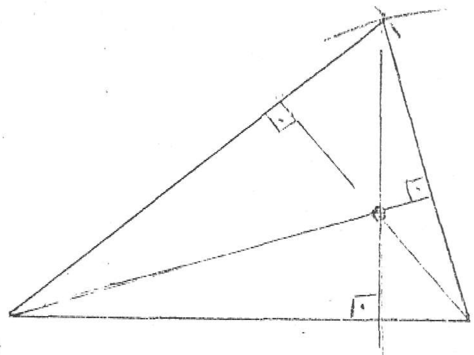
$a = 6\,cm.$



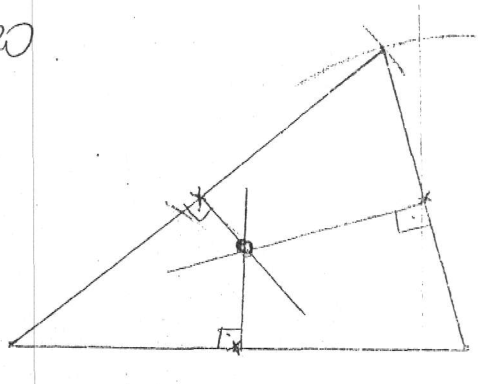
INCENTRO



ORTOCENTRO



CIRCUNCENTRO



d) $c = 3\,cm$

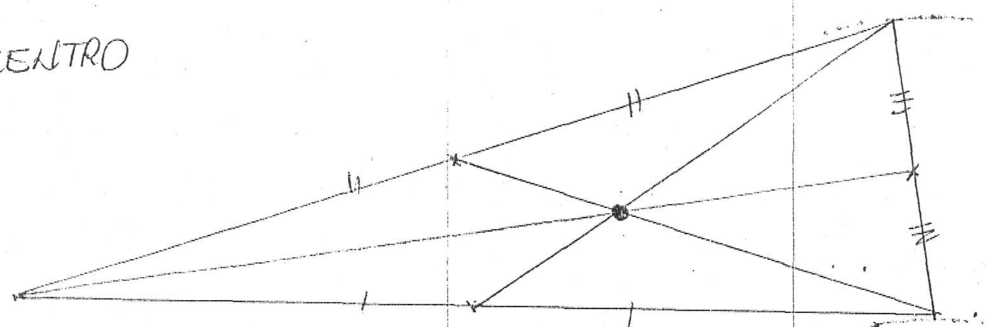
la suma de ángulos internos es 180°: $\alpha+\beta+\gamma=180°$

$\cancel{120}+\cancel{60}+\gamma=\cancel{180}$

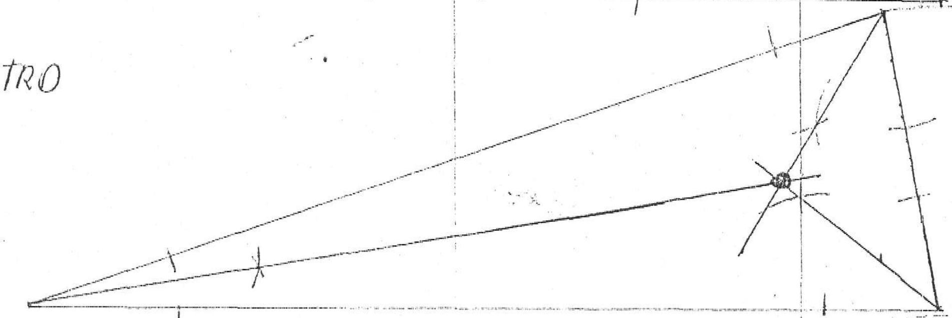
$\gamma=0°$ NO ES TRIANGULO

e) $a=12$ cm.

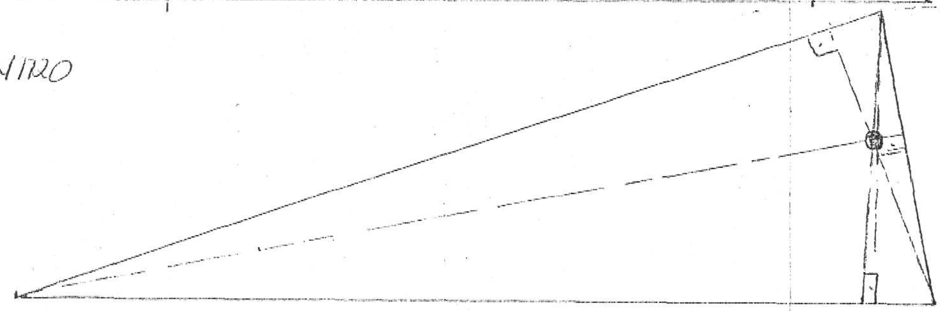
BARICENTRO



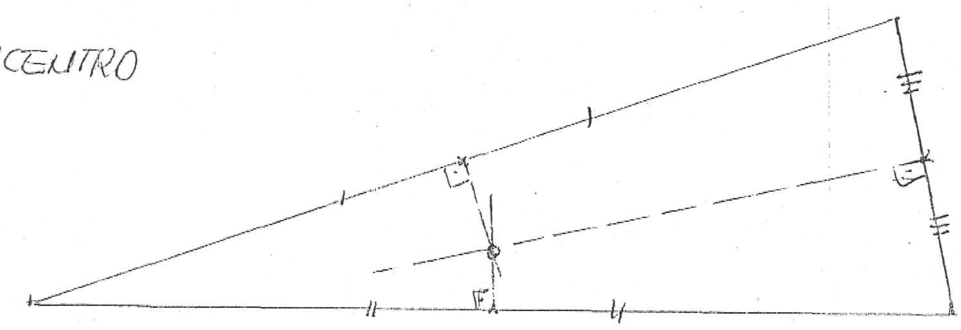
INCENTRO



ORTOCENTRO



CIRCONCENTRO



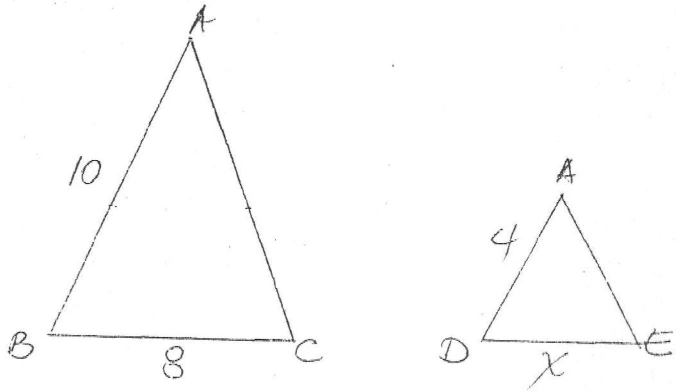
f) $b = 12cm$
$c = 4cm$
$\alpha = 180°$

$\alpha + \beta + \gamma = 180°$

$\cancel{180} + \beta + \gamma = \cancel{180}$

$\beta + \gamma = 0°$ NO ES TRIANGULO

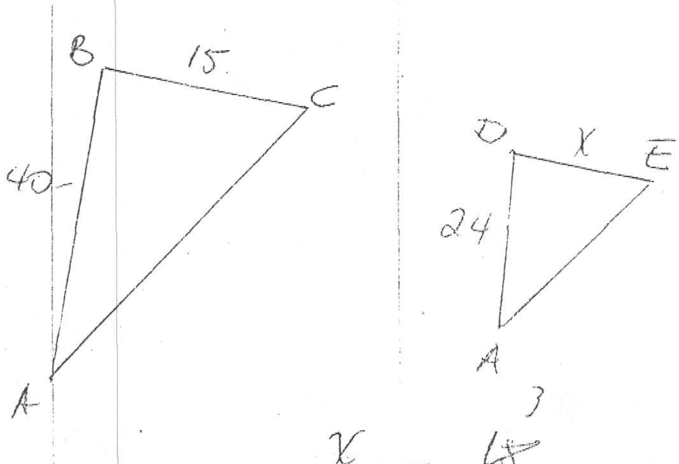
2.-



$$\frac{x}{4} = \frac{8}{10}$$

$$x = \frac{\cancel{32}^{16}}{\cancel{10}_{5}} \Rightarrow \frac{16}{5}$$

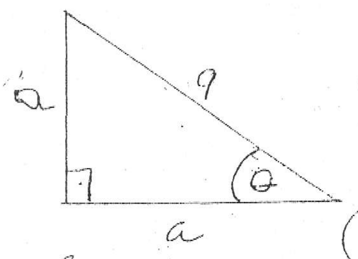
3.-



$$\frac{x}{\cancel{24}_{3}} = \frac{\cancel{15}^{3}}{\cancel{40}_{5}}$$

$x = 9$

4.-)



$$\text{sen}\,\theta = \frac{a}{9}$$

$$a = 9\,\text{sen}\,\theta$$

$$a = 9\frac{\sqrt{2}}{2}$$

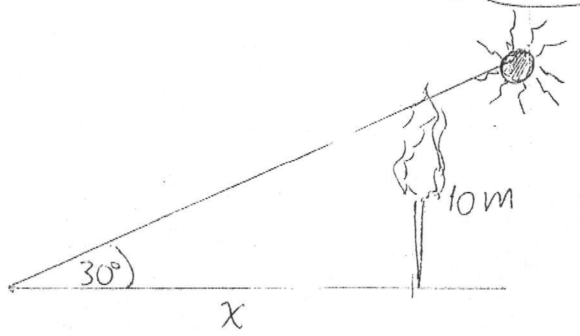
$$\cos\theta = \frac{a}{9}$$

$$\tan\theta = \frac{a}{a}$$

$$\theta = 45^\circ$$

$$a^2 + 6^2 = 81$$

5.-)

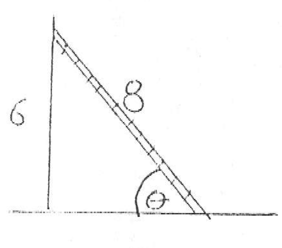


$$\tan 30^\circ = \frac{10}{x}$$

$$x = \frac{10}{\tan 30} \Rightarrow \frac{10}{\sqrt{3}/3} \Rightarrow \frac{30}{\sqrt{3}}$$

$$x = 10\sqrt{3}\ \text{m.}$$

6.-)

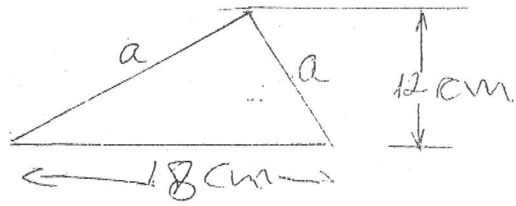


$$\text{sen}\,\theta = \frac{6}{8}$$

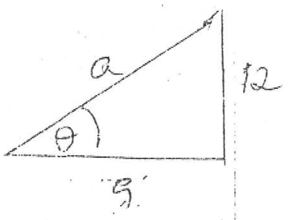
$$\theta = \text{sen}^{-1}\left(\frac{6}{8}\right)$$

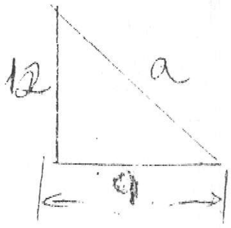
$$\theta = 48{,}6^\circ$$

7.-)



Por simetría





$$\boxed{12^2+9^2=a^2}$$

$a^2=225$

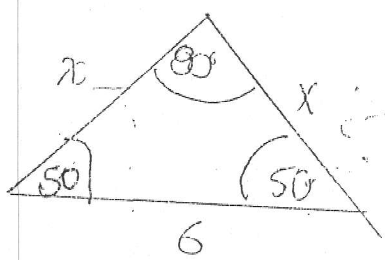
$a=15$

$\operatorname{sen}\theta=\frac{\cancel{12}^{\,4}}{\cancel{15}_{\,5}}$

$\theta=\operatorname{sen}^{-1}(4/5)$

$\theta=53{,}13°$

8.-



$\alpha+80+\beta=180$

$\beta=180-80-\alpha$

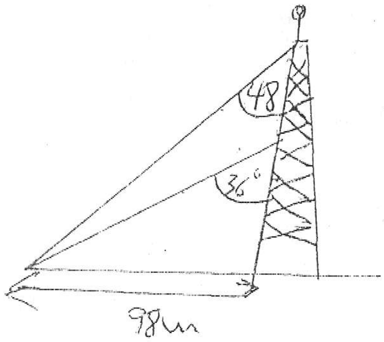
$\beta=100-\alpha$

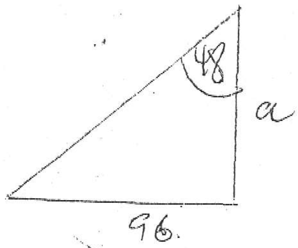
$$\frac{\operatorname{sen}80}{6}=\frac{\operatorname{sen}50}{x}$$

$$x=\frac{6\operatorname{sen}50}{\operatorname{sen}80}$$

$x=4{,}66$

9.



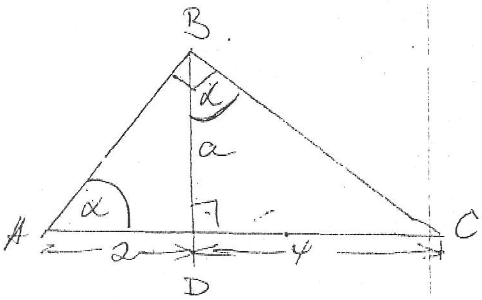


$\tan 48 = \dfrac{96}{a}$

$a = \dfrac{96}{\tan 48}$

$a = 86{,}44\,m$

10.



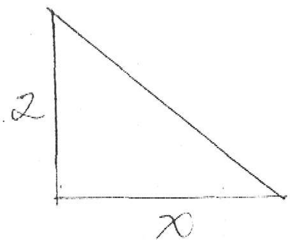
$\tan\alpha = \dfrac{a}{2} \;\Big\|\; \tan\alpha = \dfrac{4}{a}$

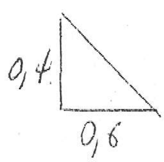
$\dfrac{a}{2} = \dfrac{4}{a}$

$a^2 = 8$

$a = \sqrt{8} \Rightarrow a = 2\sqrt{2}$ b) correcto

11.

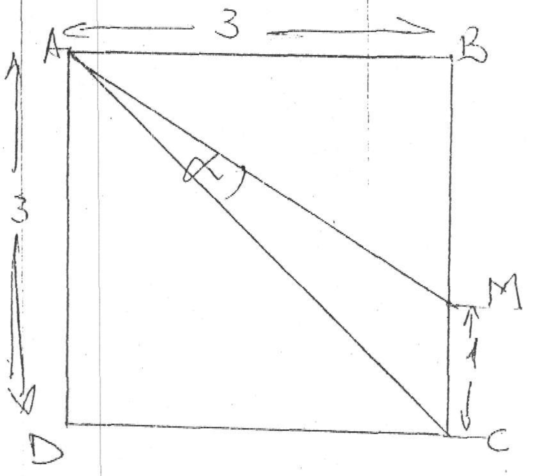


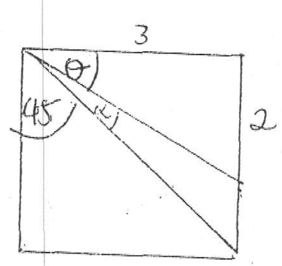


$\dfrac{x}{2} = \dfrac{0{,}6}{0{,}4} \Rightarrow x = \dfrac{1{,}2}{0{,}4} \cdot \dfrac{10}{10} \Rightarrow \dfrac{\cancel{12}^{\,3}}{\cancel{4}_{\,1}}$

$x = 3\,m$ e) correcto

12.—





$\theta + \alpha + 45 = 90$

$\alpha = 90 - 45 - \theta$

$\alpha = 45 - \theta$

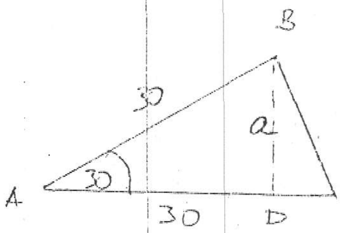
$\tan\alpha = \tan(45 - \theta)$

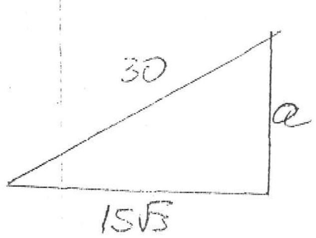
$\Rightarrow \dfrac{\tan 45 - \tan\theta}{1 + \tan\theta \tan 45}$

$\Rightarrow \dfrac{1 - 2/3}{1 + \frac{2}{3} \times 1} \Rightarrow \dfrac{\frac{1}{3}}{\frac{5}{3}}$

$\tan\alpha = 1/5$ d) CORRECTO

13.—





$AD = 30 \cos 30$

$AD = 30 \times \frac{\sqrt{3}}{2}$

$AD = 15\sqrt{3}$

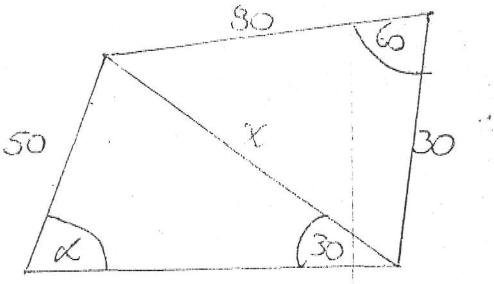
$a^2 = 30^2 - (15\sqrt{3})^2$

$a^2 = 900 - 675$

$a = \sqrt{225}$

$a = 15m$

14.-



$x^2 = 80^2+30^2-2(80)(30)\cos 60$

$x^2 = 6400+900-2400$

$x = \sqrt{4900}$

$x = 70m$

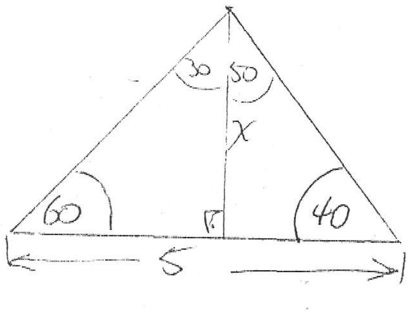
$\frac{\operatorname{sen}\alpha}{x} = \frac{\operatorname{sen}30}{50}$

$\operatorname{sen}\alpha = \frac{x \cdot \operatorname{sen}30}{50} \Rightarrow \frac{7\cancel{0}}{5\cancel{0}} \cdot \frac{1}{2}$

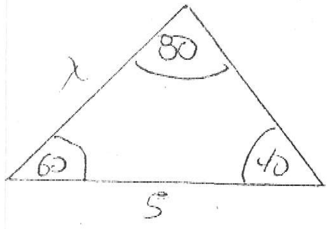
$\alpha = \operatorname{sen}^{-1}(0{,}7)$

$\alpha = 44{,}42^\circ$

15.-



a)



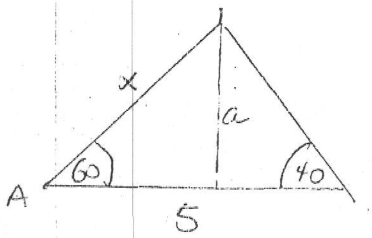
$\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$ [Angulo doble]

$\frac{x}{\operatorname{sen}40} = \frac{5}{\operatorname{sen}80}$

$x = \frac{5\operatorname{sen}40}{\operatorname{sen}80} \Rightarrow \frac{5\cancel{\operatorname{sen}40}}{2\cancel{\operatorname{sen}40}\cos 40}$

$x = \frac{5}{2\cos 40}$

6)



$$\operatorname{sen} 60 = \frac{a}{x}$$

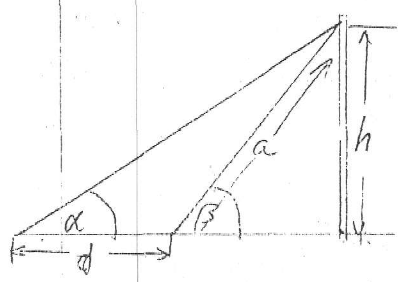
$$a = x \operatorname{sen} 60$$

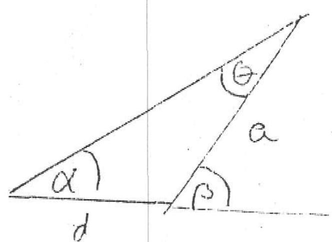
$$a = \frac{5}{2\cos 40} \cdot \operatorname{sen} 60$$

$$a = \frac{5}{2\cos 40} \cdot \frac{\sqrt{3}}{2}$$

$$a = \frac{5\sqrt{3}}{4\cos 40^\circ}$$

16.-

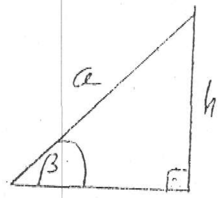




$$\beta = \alpha + \theta$$

$$\theta = \beta - \alpha$$

$$\frac{a}{\operatorname{sen}\alpha} = \frac{d}{\operatorname{sen}\theta} \Rightarrow a = \frac{d \operatorname{sen}\alpha}{\operatorname{sen}(\beta - \alpha)}$$

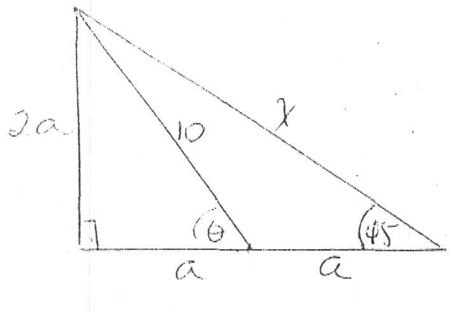


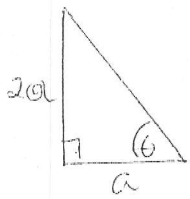
$$\operatorname{sen}\beta = \frac{h}{a}$$

$$h = a \operatorname{sen}\beta$$

$$h = \frac{d \operatorname{sen}\alpha}{\operatorname{sen}(\beta - \alpha)} \cdot \operatorname{sen}\beta$$

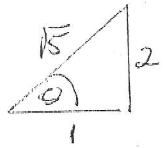
17.–



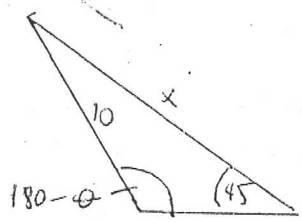


$\tan\theta = \frac{2a}{a}$

$\boxed{\tan\theta = 2}$



$\text{sen}\,\theta = \frac{2}{\sqrt{5}}$



$$\frac{x}{\text{sen}(180-\theta)} = \frac{10}{\text{sen}\,45}$$

$$\frac{x}{\cancel{\text{sen}180}\cos\theta - \text{sen}\,\theta\,\underbrace{\cos 180}_{-1}} = \frac{10}{\sqrt{2}/2}$$

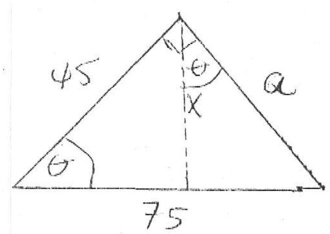
$$\frac{x}{\text{sen}\,\theta} = \frac{20}{\sqrt{2}}$$

$$x = \frac{20}{\sqrt{2}}\,\text{sen}\,\theta \Rightarrow \frac{20}{\sqrt{2}} \times \frac{2}{\sqrt{5}}$$

$$x = \frac{40}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \Rightarrow \frac{40\sqrt{10}}{10}$$

$$x = 4\sqrt{10}$$

18.–



$a^2 = 75^2 - 45^2$

$a = \sqrt{3600}$

$\boxed{a = 60}$

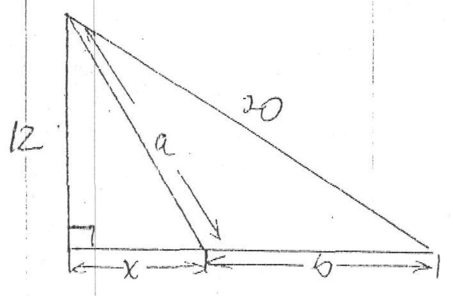
$\text{sen}\,\theta = \frac{x}{45}$ $\qquad$ $\text{sen}\,\theta = \frac{a}{75}$

$$\frac{x}{45} = \frac{a}{75}$$

$$x = a\,\frac{\cancel{45}^{\cancel{9}\,3}}{\cancel{75}_{\cancel{15}\,5}} \Rightarrow \cancel{60}^{12} \times \frac{3}{\cancel{5}_1}$$

$$x = 36$$

19.—



$\boxed{a = b + 8}$

$(x+b)^2 = 20^2 - 12^2$

$x + b = \sqrt{256}$

$\boxed{x + b = 16}$

$\Downarrow$

$b = 16 - x$

$a = b + 8$

$a = 16 - x + 8$

$\boxed{a = 24 - x}$

$a^2 = 12^2 + x^2$

$a^2 = 144 + x^2$

$(24 - x)^2 = 144 + x^2$

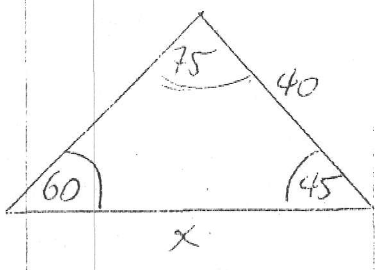
$576 - 48x + \cancel{x^2} = 144 + \cancel{x^2}$

$576 - 144 = 48x$

$432 = 48x$

$x = 9$

20.—



$\sin(75) = \sin(45 + 30)$

$\Rightarrow \sin 45 \cos 30 + \sin 30 \cos 45$

$\Rightarrow \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\Rightarrow \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\Rightarrow \frac{\sqrt{6} + \sqrt{2}}{4}$

$\frac{x}{\sin 75} = \frac{40}{\sin 60}$

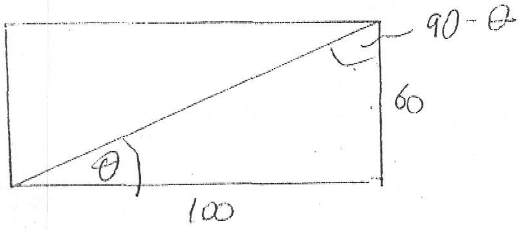
$x = 40 \frac{\sin 75}{\sin 60}$

$x = 40 \frac{(\sqrt{6} + \sqrt{2})}{\cancel{4}\,2} \cdot \frac{1}{\frac{\sqrt{3}}{\cancel{2}\,1}} \Rightarrow \frac{\overset{20}{\cancel{40}}(\sqrt{6} + \sqrt{2})}{\underset{1}{\cancel{2}}\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{20}{3}\sqrt{3}(\sqrt{6} + \sqrt{2})$

$\Rightarrow x = \frac{20\sqrt{6}}{3}(\sqrt{3} + 1)$

21.–



$\tan\theta = \frac{\cancel{60}^{\,3}}{\cancel{100}_{\,5}}$

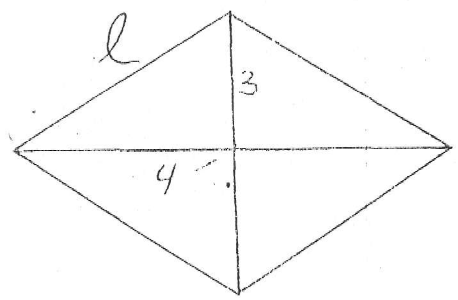
$\theta = \tan^{-1}\left(\frac{3}{5}\right)$

$\theta = 30{,}96^{\circ}$

$\phi = 90 - \theta$

$\phi = 59{,}04^{\circ}$

22.–

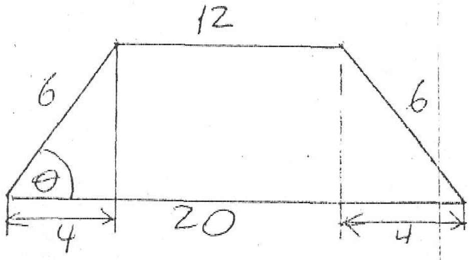


$l^2 = 3^2 + 4^2$

$l = \sqrt{9+16}$

$l = 5\text{cm}$

23.–

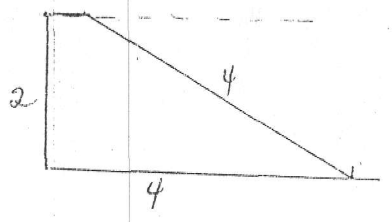


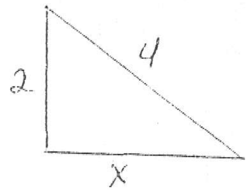
$\cos\theta = \frac{4}{6}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

$\theta = 48{,}19^{\circ}$

24-





$x^2 = 4^2 - 2^2$

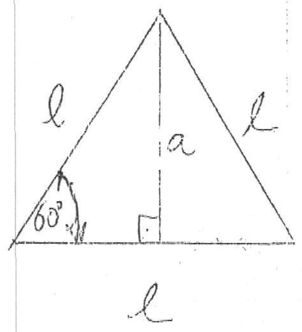
$x = \sqrt{16-4}$

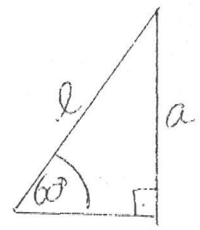
$x = \sqrt{12} \Rightarrow 2\sqrt{3}$

$b = 4 - 2\sqrt{3}$

$b = 2(2-\sqrt{3})$

25-





$\operatorname{sen} 60 = \frac{a}{l}$

$a = l \operatorname{sen} 60$

$\boxed{a = l \frac{\sqrt{3}}{2}}$

$\text{Area} = \frac{b \times a}{2}$

$\Rightarrow \frac{1}{2} l \times \frac{l\sqrt{3}}{2}$

$\Rightarrow \frac{l^2\sqrt{3}}{4}$

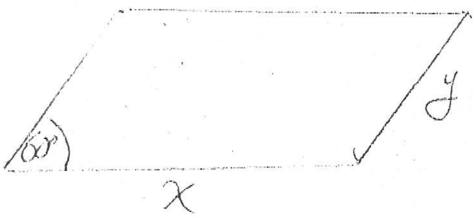
$\Rightarrow \frac{5^2\sqrt{3}}{4}$

$\Rightarrow \frac{25\sqrt{3}}{4}$

26.-
$A = 90\sqrt{3}$

$x = ¿?$

$y = ¿?$



$\boxed{x = 3y}$

$$x \cdot y \, \operatorname{sen} 60 = 90\sqrt{3}$$

$$3y \cdot y \cdot \frac{\sqrt{3}}{2} = 90\sqrt{3}$$

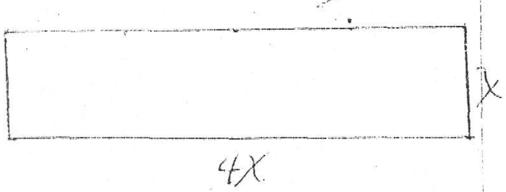
$$y^2 = \frac{2 \cdot 90^{30}}{3}$$

$$\sqrt{y^2} = \sqrt{60}$$

$$y = 2\sqrt{15}$$

$$x = 6\sqrt{15}$$

27.-



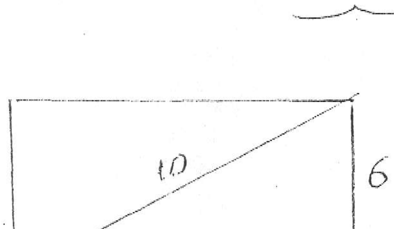
$$\text{Area} = 4x \cdot x = 100$$

$$x^2 = \frac{100}{4}$$

$$\sqrt{x^2} = \sqrt{25}$$

$$x = 5\,m$$

28.-



$$a^2 = 10^2 - 6^2$$

$$a^2 = 100 - 36$$

$$\sqrt{a^2} = \sqrt{64}$$

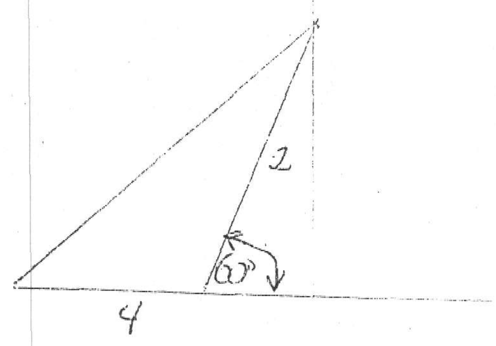
$$a = 8\,m$$

$$\text{Área} = 8 \cdot 6$$

$$\text{Área} = 48\,m^2$$

d) correcto

29.-

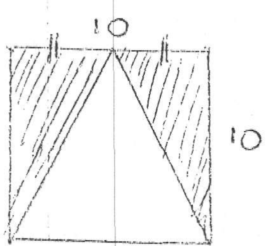


$$Área = \frac{b \times a}{2} \Rightarrow \frac{4 \times \cancel{2} \times sen60}{\cancel{2}}$$

$$\Rightarrow \frac{\cancel{4}^{2} \times \sqrt{3}}{\cancel{2}}$$

$$Área = 2\sqrt{3}\ u^2$$

30.-



$$A_R = \square - \triangle$$

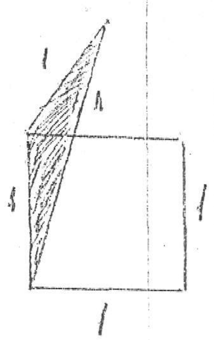
$$\Rightarrow 10^2 - \frac{b \times a}{2}$$

$$\Rightarrow 100 - \frac{10 \times 10}{2}$$

$$A_R = 50 u^2$$

b) correcto

31.-



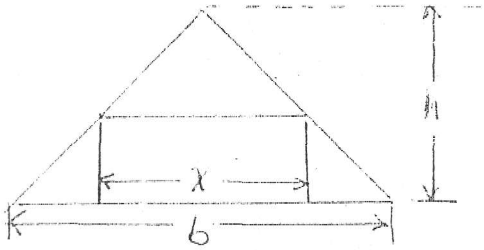
Si es $\triangle$ equilátero

$$Área = \frac{\sqrt{3}\, l^2}{4}$$

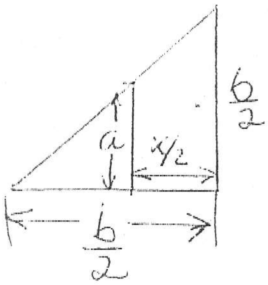
$$Área = \frac{\sqrt{3}\,(1)^2}{4}$$

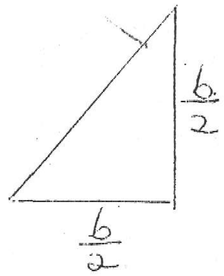
$$Área = \frac{\sqrt{3}}{4}\, u^2$$

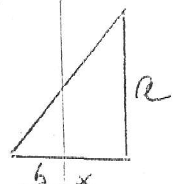
32.-



son △ isósceles







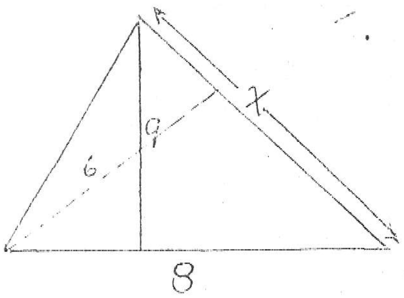
$\frac{b}{2}-\frac{x}{2}$

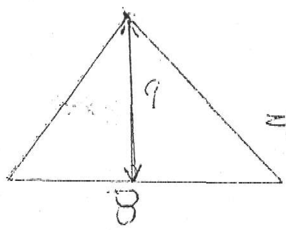
$a=\frac{b}{2}-\frac{x}{2}$

$2a=b-x$

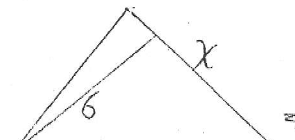
$x=b-2a$

33.-





$\Rightarrow \text{área}_1=\frac{9\times 8}{2}$



$\Rightarrow \text{área}_2=\frac{x\times 6}{2}$

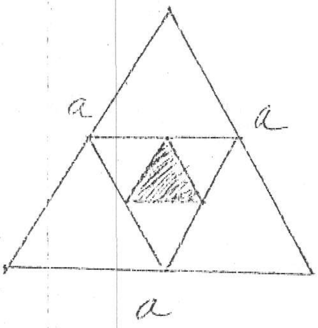
$\text{área}_1=\text{área}_2$

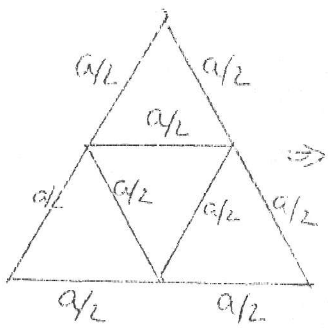
$\frac{\cancel{9}^{3}\times 8}{\cancel{2}}=\frac{x\times\cancel{6}^{2}}{\cancel{2}}$

$24=2x$

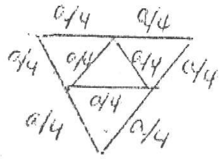
$x=12$

34.-





$\Rightarrow$ son Δ equiláteros



$$\text{Area } \Delta_1 = \frac{\sqrt{3}\,(a)^2}{4}$$

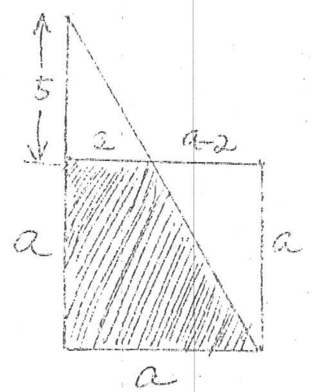
$$\Rightarrow \frac{\sqrt{3}a^2}{4}$$

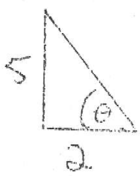
$$\text{Area}\Delta_2 = \frac{\sqrt{3}\,(a/4)^2}{4}$$

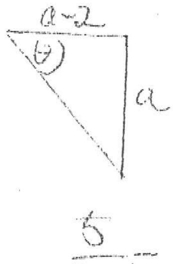
$$\Rightarrow \frac{1}{16}\left(\frac{\sqrt{3}a^2}{4}\right) \text{ area } 1$$

$$\text{Area } 2 = \frac{1}{16} \text{ area } 1$$

35.-







$$\tan\theta = \frac{a}{a-2}$$

$$\tan\theta = \frac{5}{2}$$

$$\frac{5}{2} = \frac{a}{a-2}$$

$$5(a-2) = 2a$$

$$5a - 10 = 2a$$

$$3a = 6$$

$$a = 10/3$$

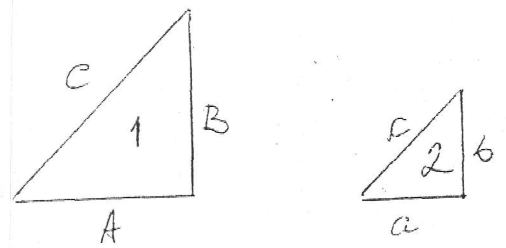
$$A_R = \square - \Delta$$

$$A_R = \left(\frac{10}{3}\right)^2 - \left(\frac{10}{3}\right)\left(\frac{10}{3} - 2\right) \cdot \frac{1}{2} \quad 4/3$$

$$= \frac{100}{9} - \frac{\cancel{40}\ 20}{\cancel{18}\ 9}$$

$$A_R = 80/9$$

36.-



$$\left.\begin{aligned} A &= 2a \\ B &= 2b \\ C &= 2c \end{aligned}\right\}$$

$A = 2a$

$\cancel{AB} = 2a \cdot 2b$

$\dfrac{AB}{2} = \dfrac{4ab}{2}$

$A_{\triangle 1} = 4 A_{\triangle 2}$

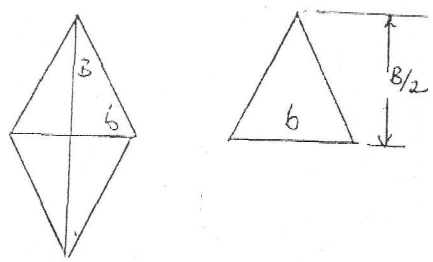
$\boxed{A_{\triangle 2} = 4u^2}$

$A_{\triangle 1} = 4(4)$

$\boxed{A_{\triangle 1} = 16u^2}$

b) FALSO

37.-



$A_\triangle = \frac{1}{2} b \left(\frac{B}{2}\right)$

$A_\triangle = \dfrac{bB}{4}$

$A_\Diamond = 2 A_\triangle$

$A_\Diamond = \cancel{2}^{1} \left(\dfrac{bB}{\cancel{4}_{2}}\right)$

$A_\Diamond = \dfrac{bB}{2}$

$A_\Diamond = \dfrac{\cancel{6}^{3} \times 8}{\cancel{2}_{1}}$

$A_\Diamond = 24u^2$

b) FALSO

38.- $A_\square = l^2$ $A_\triangle = \frac{\sqrt{3}}{4} l^2$

$A_\square = 2A_\triangle$

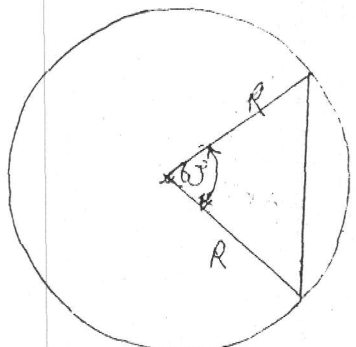
$l^2 = \cancel{2}\left(\frac{\sqrt{3}}{\cancel{4}_2} l_1^2\right)$

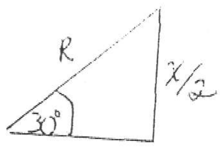
$l^2 = \frac{\sqrt{3}}{2} l_1^2$

$\sqrt{l^2} = \sqrt{\frac{\sqrt{3}}{2} l_1^2}$

$l = \frac{\sqrt[4]{3}}{\sqrt{2}} l_1$ b) FALSO

39.-

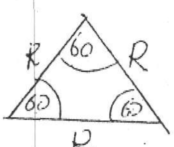




$\sin 30 = \frac{x/2}{R}$

$R = \frac{x/2}{\sin 30}$

$R = \frac{x/2}{1/2}$

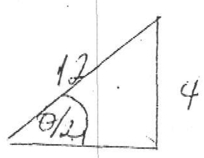


Δ equilátero

$x = R$

$x = 4\,cm$

40.-



$\sin\left(\frac{\theta}{2}\right) = \frac{\cancel{4}}{\cancel{12}_3}$

$\frac{\theta}{2} = \sin^{-1}\left(\frac{1}{3}\right)$

$\theta = 2 \sin^{-1}\left(\frac{1}{3}\right)$

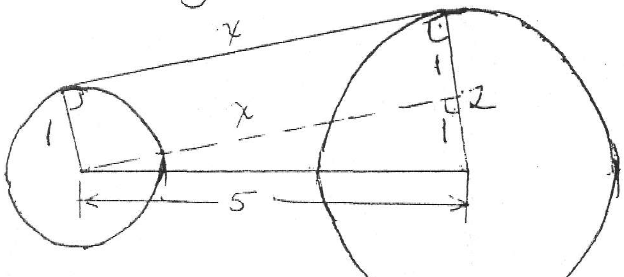
41.- Los datos corresponden al ejercicio anterior,

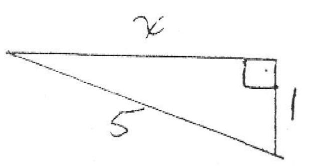
$$\theta = 2\,\text{sen}^{-1}\left(\tfrac{1}{3}\right)$$

$$\theta \neq \pi/3$$

b) FALSO

42.-



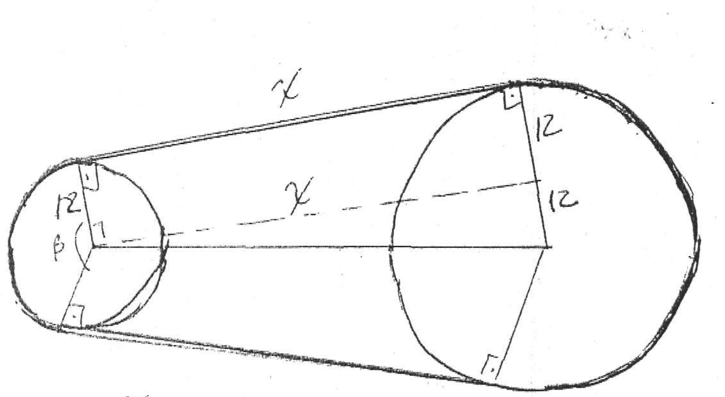


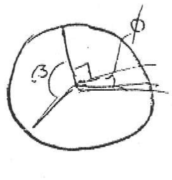
$$x = \sqrt{5^2 - 1^2}$$

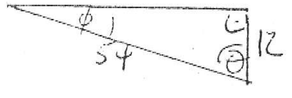
$$x = \sqrt{24}$$

$$x = 2\sqrt{6}$$

43.-







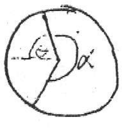
$$x = \sqrt{54^2 - 12^2}$$

$$x = \sqrt{2772}$$

$$x = 6\sqrt{77}$$

$$\theta = \cos^{-1}\left(\frac{\overset{2}{\cancel{12}}}{\underset{9}{\cancel{54}}}\right)$$

$$\theta = 1{,}35 \text{ rad}$$



$$\alpha = 2\pi - 2\theta$$

$$\alpha = 3{,}589 \text{ rad}$$

$$S_1 = \theta R$$

$$S_1 = 3{,}599(24)$$

$$S_1 = 86{,}15 \text{ cm.}$$

$$\phi = \pi - \frac{\pi}{2} - \theta$$

$$\phi = 0{,}22 \text{ rad}$$

$$\beta + \frac{\pi}{2} + \phi + \frac{\pi}{2} + \phi = 2\pi$$

$$\beta = 2\pi - \pi - 2\phi$$

$$\beta = 2{,}7 \text{ rad}$$

$$S_2 = \theta R \Rightarrow 2{,}7(12)$$

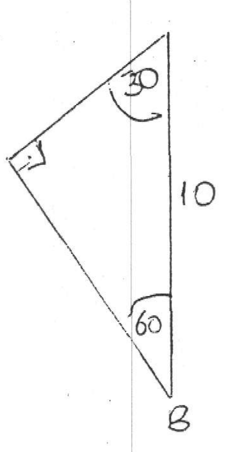
$$S_2 = 32{,}4 \text{ cm}$$

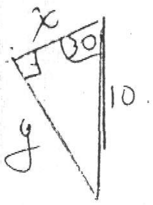
$L = 2x + S_1 + S_2$

$L = 2(6\sqrt{77}) + 86{,}15 + 32{,}4$

$L = 223{,}85\,cm$

44.-





$\cos 30 = \dfrac{x}{10}$

$x = 10\cos 30$

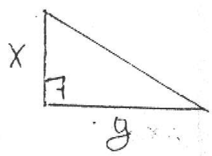
$x = 10 \cdot \dfrac{\sqrt{3}}{2}$

$x = 5\sqrt{3}$

$\operatorname{sen} 30 = \dfrac{y}{10}$

$y = 10\operatorname{sen} 30$

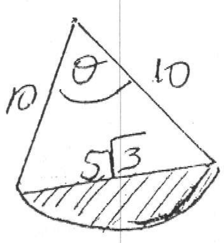
$y = 5$



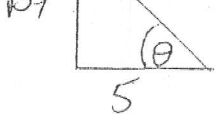
$\text{Area} = \dfrac{5 * 5\sqrt{3}}{2}$

$\Rightarrow \dfrac{25\sqrt{3}}{2}$

45.-



$S_1 =$



$\theta = 0{,}89561\ldots$

$(5\sqrt{3})^2 = 10^2 + 10^2 - 2(10)(10)\cos\theta$

$75 = 200 - 200\cos\theta$

$200\cos\theta = 200 - 75$

$\cos\theta = \dfrac{\cancel{125}^{\,5}}{\cancel{200}_{\,8}}$

$\cos\theta = \dfrac{5}{8}$

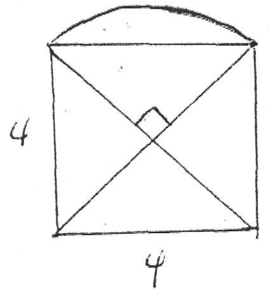
⌓ = ⌔ − △

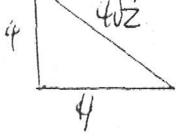
$⌓ = \dfrac{\theta R^2}{2} - \dfrac{1}{2} l^2 \operatorname{sen}\theta$

$\Rightarrow \dfrac{(0{,}8956)10^2}{2} - \dfrac{1}{2}(10^2)\dfrac{\sqrt{39}}{8}$

$\Rightarrow 5{,}75\,m^2$

46.-



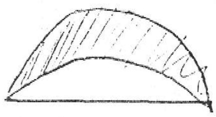


$= \frac{\theta R^2}{2} - \frac{ba}{2}$

$= \frac{1}{\cancel{2}}\frac{\pi}{2}(2\sqrt{2})^2 - \frac{1}{\cancel{2}}(4)(2)$

$= \frac{8\pi}{4} - 4$

$\boxed{= 2\pi - 4}$



$\Rightarrow \frac{\pi R^2}{2} -$

$\Rightarrow \frac{\pi(2)^2}{2} - (2\pi - 4)$

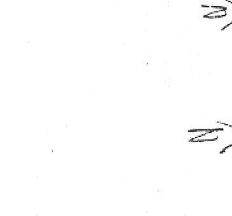
$\Rightarrow \cancel{2\pi} - \cancel{2\pi} + 4$

$\Rightarrow 4$

$A_R = 4 * 4$

$\Rightarrow 16u^2$

47.-



$\frac{x}{2} = \frac{2}{1}$

$x = 4u^2$

48.— b) FALSO

$\Delta_{ACE} \approx \Delta_{DBE}$

49.— $R = 3$ cm

$\widehat{AB} = 2\pi$ cm

$\theta = \frac{2\pi}{3}$ rad

$S = \frac{\theta R}{2}$

$\frac{2S}{R} = \theta$

$\theta = \frac{2(2\pi)}{3}$

$\theta = \frac{4\pi}{3}$ rad

b) FALSO

50.—

$A_R = (\bigcirc - \square)/4$

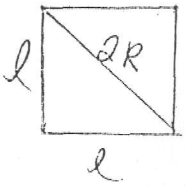
$A_R = \frac{\pi R^2 - l^2}{4}$

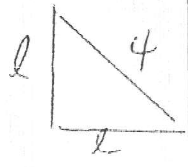
$A_R = \frac{\pi(2)^2 - (2\sqrt{2})^2}{4}$

$A_R = \frac{4\pi - 8}{4}$

$A_R = \pi - 2$

c) CORRECTO





$4 = \sqrt{2}\, l$

$l = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$l = 2\sqrt{2}$

51.-

$R = 2\,m$

$r = 1\,m$

$\theta = \frac{\pi}{3}$

$A_R = \quad - \quad$

$A_R = \frac{\theta R^2}{2} - \frac{\theta r^2}{2}$

$A_R = \frac{\theta}{2}(R^2 - r^2)$

$A_R = \frac{1}{2}\frac{\pi}{3}(2^2 - 1^2)$

$A_R = \frac{\pi}{\cancel{6}_2}\,\cancel{3}^1$

$A_R = \frac{\pi}{2}\,m^2$ a) CORRECTO

52.-

$l = 12\,cm.$

$R = 6\,cm$ $A_R = \quad - \quad$

$A_R = l^2 - \pi R^2$

$A_R = 12^2 - \pi(6)^2$

$A_R = 144 - 36\pi$

$A_R = 36(4-\pi)$ c) CORRECTO

53.-

$\theta = \frac{\pi}{6}\,rad$

$A_S = \frac{4\pi}{3}\,u^2$

$P = ¿?$

$A_S = \frac{\theta R^2}{2}$

$\frac{2A_S}{\theta} = R^2$

$R = \sqrt{\frac{2A_S}{\theta}}$

$R = \sqrt{\frac{2\cdot 4\pi\cdot 6}{3\cdot \pi}}$

$R = 4\,u$

$Arco = \theta R$

$Arco = \frac{\pi}{\cancel{6}_3}(4)^2$

$Arco = \frac{2\pi}{3}$

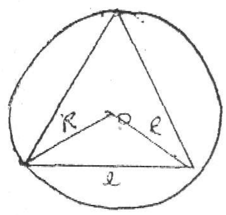
$Perimetro = 2R + arco$

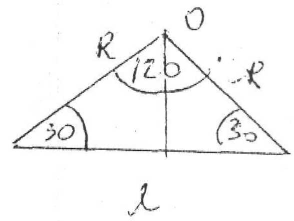
$\Rightarrow 2(4) + \frac{2\pi}{3}$

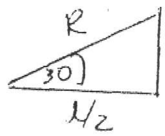
$\Rightarrow \frac{24 + 2\pi}{3}$

54.- $R = 8\,cm.$

$A\triangle = ?$
equilatero







$\cos 30 = \frac{l/2}{R}$

$R\cos 30 = \frac{l}{2}$

$\frac{l}{\cancel{2}} = 8\left(\frac{\sqrt{3}}{\cancel{2}}\right)$

$l = 8\sqrt{3}$

$A\triangle = \frac{\sqrt{3}\,l^2}{4} \Rightarrow \frac{\sqrt{3}(8\sqrt{3})^2}{4}$

$A\triangle = \frac{\cancel{64}^{16} \cdot 3\sqrt{3}}{\cancel{4}_1} \Rightarrow 48\sqrt{3}$

55.—

$A_{hex} = 2\,A_\Delta$

$A_{hex} = 2(48\sqrt{3})$

$\cancel{6}\Delta = 2(\cancel{48})^{8}\sqrt{3}$

$A_\Delta = 16\sqrt{3}$

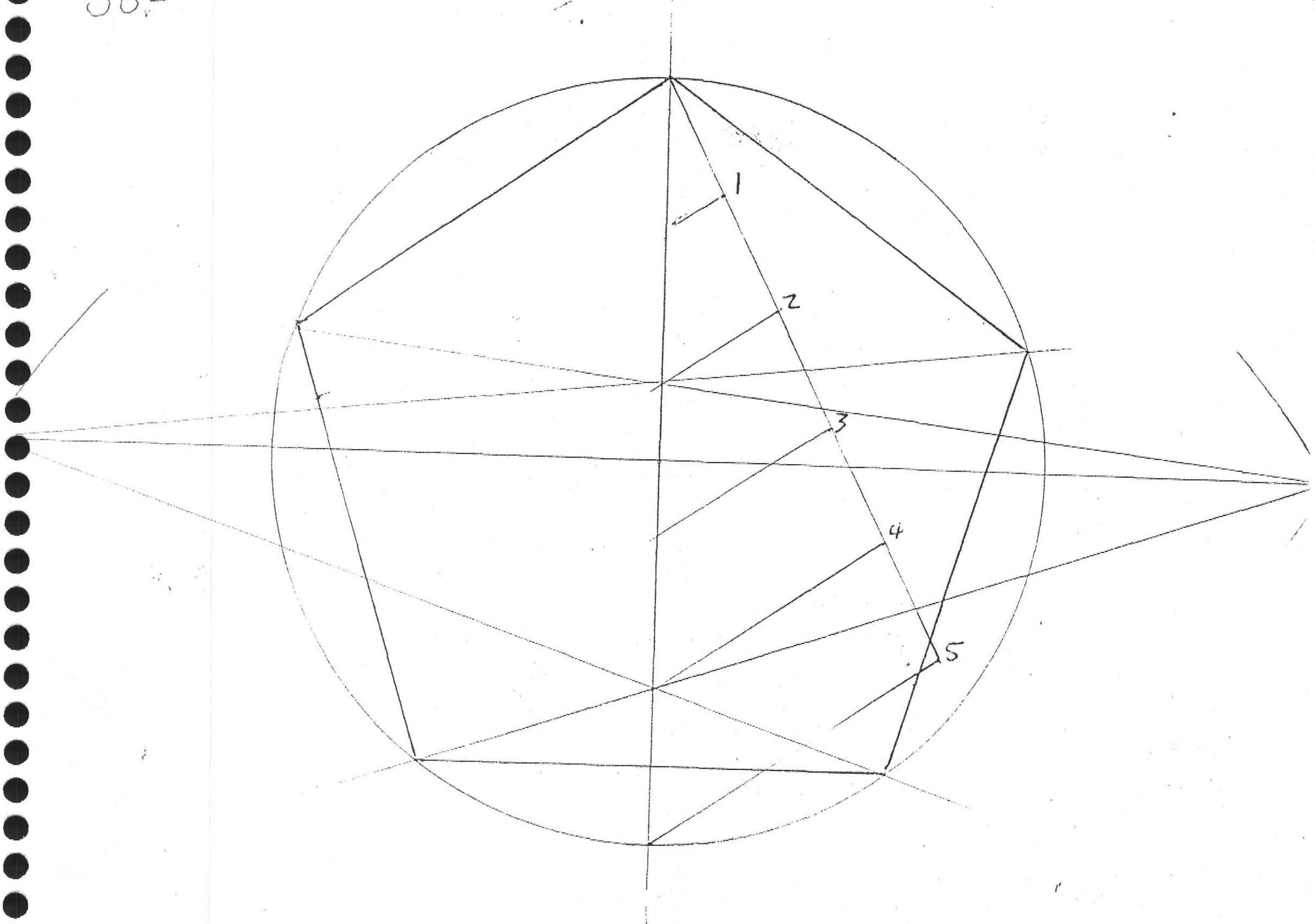
$\frac{\cancel{\sqrt{3}}L^2}{4} = 16\cancel{\sqrt{3}}$

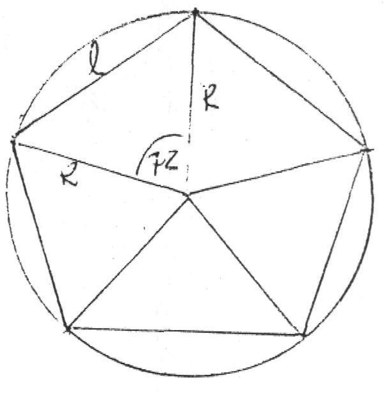
$\sqrt{L^2} = \sqrt{16(4)}$

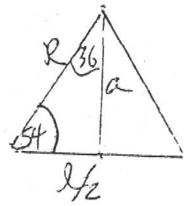
$L = 4(2)$

$L = 8cm$

56.—







$\cos 54 = \frac{l}{2R}$

$R = \frac{l}{2\cos 54}$

$R = 6\,cm$

$a = \sqrt{R^2 - 3{,}5^2}$

$a = \sqrt{36 - 3{,}5^2}$

$a = 4{,}87\,cm$

$P = 5l$

$P = 35\,cm$

$Area = 5\,A_i$

$\Rightarrow 5\,\frac{1}{2}\,l^2 \operatorname{sen}\theta$

$\Rightarrow 5\left(\frac{1}{2}\right)(6)^2 \operatorname{sen} 72$

$\Rightarrow 85{,}6\,cm^2$

Pentágono circunscrito

radio = apotema

$r = 4{,}87\,cm$

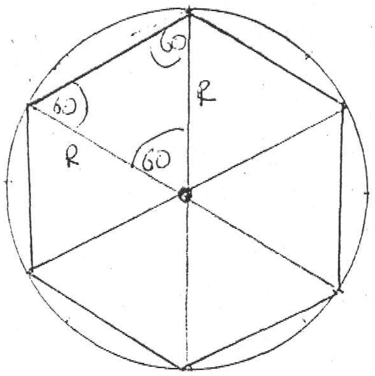
las demás mediciones son iguales

$P = 35\,cm$

$a = 4{,}87\,cm$

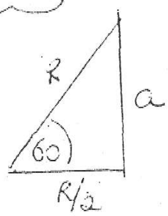
$Area = 85{,}6\,cm^2$

b)



$R = l$

$R = 5cm$



$a = \sqrt{R^2 - \left(\frac{R}{2}\right)^2}$

$a = \sqrt{5^2 - 2{,}5^2}$

$a = \sqrt{18{,}75}$

$a = 4{,}33cm.$

Perímetro = 6 R

$\Rightarrow 6\ (5)$

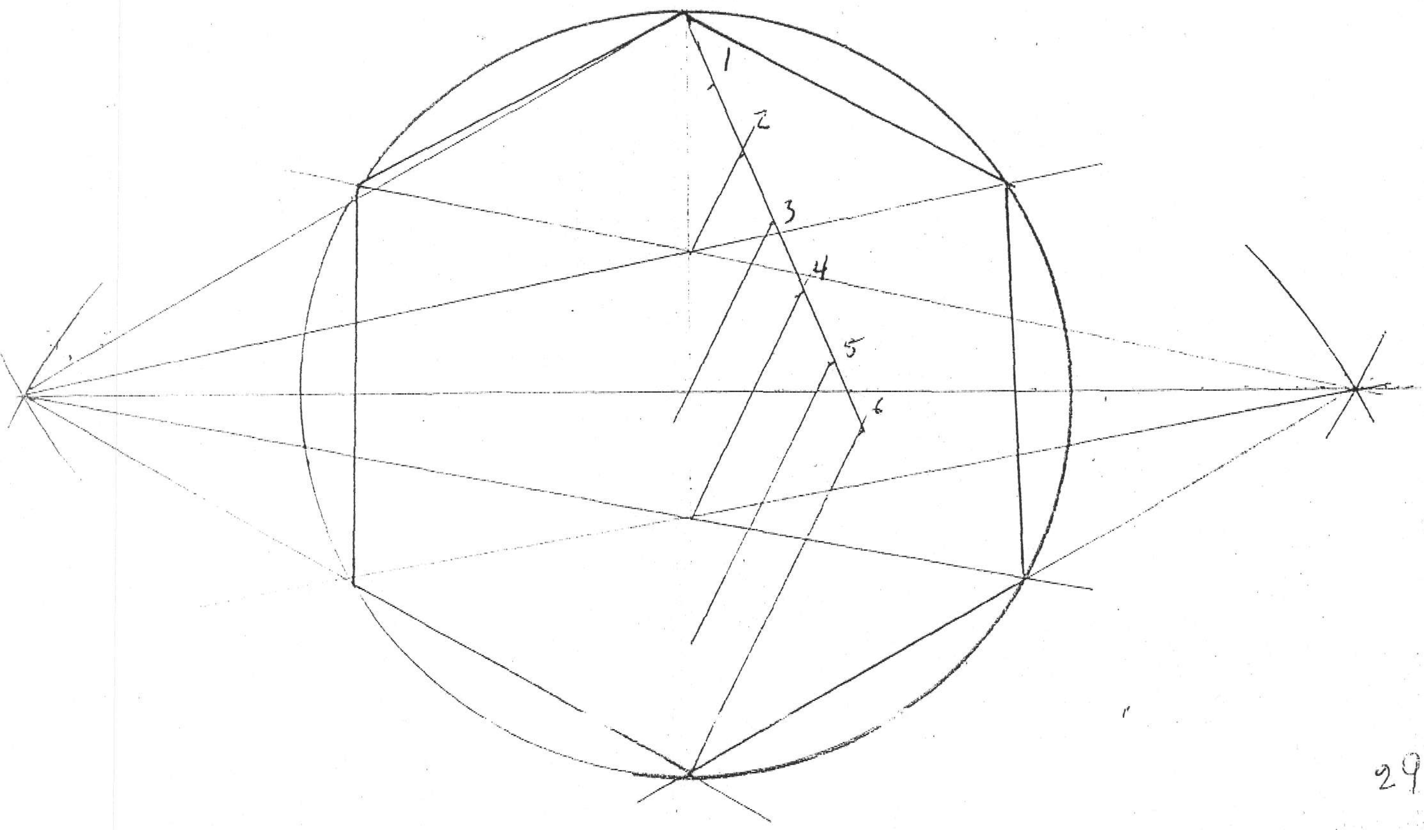
$\Rightarrow 30cm$

Area = 6 A△

$\Rightarrow 6\ \frac{\sqrt{3}}{4} l^2$

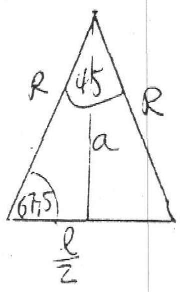
$\Rightarrow \cancel{6}^{3} \frac{\sqrt{3}}{\cancel{4}_{2}} (5)^2$

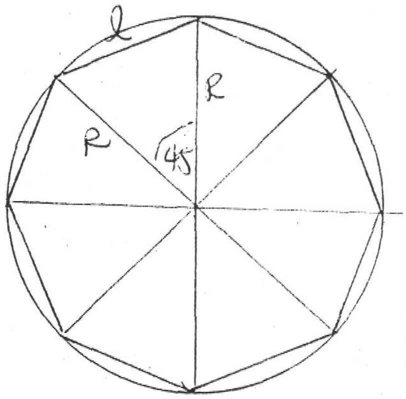
$\Rightarrow \frac{75\sqrt{3}}{2}$



c)

$l = 6\,cm$





$\cos 67{,}5 = \frac{l}{2R}$

$R = \frac{l}{2\cos 67{,}5}$

$\boxed{R = 7{,}84\,cm}$

$\sin 67{,}5 = \frac{a}{R}$

$a = R \sin 67{,}5$

$\boxed{a = 7{,}24\,cm}$

Perímetro $= 8l$

$\Rightarrow 8(6)$

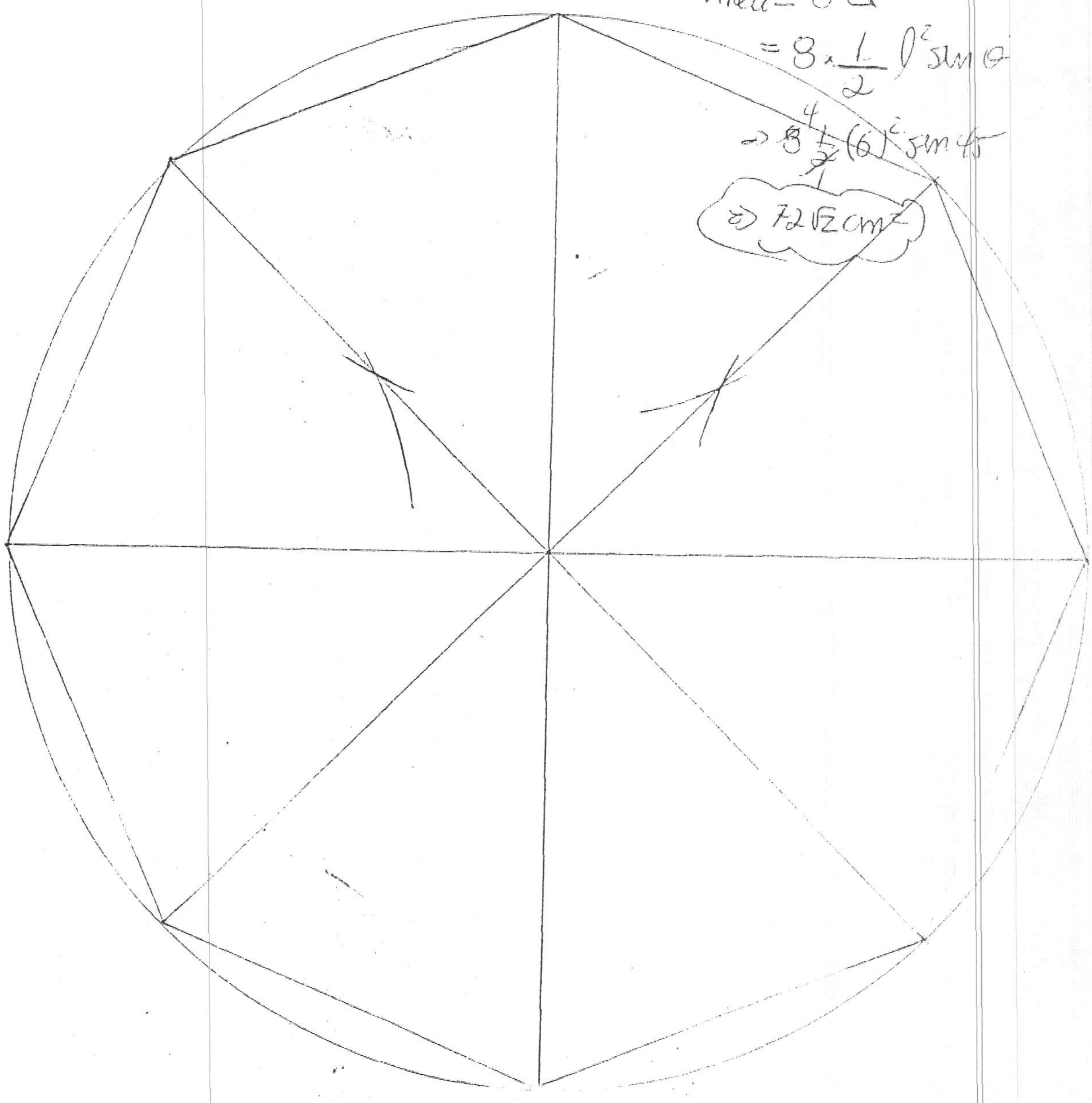
$\Rightarrow 48\,u^2$

Area $= 8\,\Delta$

$= 8 \times \frac{1}{2} l^2 \sin\theta$

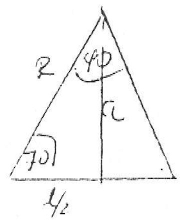
$\Rightarrow \cancel{8}^{4}\, \frac{1}{\cancel{2}_{1}} (6)^2 \sin 45$

$\Rightarrow 72\sqrt{2}\,cm^2$



d)

$\ell = 4\,cm$

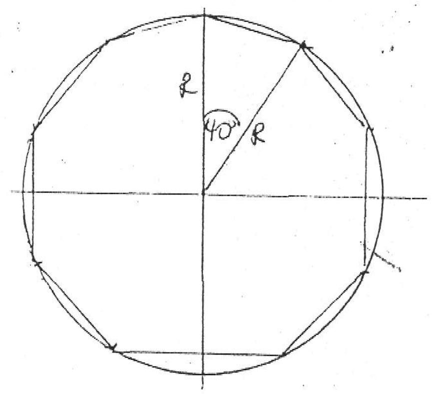


$\cos 70 = \frac{\ell}{2R}$

$R = \frac{\ell}{2\cos 70}$

$R = \frac{4}{2\cos 70}$

$R = 5{,}87\,cm$



$\text{sen}70 = \frac{a}{R}$

$a = R\,\text{sen}70$

$a = 5{,}5\,cm$

Perímetro $= 9.\ell$

$\Rightarrow 9(4)$

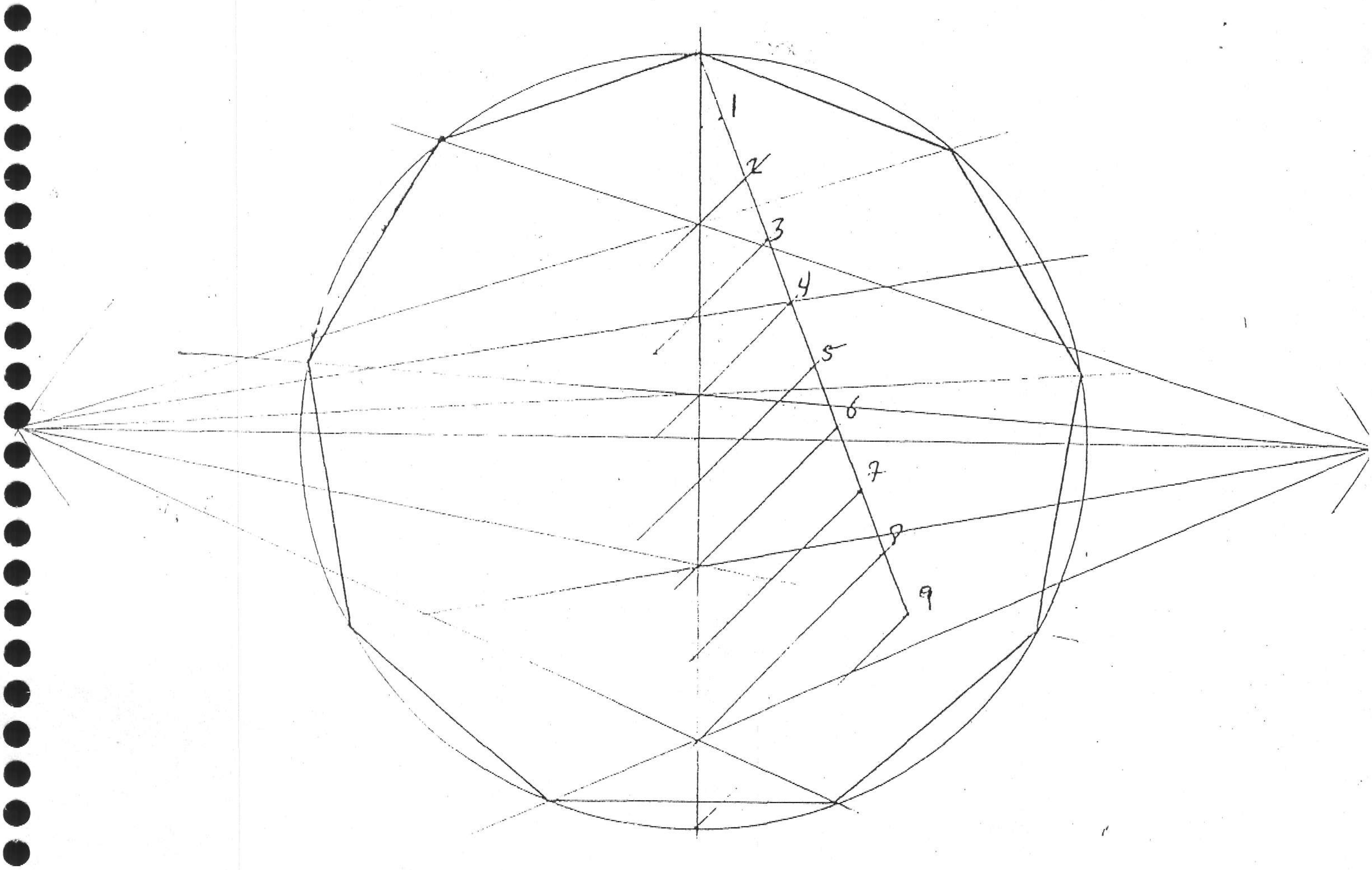
$\Rightarrow 36\,cm^2$

Área $= 9\,A_\Delta$

$\Rightarrow 9 \times \frac{1}{2}\ell^2 \,\text{sen}\,\theta$

$\Rightarrow \frac{9}{2}(5{,}87)^2\,\text{sen}40$

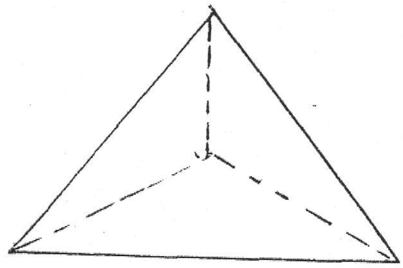
Área $= 99{,}67\,u^2$



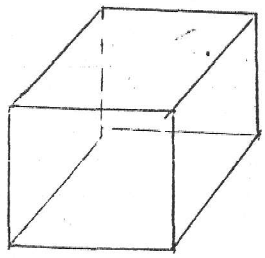
0.60

SOLUCION
EJERCICIOS PROPUESTOS
CAPITULO OCHO
MATEMATICAS
GEOMETRIA DEL ESPACIO.

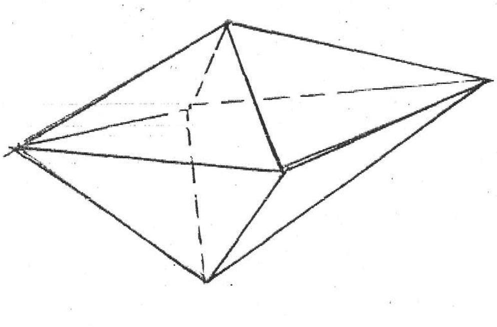
1.- TETRAEDRO REGULAR



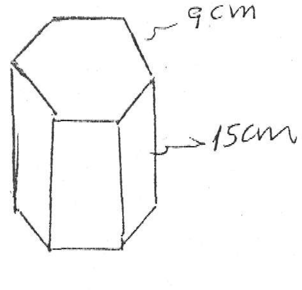
2.- HEXAEDRO REGULAR



3.- OCTAEDRO

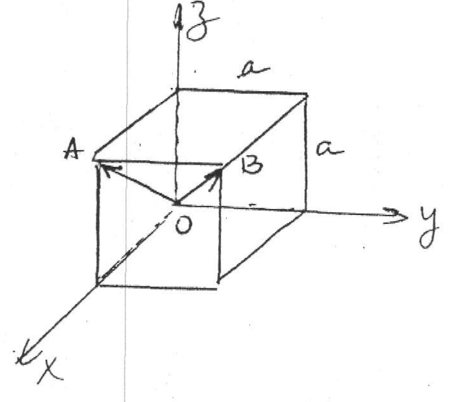


4.-



las aristas de las bases ≠ aristas laterales

5.–



$O(0,0,0)$

$A=(a,0,a)$

$B=(a,a,a)$

$\vec{A}=a\hat{\imath}+a\hat{k}$

$\vec{B}=a\hat{\imath}+a\hat{\jmath}+a\hat{k}$

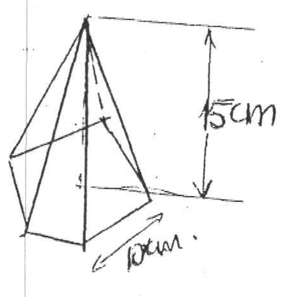
$$\theta=\cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$$

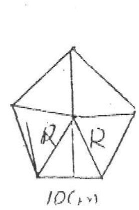
$$\theta=\cos^{-1}\frac{a^2+a^2}{\sqrt{a^2+a^2}\cdot\sqrt{a^2+a^2+a^2}}$$

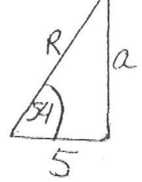
$$\theta=\cos^{-1}\frac{2a^2}{\sqrt{2a^2\times 3a^2}}\Rightarrow\cos^{-1}\frac{2\cancel{a^2}}{\sqrt{6}\,\cancel{a^2}}$$

$$\boxed{\theta=\cos^{-1}\left(\frac{2}{\sqrt{6}}\right)}$$

6.–



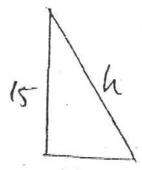




$\tan 54=\frac{a}{5}$

$a=5\cdot\tan 54$

$\{a=6{,}88\}$



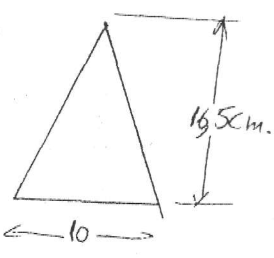
$h=\sqrt{15^2+6{,}98^2}$

$h=16{,}5\text{cm}$

$\cos 54=\frac{5}{R}$

$\left\{R=\frac{5}{\cos 45}\right\}$

caras laterales



c) Octaedro ⇒ 8△ equiláteros

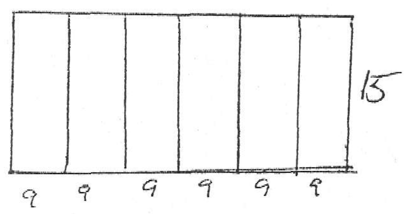
$$A_L = A_T = 8\,\frac{\sqrt{3}\,l^2}{4}$$

$$\Rightarrow 2\sqrt{3}\,l^2$$

$$\Rightarrow 2\sqrt{3}(8)^2$$

$$\Rightarrow 128\sqrt{3}\ cm^2$$

d) A_L



$$A_L = 15 \times 9(6)$$

$$A_L = 810\,cm^2$$

$$\therefore\ A_T = A_L + 2\,⬡$$

$$A_T = 810 + 2(6)\triangle$$

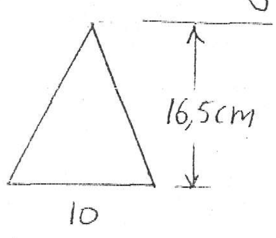
$$A_T = 810 + \cancel{12}^{3}\,\frac{\sqrt{3}\,l^2}{\cancel{4}_{1}}$$

$$A_T = 810 + 3\sqrt{3}(9)^2$$

$$A_T = 243\sqrt{3} + 810$$

$$A_T = 81(3\sqrt{3} + 10)$$

e) Tomando datos del ejercicio #6



$$A_L = 5\triangle$$

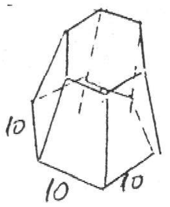
$$A_L = 5 \times \frac{10 \times 16{,}5}{2}$$

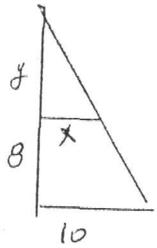
$$A_L = 412{,}5\,cm^2$$

$$A_T = A_L + 5\triangle$$

$$A_T = 412{,}5 + 5\cdot\frac{1}{2}\,l^2 \operatorname{sen}\theta$$

7.-





$$\frac{y+8}{10} = \frac{y}{x}$$

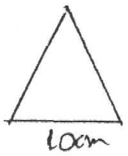
$$x(y+8) = 10y$$

$$xy + 8x = 10y$$

$$x = \frac{10y}{y+8}$$

Las aristas de las bases dependen de la inclinación. de las caras laterales.

8.-a) $A_{Lateral} \Rightarrow$



$$\rightarrow A = \frac{\sqrt{3}\,l^2}{4}$$

$$A = \frac{\sqrt{3}(10)^2}{4}$$

$$A = 25\sqrt{3}$$

$$A_L = 25\sqrt{3}(3)$$

$$A_L = 75\sqrt{3}$$

$$A_T = 4A$$

$$\Rightarrow 4(25\sqrt{3})$$

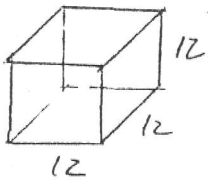
$$\Rightarrow 100\sqrt{3}\,cm^2$$

b) $A_L \Rightarrow$ cuadrado

$$A = l^2$$

$$A = (12)^2$$

$$A = 144\,cm^2$$

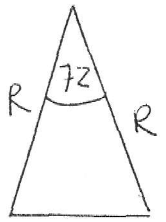


$$A_L = 4(144)$$

$$A_L = 576\,cm^2$$

$$A_T = 6(144)$$

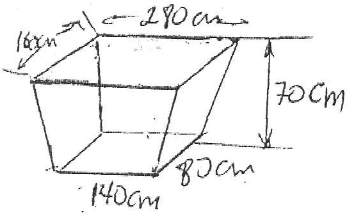
$$A_T = 864\,cm^2$$



$$A_T = 42{,}54 + \frac{5}{2}(R^2)\,\text{sen}\,72$$

$$A_T = 161{,}38$$

9.-



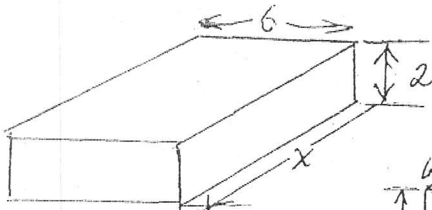
$$V_T = \frac{h}{3(A-a)}\left(A^2B - a^2b\right)$$

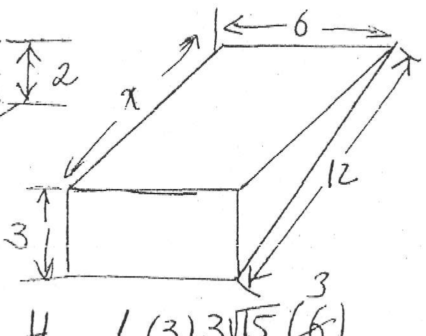
$$V_T = \frac{70}{3(280-140)}\left(280^2(160) - 140^2(80)\right)$$

$$V_T = \frac{\cancel{70}^{1}}{3(\cancel{140}_{2})}\left(\overset{548800}{\cancel{10976000}}\right)$$

$$V_T = \frac{548800}{3}$$

10.-





$$x = \sqrt{12^2 - 3^2}$$

$$x = \sqrt{144 - 9}$$

$$x = \sqrt{135}$$

$$x = 3\sqrt{15}$$

$$V_1 = 6(2)(3\sqrt{15})$$

$$V_1 = 36\sqrt{15}$$

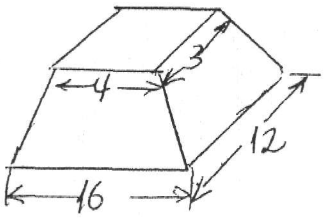
$$V_2 = \frac{1}{\cancel{2}}(3)\,3\sqrt{15}\,(\cancel{6})$$

$$V_2 = 27\sqrt{15}$$

$$V_T = V_1 + V_2$$

$$V_T = 63\sqrt{15}$$

11.—



$$V_T = \frac{h}{3(A-a)}\left(A^2B - a^2b\right)$$

$$V_T = \frac{\overset{6}{\cancel{18}}}{\underset{1}{\cancel{3}}(16-4)}\left(16^2(12) - 4^2(3)\right)$$

$$V_T = \frac{\cancel{6}^{\,1}}{\cancel{12}_{\,2}}\left(256(12) - 16(3)\right)$$

$$V_T = \frac{1}{2}(3024)$$

$$V_T = 1512\ u^3$$

12.— $V_C = 512\ cm^3$

$$\sqrt[3]{l^3} = \sqrt[3]{V_C}$$

$$l = \sqrt[3]{512}$$

$$l = 8\ cm$$

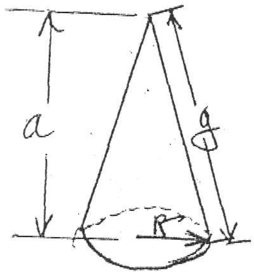
13.— $V = V_1 - V_2$

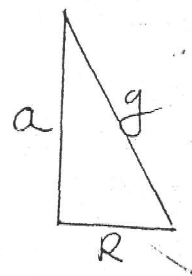
$$V = a^3 - \left(\frac{a}{2}\right)^3$$

$$V = a^3 - \frac{a^3}{8}$$

$$V = \frac{7a^3}{8}$$

14.-





$$g = \sqrt{a^2 + R^2}$$

$$g = \sqrt{8^2 + 6^2}$$

$$g = 10\ cm$$

15.- $R_1 = 4 cm$

$R_2 = 7 cm$

$V_1 = \frac{4}{3}\pi(4)^3$

$V_1 = \frac{256}{3}\pi\ cm^3$

$V_2 = \frac{4}{3}\pi(7)^3$

$V_2 = \frac{1372}{3}\pi\ cm^3$

$V = V_1 + V_2$

$\Rightarrow \frac{256}{3}\pi + \frac{1372}{3}\pi$

$V = \frac{1628}{\cancel{3}}\cancel{\pi} = \frac{4}{\cancel{3}}\cancel{\pi}R^3$

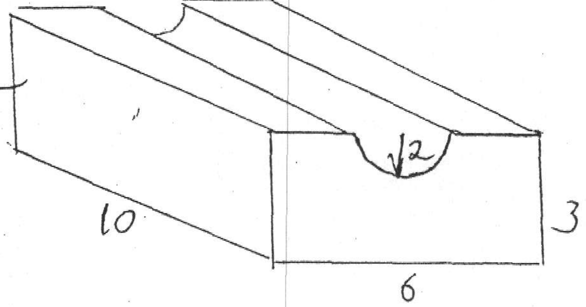
$4R^3 = \frac{1628}{1} \Rightarrow R^3 = \frac{\cancel{1628}\ \cancel{814}\ 407}{\cancel{4}\ \cancel{2}\ 1}$

$R^3 = 407$

$\sqrt[3]{R^3} = \sqrt[3]{407}$

$R = \sqrt[3]{407}$

16.—



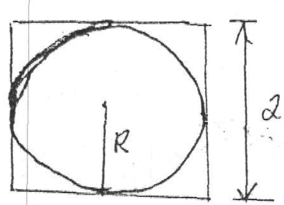
$V =$ 

$V = 6\times3\times10 - \frac{\pi(2)^2}{2}\times10$

$V = 180 - 20\pi$

$V = 20(9-\pi)$ a) CORRECTO

17.—



$V_C = l^3$

$V_C = a^3$

$V_E = \frac{4}{3}\pi R^3$

$V_E = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

$V_E = \frac{4\pi a^3}{3(8)}$

$V_E = \frac{\pi a^3}{6}$

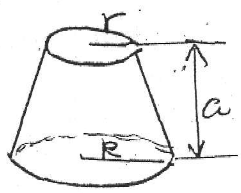
$2R = a$

$R = a/2$

18.— $\Delta V = a^3 - \frac{\pi a^3}{6}$

$\Rightarrow \frac{6a^3 - \pi a^3}{6} \Rightarrow \Delta V = \frac{a^3}{6}(6a^3 - \pi)$

19.— $V = \dfrac{a\pi}{3(R-r)}(R^3 - r^3)$



$V = \dfrac{\cancel{6}^{\cancel{2}\,1}\pi}{\cancel{3}_{1}(\cancel{4-2})_{1}}(4^3 - 2^3)$

$V = \pi(64 - 8)$

$V = 56\pi$ b) CORRECTO

20.— $V = V_{C1} - V_{C2}$

$V = \pi R^2 \cdot \ell - \pi r^2 \ell$

$V = \pi \ell (R^2 - r^2)$

$V = \pi(10)(8^2 - 6^2)$

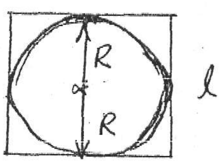
$V = 280\pi\, cm^3$

21.— $V_c = 64\, cm^3$

$\ell^3 = 64$

$\sqrt[3]{\ell^3} = \sqrt[3]{64}$

$\boxed{\ell = 4cm}$



$\ell = 2R$

$R = \dfrac{\ell}{2}$

$\boxed{R = 2cm}$

$V_e = \dfrac{4}{3}\pi R^3$

$V_e = \dfrac{4}{3}\pi (2)^3$

$V_e = \dfrac{32}{3}\pi\, cm^3$

c) CORRECTO

22.-

$\forall_T = \forall_{\ominus} + 2\forall_{\Delta} + \forall_{\square}$

$\forall_T = \frac{4}{3}\pi R^3 + 2\left(\frac{1}{3}\pi R^2 h\right) + \pi R^2 a$

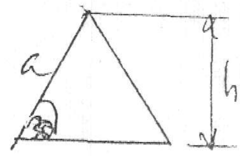
$\forall_T = \frac{4}{3}\pi a^3 + \frac{\cancel{2}}{3}\pi (a^2) \times \frac{a}{\cancel{2}} + \pi a^2 a$

$\forall_T = \frac{4}{3}\pi a^3 + \frac{a^3\pi}{3} + \pi a^3$

$\forall_T = \pi a^3\left(\frac{4}{3} + \frac{1}{3} + 1\right)$

$\forall_T = \pi a^3\left(\frac{5}{3} + 1\right)$

$\forall_T = \frac{8}{3}\pi a^3$

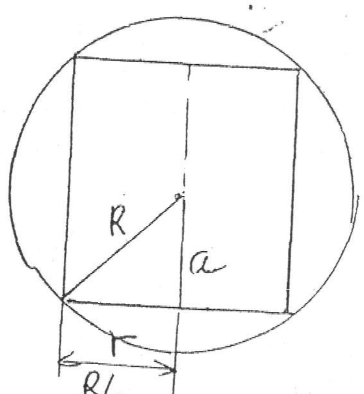


$\text{sen}30 = \frac{h}{a}$

$h = a\,\text{sen}30$

$\boxed{h = \frac{a}{2}}$

23.-



$a = \sqrt{R^2 - \left(\frac{R}{3}\right)^2}$

$a = \sqrt{R^2 - \frac{R^2}{9}}$

$a = \sqrt{\frac{9R^2 - R^2}{9}}$

$a = \sqrt{\frac{8R^2}{9}}$

$\boxed{a = \frac{2R}{3}\sqrt{2}}$

$\forall_c = \pi r^2 (2a)$

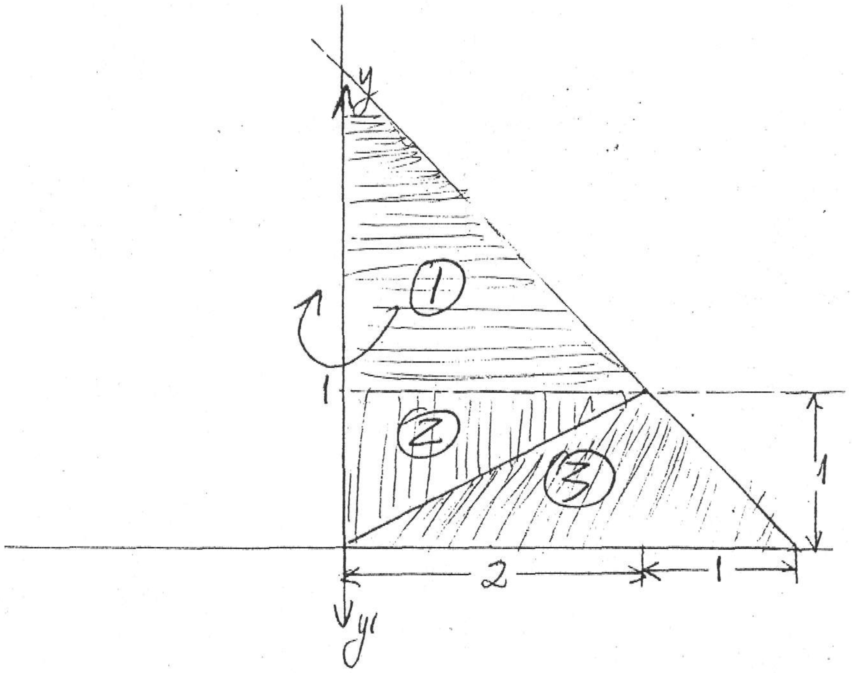
altura

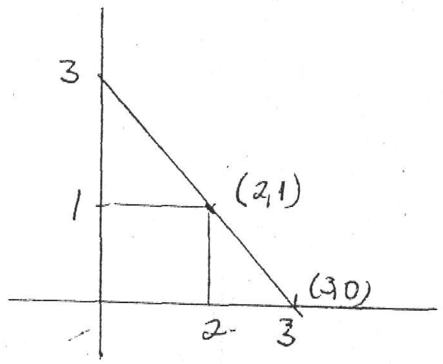
$\forall_c = \pi\left(\frac{R}{3}\right)^2\left(2\left(\frac{2R}{3}\sqrt{2}\right)\right)$ d) CORREC

$\forall_c = \frac{R^2}{9}\pi \cdot \frac{4R\sqrt{2}}{3}$

$\forall_c = \frac{4\sqrt{2}\pi R^3}{27}$

-24.-





$m = \frac{0-1}{3-2} = -1$

$y = -x+b$

$0 = -3+b$

$\boxed{b=3}$

$V = V_T - V_1 - V_2$

$V = \frac{1}{3}\pi R_1^2 a_1 - \frac{1}{3}\pi R_2^2 a_2 - \frac{1}{3}\pi R_3^2 a_3$

$V = \frac{\pi}{3}\left(3^2(3) - 2^2(2) - (2)^2(1)\right)$

$V = \frac{\pi}{3}(27 - 8 - 4)$

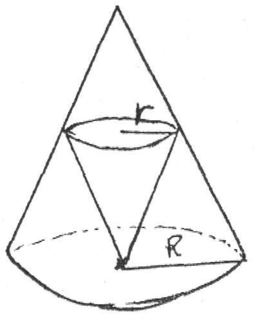
$V = \frac{\cancel{15}^{5}}{\cancel{3}_{1}}\pi$

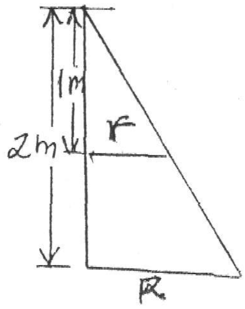
$V = 5\pi$

a) CORRECTO.

25.- $\begin{cases} \Phi = 2m \Rightarrow R = 1m \\ a = 2m \end{cases}$

$a_1 = 1m.$





$V_c = \frac{1}{3}\pi r^2 a_1$

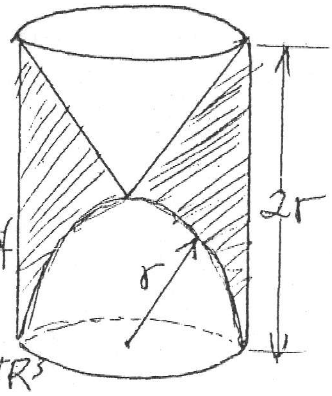
$\frac{r}{1} = \frac{R}{2}$

$V_c = \frac{1}{3}\pi \left(\frac{1}{2}\right)^2 (1)$

$\boxed{r = \frac{1}{2} m}$

$V_c = \frac{\pi}{12} m^3$ b) CORRECTO

26.-



$V = V_{cil} - V_{cono} - V_{semiesf}$

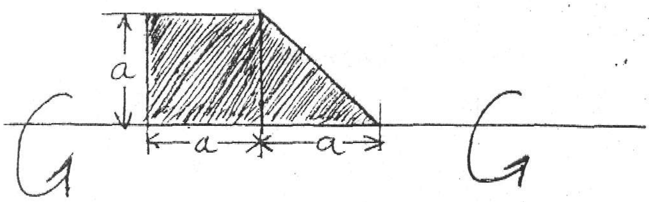
$V = \pi R^2 a - \frac{1}{3}\pi R^2 a - \frac{2}{3}\pi R^3$

$V = \pi (r)^2 (2r) - \frac{1}{3}\pi r^2 \cdot r - \frac{2}{3}\pi r^3$

$V = \pi \left(2r^3 - \frac{r^3}{3} - \frac{2}{3} r^3\right)$

$V = \pi r^3$ d) CORRECTO

27.-



$V_{cilin} + V_{cono}$

$\pi R^2 a + \frac{1}{3}\pi R^2 a$

$\pi (a^2) a + \frac{1}{3}\pi a^2 a$

$a^3\pi + \frac{1}{3}a^3\pi$

$\frac{4}{3}a^3\pi$

28.- $R = 7$ pulg

$r = 5$ pulg.

$a = 8$ pulg

$V_{hueco} = V_{C1} - V_{C2}$

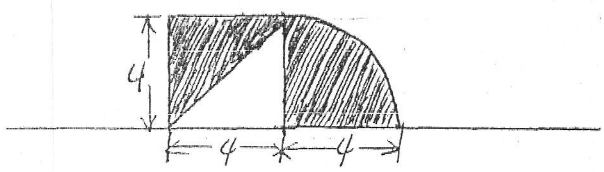
$\Rightarrow \pi R^2 a - \pi r^2 a$

$\Rightarrow \pi a (R^2 - r^2)$

$\Rightarrow \pi (8)(7^2 - 5^2)$

$\Rightarrow 192\pi$ d) CORRECTO.

29.-



$V = V_{cil} - V_{cono} + \frac{1}{2} V_{esfera}$

$V = \pi R^2 a - \frac{1}{3}\pi R^2 a + \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{\,2}}{3}\pi R^3$

$V = \pi (4)^2(4) - \frac{\pi}{3}(4)^2(4) + \frac{2\pi}{3}4^3$

$V = 64\pi\left(1 - \frac{1}{3} + \frac{2}{3}\right)$

$V = \frac{256\pi}{3}$ b) CORRECTO

30.- $R = 2u.$

$V = 16\pi.$

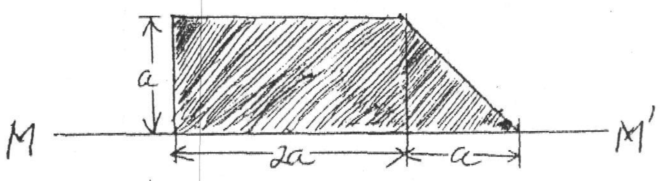
$V_{esfera} = \frac{4}{3}\pi R^3$

$\Rightarrow \frac{4}{3}\pi(2)^3$

$\Rightarrow \frac{32}{3}\pi$

b) CORRECTO

31.-



$V = V_{cil} + V_{cono}$

$\Rightarrow \pi R^2 a + \frac{1}{3}\pi R^2 a$

$\Rightarrow \pi(a)^2(2a) + \frac{1}{3}\pi a^2 a$

$\Rightarrow 2\pi a^3 + \frac{\pi}{3}a^3$

$\Rightarrow \frac{7\pi a^3}{3}$ a) CORRECTO

32.- $V = 125cm^3$

$a = 10cm$

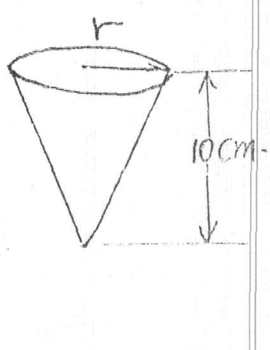
$R = ¿?$

$V = \frac{1}{3}\pi R^2 a$

$\frac{3V}{\pi a} = R^2$

$R = \sqrt{\frac{3(125)}{\pi(10)}}$

$R = 3.45cm$



33.-

$V_{cil} = 400 cm^3$

$R = 7,5 cm$

$a = ¿?$

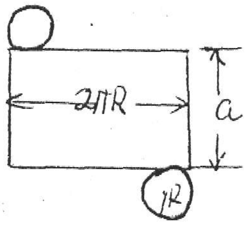
$A_L = ¿?$

$V_{cil} = \pi R^2 a$

$$a = \frac{V_{cil}}{\pi R^2}$$

$$a = \frac{400}{\pi (7,5)^2}$$

$$a = 2,26 cm$$



$A_L = 2\pi R a + 2\pi R^2$

$A_L = 2\pi (Ra + R^2)$

$A_L = 2\pi (7,5(2,26) + 7,5^2)$

$$A_L = 460 cm^2$$

0.15
1.15

SOLUCION EJERCICIOS PROPUESTOS

CAPITULO NUEVE

VECTORES EN EL PLANO Y ESPACIO

1.-

a) $A(2,1)$; $B(3,4)$

$\overrightarrow{AB} = (1, 3)$

$A(x_1, y_1)$

$B(x_2, y_2)$

$\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$

b) $A:(-3,0)$; $B(10,-5)$

$\overrightarrow{AB} = (13, -5)$

c) $A:(-1,-2)$; $B(2,1)$

$\overrightarrow{AB} = (3, 3)$

d) $A(0,1)$; $B(0,4)$

d) $\overrightarrow{AB} = (0, 3)$

e) $A:(2,0)$; $B:(-7,0)$

$\overrightarrow{AB} = (-9, 0)$

f) $A:(1,1,0)$; $B:(3,4,2)$

$\vec{AB}=(2,3,2)$

g) $A:(0,-3,4)$; $B:(1,-1,1)$

$\vec{AB}=(1,2,-3)$

h) $A:(3,2,3)$; $B:(3,1,3)$

$\vec{AB}=(0,-1,0)$

i) $A:(0,1,0)$; $B:(0,1,0)$

$\vec{AB}=(0,0,0)$ (vector nulo)

j) $A:(8,0,0)$; $B:(0,-1,2)$

$\vec{AB}=(-8,-1,2)$

2.- a) $\vec{V}_1=(\underbrace{2a+b-1}, \underbrace{3a-b})$ $\quad \vec{V}_2=(2b+3, 2)$

$2a+b-1=2b+3$ $\qquad$ (2) $3a-b=2$

$2a+b-2b=3+1$ $\qquad$ (1)(-1)+(2)

(1) $2a-b=4$ $\qquad$ $-2a+b=-4$

$3a-b=2$

$\boxed{a=-2}$

$3a-b=2$

$3a-2=b$

$b=3(-2)-2$

$\boxed{b=-8}$

b) $\vec{V}_1 = (c-2b+1,\ 4a,\ c)$ $\quad V_2 = (a-c, -a, 1-b)$

$c-2b+1 = a-c$

$1 = a-c-c+2b$

(1) $a+2b-2c=1$

$4a=-a$

$4a+a=0$

$5a=0$

$a=0$ (3)

$c=1-b$

$b+c=1$ (2)

(3) en (1)

(4) $2b-2c=1$

(2) $*(+2)+$ (4)

$$\begin{array}{r} +2b+2c=2 \\ 2b-2c=1 \\ \hline 4b=3 \end{array}$$

$b=3/4$

c) $\vec{V}_1 = (-a-b,\ -a-b+1)$ $\quad \vec{V}_2 = (2,3)$

$-a-b=2$

$-a-b+1=3$

$-a-b=2$

Son la misma ecuación

$-a=2+b$

$a=-(2+b)$

d) $\vec{V_1}=(4a+3b, a+2b)$; $\vec{V_2}=(3a, -b-1)$

$4a+3b=3a$

$4a-3a+3b=0$

$a+3b=0$

$a+2b=-b-1$

$a+2b+b=-1$

$\boxed{a+3b=-1}$

$a+3b=0 \longleftrightarrow a+3b=-1$

Es inconsistente el ejercicio

3.-

a) $\vec{V_1}=(a+b-3+c, -2a-b+2c, 3c)$

$\vec{V_2}=(b+c, 2c-3, \phi)$

No se puede igualar un vector en el espacio ($\vec{V_1}$) con un vector en el plano ($\vec{V_2}$)

b) $\vec{V_1}=(c-2b+1, 4a, c)$ $\quad$ $\vec{V_2}=(a-c, -a, 1-b)$

$1+\cancel{c}-2b=a-\cancel{c}$

$a+2b-c-c=0+1$

$a+2b-2c=1$

$\Downarrow$

$0+2b-2c=1$

$2(b-c)=1$

$b-c=1/2$ (1)

$4a=-a$

$4a+a=0$

$5a=0$

$a=0$

$c=1-b$

(2) $c+b=1$

(1) + (2)

$$\begin{array}{r} b-c=0{,}5 \\ b+c=1 \\ \hline 2b=1{,}5 \end{array}$$

$b=1{,}5/2$

$b=0{,}75$

$c+b=1$

$c=1-b$

$c=1-0{,}75$

$c=0{,}25$

c) $\vec{V}_1 = (10a + 2b - c,\ -a-b,\ 1)$ $\qquad$ $\vec{V}_2 = (b - 2c - 1,\ 3,\ b)$

$10a + 2b - c = b - 2c - 1$ $\qquad$ $-a-b=3$ $\qquad$ $1=b$

$10a + 2b - b - c + 2c = -1$ $\qquad \Leftarrow$ $b=1$

$\underline{10a + b + c = -1}$ $\qquad$ $-a = 3+b$

$a = -(3+b)$

$c = -1 - 10a - b$ $\qquad$ $a=-4$

$c = -1 - 10(-4) - 1$

$c = -2 + 40$

$c = 38$

d) $\vec{V} = (ab,\ ac,\ bc)$ $\qquad$ $\vec{V}_2 = (2,\ -b,\ -c)$

$ab = 2$ $\qquad$ $ac = -b$ $\qquad$ $bc = -c$

$a = \frac{2}{b}$ $\qquad$ $ac = -1$ $\qquad$ $bc + c = 0$

$a = -2$ $\qquad$ $c = -\frac{1}{a}$ $\qquad$ $c(b+1) = 0$

$c \neq 0 \therefore b+1 = 0$

$c = -\frac{1}{(-2)}$ $\qquad$ $b = -1$

$c = 1/2$

4.- $\vec{A} = (2, 1, 1)$

$\vec{B} = (-3, 5, 1)$

$\vec{C} = (2, -1, 0)$

$\vec{D}\ (-5, 6, 4)$

a) $2\vec{A} = (4, 2, 2)$

$-3\vec{B} = (9, -15, -3)$

$2(\vec{C} + \vec{D}) = (-6, 10, 8)$

$\vec{R} = (7, -3, 7)$

b) $\vec{A} = (2, 1, 1)$

$2\vec{C} = (4, -2, 0)$

$-2(\vec{C}+\vec{A}) = (-8, 0, -2)$

$-\vec{B} = (3, -5, -1)$

$\vec{R} = (1, -6, -2)$

c) $A-D = (7, -5, -3)$

$-2(A-D) = (-14, 10, 6)$

$B-2(A-D) = (-17, 15, 7)$

$3(B-2(A-D)) = (-51, 45, 21)$

$3C = (6, -3, 0)$

$3(B-2(A-D)) + 3C \Rightarrow (-51, 45, 21) + (6, -3, 0)$

$\Rightarrow (-45, 42, 21)$

d) $3A = (6, 3, 3)$

$\vec{X}+\vec{D} = (x_1-5;\ x_2+6;\ x_3+4)$

$3A-C = (4, 4, 0)$

$4(X+B) = 4(x_1-3, x_2+5, x_3+1)$

$\Rightarrow (4x_1-12;\ 4x_2+20;\ 4x_3+4)$

$(1, -6, 0)$

$$
\begin{array}{l}
\ \ 3A-C = (4, 4, 0) \\
+ \\
\ \ 4(X+B) = (4x_1-12;\ 4x_2+20;\ 4x_3+4) \\
\hline
\end{array}
$$

$3A-C+4(X+B) = (4x_1-8;\ 4x_2+24;\ 4x_3+4) = (x_1-5;\ x_2+6;\ x_3+4)$

$4x_1-8 = x_1-5$ $\quad$ $4x_2+24 = x_2+6$ $\quad$ $4x_3+\cancel{4} = x_3+\cancel{4}$

$4x_1-x_1 = -5+8$ $\quad$ $4x_2-x_2 = 6-24$ $\quad$ $4x_3-x_3 = 0$

$3x_1 = 3$ $\quad$ $3x_2 = -18$ $\quad$ $3x_3 = 0$

$x_1 = 1$ $\quad$ x_2 [illegible] $\quad$ x_3 [illegible]

e) $y+A=(y_1+2;\ y_2+1;\ y_3+1)$

$2y-C=(2y_1-2;\ 2y_2+1;\ 2y_3)$

$2(B+y)\Rightarrow 2(y_1-3;\ y_2+5;\ y_3+1)$

$2y-C+D=(2y_1-7;\ 2y_2+7;\ 2y_3+4)$

$\Rightarrow(2y_1-6;\ 2y_2+10;\ 2y_3+2)$

$3(2y-C+D)=(6y_1-21;\ 6y_2+21;\ 6y_3+12)$

$$\begin{array}{rl} & y+A=(y_1+2;\ y_2+1;\ y_3+1) \\ - & \underline{2(B+y)=(2y_1-6;\ 2y_2+10;\ 2y_3+2)} \\ & y+A-2(B+y)=(-y_1+8;\ -y_2-9;\ -y_3-1) \end{array}$$

$$(-y_1+8;\ -y_2-9;\ -y_3-1)=(6y_1-21;\ 6y_2+21;\ 6y_3+12)$$

$-y_1+8=6y_1-21$

$-y_1-6y_1=-21-8$

$-7y_1=-29$

$y_1=\frac{29}{7}$

$-y_2-9=6y_2+21$

$-y_2-6y_2=21+9$

$-7y_2=30$

$y_2=-30/7$

$-y_3-1=6y_3+12$

$-y_3-6y_3=12+1$

$-7y_3=13$

$y_3=-13/7$

$$y=\left(\frac{29}{7},-\frac{30}{7},-\frac{13}{7}\right)$$

5.- $\underbrace{a(2,1,1)\ ;\ b(-3,5,1)\ ;\ c(2,-1,0)\ ;\ d(-5,6,4)}_{(2,-10,7)}$

$2a-3b+2c-5d=2$

$a+5b-c+6d=-10$

$a+b\qquad+4d=7$

$$\Rightarrow \begin{array}{c} \\ (-2) \\ (-2) \end{array}\left(\begin{array}{cccc|c} 2 & -3 & 2 & -5 & 2 \\ 1 & 5 & -1 & 6 & -10 \\ 1 & 1 & 0 & 4 & 7 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 2 & -3 & 2 & -5 & 2 \\ 0 & -13 & 4 & -17 & 22 \\ 0 & -5 & 2 & -13 & -12 \end{array}\right)$$

Si falta una ecuación el sistema tendrá infinitas soluciones, si es combinación lineal.

6.- $\vec{V}=(2x, 3, y+1)$ $\quad \vec{W}=(x^2+1, 3, 2y)$

$$\vec{V}=\vec{W}$$

$$(2x, 3, y+1)=(x^2+1, 3, 2y)$$

$x^2+1=2x$ $\quad 3=3$ $\quad y+1=2y$

$1=2y-y$

$x^2-2x+1=0$ $\quad y=1$

$(x-1)^2=0$

$x=1$

$\vec{V}=(2, 3, 2)$ $\quad \vec{W}=(2, 3, 2)$

$\vec{V}+\vec{W}=(4, 6, 4)$

$\Rightarrow 2(2, 3, 2)$

a) CORRECTO

7.- $\vec{V}_1=(-3, 2, 4)$

$\vec{V}_2=(3/2, -1, -2)$ $\quad$ Si $\vec{V}_1 \parallel \vec{V}_2$

$\theta = 0°$

$$\theta = \cos^{-1} \frac{\vec{V}_1 \cdot \vec{V}_2}{V_1 \times V_2}$$

$$\theta = \cos^{-1} \frac{-\frac{9}{2}-2-8}{\sqrt{9+4+16}\cdot\sqrt{\frac{9}{4}+1+4}}$$

$$\theta = \cos^{-1} \frac{-\frac{29}{2}}{\sqrt{29}\sqrt{\frac{29}{4}}} \Rightarrow \theta = \cos^{-1} \frac{-\frac{29}{2}}{\frac{29}{2}}$$

$\theta = \cos^{-1}(-1) \Rightarrow \theta = 180°$

b) CORRECTO.

8.- a) $\vec{P} \parallel Q$

$\vec{P} \cdot \vec{Q} = PQ$

$\vec{P} = (K, 2, K)$ $\qquad \vec{P} \cdot \vec{Q} = 2K - 2 + 2K \Rightarrow 4K - 2$

$\vec{Q} = (2, -1, 2)$ $\qquad P = \sqrt{K^2 + 4 + K^2} \Rightarrow \sqrt{2K^2 + 4}$

$Q = \sqrt{2^2 + 1^2 + 2^2} = 3$

$\vec{P} \cdot \vec{Q} = PQ$

$4K - 2 = \sqrt{2K^2 + 4} \times 3$

$(4K-2)^2 = (3\sqrt{2K^2+4})^2$

$16K^2 - 16K + 4 = 9(2K^2 + 4)$

$16K^2 - 16K + 4 = 18K^2 + 36$

$18K^2 - 16K^2 + 16K + 36 - 4 = 0$

$2K^2 + 16K - 32 = 0$

$2(K^2 + 8K - 16) = 0$

$K^2 + 8K - 16 = 0$

$K = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(-16)}}{2(1)} \Rightarrow \dfrac{-8 \pm \sqrt{64 + 64}}{2}$

$K = \dfrac{\overset{4}{\cancel{-8}} \pm \overset{4}{\cancel{8}}\sqrt{2}}{\cancel{2}} \Rightarrow -4 \pm 4\sqrt{2}$

b) $\vec{A} \parallel \vec{B}$

$\vec{A} \cdot \vec{B} = AB$

$\vec{A} = (K, 1, -1)$ $\qquad \vec{A} \cdot \vec{B} = 3K + 1 - 4$ $\qquad |\vec{A}| = \sqrt{K^2 + 1 + 1} = \sqrt{K^2 + 2}$

$\vec{B} = (3, 1, 4)$ $\qquad \Rightarrow 3K - 3$ $\qquad B = \sqrt{3^2 + 1 + 16} = \sqrt{26}$

$\vec{A} \cdot \vec{B} = AB$

$(3K-3)^2 = (\sqrt{26}\sqrt{K^2+2})^2$

$9K^2 - 18K + 9 = 26(K^2 + 2)$

$9K^2 - 18K + 9 = 26K^2 + 52$

$26K^2 - 9K^2 + 18K + 52 - 9 = 0$

$17K^2 + 18K + 43 = 0$

$$K = \frac{-18 \pm \sqrt{18^2 - 4(17)(43)}}{2(17)}$$

NO HAY SOLUCION.

9.- $\vec{A} = (1, 1, 5)$; $\vec{B} = (3, 2, -3)$

(a)

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\theta = \cos^{-1} \frac{3 + 2 - 15}{\sqrt{1+1+25}\,\sqrt{9+4+9}} \Rightarrow \cos^{-1} \frac{-10}{\sqrt{27}\,\sqrt{22}}$$

$$\theta = \cos^{-1} \frac{-10}{\sqrt{27 \times 22}} \Rightarrow \cos^{-1} \frac{-10}{3\sqrt{66}}$$

$\theta = 114{,}22^\circ$

b) $\vec{C} = (2, 10, -2)$; $\vec{D} = (2, 2, 1)$

$$\theta = \cos^{-1} \frac{4 + 20 - 2}{\sqrt{4+100+4}\,\sqrt{4+4+1}} \Rightarrow \cos^{-1} \frac{22}{\sqrt{108}\,\sqrt{9}}$$

$$\theta = \cos^{-1} \frac{22}{3\sqrt{108}}$$

$\theta = 45{,}11^\circ$

c) $\vec{E}=(-2,-10,2)$ $\qquad$ $\vec{F}=(2,2,1)$

$$\theta = \cos^{-1} \frac{-4-20+2}{\sqrt{4+100+4}\,\sqrt{4+4+1}}$$

$$\theta = \cos^{-1} \frac{-22}{\sqrt{108}\,\sqrt{9}}$$

$$\theta = \cos^{-1} \frac{-22}{3\sqrt{108}}$$

$$\theta = 134{,}88^\circ$$

d) $\vec{P}=(0,0,1)$

$\vec{Q}=(1,0,1)$

$$\theta = \cos^{-1} \frac{0+0+1}{\sqrt{1}\,\sqrt{1+1}}$$

$$\theta = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

e) $\vec{R}=(1,0,0)$

$\vec{S}=(0,1,0)$

$$\theta = \cos^{-1} \frac{0+0+0}{\sqrt{1}\,\sqrt{1}}$$

$$\theta = \cos^{-1} 0$$

$$\theta = 90^\circ$$

10.- $\vec{V_1} = (10, 2, -1)$

$\vec{V_2} = (2, -8, 4)$

$$\vec{V_1} \perp \vec{V_2} \Rightarrow \vec{V_1} \cdot \vec{V_2} = 0$$

$$\Rightarrow 20 - 16 - 4$$

$$\Rightarrow 0$$

a) CORRECTO

11.- a) $\vec{A} = (4, 2K, 5)$

$\vec{B} = (3K, -10, 1)$ $\quad \vec{A} \cdot \vec{B} = 0$

$$12K - 20K + 5 = 0$$

$$-8K = -5$$

$$K = 5/8$$

b.) $\vec{P} = (5K+1, K-1, 0)$

$\vec{Q} = (1, K, 4)$

$$\vec{P} \cdot \vec{Q} = 0$$

$$5K + 1 + K(K-1) + 0 = 0$$

$$5K + 1 + K^2 - K = 0$$

$$K^2 + 4K + 1 = 0$$

$$K = \frac{-4 \pm \sqrt{4^2 - 4(1)(1)}}{2(1)} \Rightarrow \frac{-4 \pm \sqrt{12}}{2}$$

$$K = \frac{-\cancel{4}^{2} \pm \cancel{2}^{1}\sqrt{3}}{\cancel{2}} \Rightarrow K = -2 \pm \sqrt{3}$$

12.- $\vec{V} = (4, 2, t)$

$\vec{W} = (-1, 1, 2)$

$\vec{V} \perp \vec{W}$

$\vec{V} \cdot \vec{W} = 0$

$-4 + 2 + 2t = 0$

$2t = 2$

$t = 1$ b) CORRECTO

13.- $\vec{V_1} = (1, 2, x)$

$\vec{V_2} = (3, -1, -2)$

$\vec{V_1} \perp \vec{V_2}$

$\vec{V_1} \cdot \vec{V_2} = 0$

$3 - 2 - 2x = 0$

$-2x = -1$

$x = 1/2$ a) CORRECTO

14.- $\vec{V_1} = (V_{1x}, V_{1y}, V_{1z})$

$\vec{V_2} = (V_{2x}, V_{2y}, V_{2z})$

$\vec{V_1} \perp \vec{V_2}$

$\vec{V_1} \cdot \vec{V_2} = 0$

$V_{1x} V_{2x} + V_{1y} + V_{2y} + V_{1z} V_{2z} = 0$

$\vec{U} = \vec{V_1} + \lambda \vec{V_2}$ $\vec{W} = \vec{V_1} - \vec{V_2}$

$\vec{U} = (V_{1x} + V_{1y}, V_{1z}) + (\lambda V_{2x}, \lambda V_{2y}, \lambda V_{2z})$

$\vec{U} = (V_{1x} + \lambda V_{2x}, V_{1y} + \lambda V_{2y}, V_{1z} + \lambda V_{2z})$

$\vec{W} = (V_{1x} - V_{2x}, V_{1y} - V_{2y}, V_{1z} - V_{2z})$

$$U \perp W$$

$$\vec{U} \cdot \vec{W} = 0$$

$$(V_{1x} + \lambda V_{2x})(V_{1x} - V_{2x}) + (V_{1y} + \lambda V_{2y})(V_{1y} - V_{2y}) + (V_{1z} + \lambda V_{2z})(V_{1z} - V_{2z}) = 0$$

$$V_{1x}^2 - V_{1x}V_{2x} + \lambda V_{1x}V_{2x} - \lambda V_{2x}^2 + V_{1y}^2 - V_{1y}V_{2y} + \lambda V_{1y}V_{2y} - \lambda V_{2y}^2 + V_{1z}^2 - V_{1z}V_{2z} + \lambda V_{1z}V_{2z} - \lambda V_{2z}^2 = 0$$

$$V_{1x}^2 + V_{2y}^2 + V_{1z}^2 - \lambda V_{2x}^2 + \lambda V_{2y}^2 - \lambda V_{2z}^2 - \underbrace{(V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z})}_{0} + \lambda \underbrace{(V_{1x}V_{2x} + V_{1y}V_{2y} + V_{1z}V_{2z})}_{0} = 0$$

$$V_{1x}^2 + V_{1y}^2 + V_{1z}^2 - \lambda(V_{2x}^2 + V_{2y}^2 + V_{2z}^2) = 0$$

$$-\lambda\underbrace{(V_{2x}^2 + V_{2y}^2 + V_{2z}^2)}_{V_2^2} = -\underbrace{(V_{1x}^2 + V_{1y}^2 + V_{1z}^2)}_{V_1^2}$$

$$\lambda = \frac{V_1^2}{V_2^2} \Rightarrow \lambda = \left(\frac{V_1}{V_2}\right)^2$$

15.— $\vec{X} = -\vec{y}$

$\vec{X} + \vec{y} = 0 \qquad X + y \Rightarrow 2X$

$|\vec{x} + \vec{y}| \neq |\vec{x}| + |\vec{y}|$

b) CORRECTO

16.— $K\vec{X} \Rightarrow \vec{X} = (X_1K, X_2K, X_3K)$

$|\vec{X}| = \sqrt{X_1^2K^2 + X_2^2K^2 + X_3^2K^2}$

$|\vec{x}| = \sqrt{K^2(X_1^2 + X_2^2 + X_3^2)}$

$\Rightarrow K\sqrt{X_1^2 + X_2^2 + X_3^2}$

$\Rightarrow K|\vec{x}|$

a) CORRECTO

17.- $\vec{X} = (X_1, X_2, X_3)$

$X = \sqrt{X_1^2 + X_2^2 + X_3^2}$

$X \neq \vec{X_1} \cdot \vec{X_1}$

$\vec{X} \cdot \vec{X} = X_1^2 + X_2^2 + X_3^2$

b) CORRECTO

18.- $\vec{X} + \vec{Y} = (X_1 + Y_1, X_2 + Y_2, X_3 + Y_3)$

$\vec{X} - \vec{Y} = (X_1 - Y_1, X_2 - Y_2, X_3 - Y_3)$

$|\vec{X} + \vec{Y}|^2 + |\vec{X} - \vec{Y}|^2 = (X_1+Y_1)^2 + (X_2+Y_2)^2 + (X_3+Y_3)^2 + (X_1-Y_1)^2 + (X_2-Y_2)^2 + (X_3-Y_3)^2$

$4\vec{X} \cdot \vec{Y} = 4(X_1Y_1 + X_2Y_2 + X_3Y_3)$

$|\vec{X} + \vec{Y}|^2 + |\vec{X} - \vec{Y}|^2 \neq 4\vec{X} \cdot \vec{Y}$

b) CORRECTO

19.- a) $\vec{V} = (1, 1)$

$V = \sqrt{1+1} \Rightarrow \sqrt{2}$

$\vec{\mu}_V = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \Rightarrow \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

b) $\vec{V} = (-1, 3)$

$V = \sqrt{1 + 3^2} = \sqrt{10}$

$\vec{\mu}_V = \left(-\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\right) \Rightarrow \left(-\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10}\right)$

c) $\vec{v} = (10, 0)$

$v = \sqrt{10^2} = 10$ $\quad \vec{u}_v = (1, 0)$

d) $\vec{v} = (0, -2)$

$v = \sqrt{0 + 2^2} = 2$

$\vec{u}_v = (0, -1)$

e) $\vec{v} = (2, 0, 1)$

$v = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\vec{u}_v = \left(\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right) \Rightarrow \left(\frac{2\sqrt{5}}{5}, 0, \frac{\sqrt{5}}{5}\right)$

f) $\vec{v} = (0, -10, 1)$

$v = \sqrt{10^2 + 1^2} \Rightarrow \sqrt{101}$

$\vec{u}_v = \left(0, \frac{-10}{\sqrt{101}}, \frac{1}{\sqrt{101}}\right) \Rightarrow \left(0, \frac{-10\sqrt{101}}{101}, \frac{\sqrt{101}}{101}\right)$

g) $\vec{v} = (0, 0, 0) \Rightarrow$ NO HAY VECTOR

h) $\vec{v} = (0, -2, -2)$

$|\vec{v}| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

$\vec{u}_v = \left(0, \frac{-\cancel{2}}{\cancel{2}\sqrt{2}}, \frac{-\cancel{2}}{\cancel{2}\sqrt{2}}\right) \Rightarrow \left(0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

20.— a) $\vec{U}=(2,3)$ $\quad\quad$ $\vec{V}=(5,-1)$

$V=\sqrt{5^2+1}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V}\Rightarrow\frac{10-3}{\sqrt{25+1}}\Rightarrow\frac{7}{\sqrt{26}}\Rightarrow\frac{7\sqrt{26}}{26}$$

$$\vec{P}_{U/V}=P_{U/V}*\frac{\vec{V}}{V}\Rightarrow\frac{7\cancel{\sqrt{26}}}{26}\cdot\frac{(5,-1)}{\cancel{\sqrt{26}}}$$

$$\Rightarrow\left(\frac{35}{26},-\frac{7}{26}\right)$$

b) $\vec{U}=(-3,10)$ $\quad\quad$ $\vec{V}(2,0)$

$V=\sqrt{2^2+0}$

$V=2$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V}\Rightarrow\frac{-6+0}{2}\Rightarrow 3$$

$$\vec{P}_{U/V}=P_{U/V}\cdot\frac{\vec{V}}{V}\Rightarrow 3\,\frac{(2,0)}{2}\Rightarrow(3,0)$$

c) $\vec{U}=(1,8)$ $\quad\quad$ $\vec{V}=(0,3)$

$V=\sqrt{0+3^2}$

$V=3$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V}\Rightarrow\frac{0+24}{3}\Rightarrow 8$$

$$\vec{P}_{U/V}=P_{U/V}\cdot\frac{\vec{V}}{V}\Rightarrow 8\,\frac{(0,3)}{3}$$

$$\Rightarrow(0,8)$$

d) $\vec{U}=(3,1,3)$; $\vec{V}=(0,5,-1)$ $V=\sqrt{5^2+1}$

$V=\sqrt{26}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{0+5-3}{\sqrt{26}}$$

$$\Rightarrow \frac{2}{\sqrt{26}}$$

$$\vec{P}_{U/V}= P_{U/V} \cdot \frac{\vec{V}}{V} \Rightarrow \frac{2}{\sqrt{26}} \cdot \frac{(0,5,-1)}{\sqrt{26}}$$

$$\Rightarrow \frac{2}{26}(0,5,-1)$$

$$\Rightarrow \left(0, \frac{5}{13}, -\frac{1}{13}\right)$$

e) $\vec{U}=(1,-5,2)$; $\vec{V}=(2,0,4)$; $V=\sqrt{2^2+4^2}$

$\boxed{V=\sqrt{20}}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{2+0+8}{\sqrt{20}}$$

$$\Rightarrow \frac{10}{\sqrt{20}} \Rightarrow \frac{\cancel{10}\sqrt{20}}{2\cancel{0}}$$

$$\vec{P}_{U/V}= P_{U/V} \times \frac{\vec{V}}{V} \Rightarrow \frac{10}{\sqrt{20}}\,\frac{(2,0,4)}{\sqrt{20}} \Rightarrow \frac{10(2,0,4)}{2\cancel{0}}$$

$$\Rightarrow (1,0,2)$$

f) $\vec{U}=(7,1,3)$; $\vec{V}=(4,-3,2)$ $V=\sqrt{4^2+3^2+2^2}$

$V=\sqrt{29}$

$$P_{U/V}=\frac{\vec{U}\cdot\vec{V}}{V} \Rightarrow \frac{28-3+6}{\sqrt{29}} \Rightarrow \frac{31}{\sqrt{29}}$$

$$\vec{P}_{U/V} = P_{U/V} \times \frac{\vec{V}}{V} \Rightarrow \frac{31}{\sqrt{29}} \times \frac{(4,-3,2)}{\sqrt{29}} \Rightarrow \frac{31}{29}(4,-3,2)$$

21.- $\vec{U} = (1,0)$

$\vec{V} = (-1,0)$

$\underline{\underline{\vec{U}+\vec{V}=0}}$

$|\vec{U}+\vec{V}| \neq 2$

b) CORRECTO

22.- $\vec{U}=(1,0)$

$\vec{V}=(0,1)$

$\vec{U}\cdot\vec{V}=0$

entonces $\vec{U}\perp\vec{V}$

$\vec{U}+\vec{V}=(1,1)$

$|\vec{U}+\vec{V}|=\sqrt{1+1}$

$=\sqrt{2}$

a) CORRECTO

23.- $\vec{V}=(4,2,t)$

$\vec{W}=(-1,1,2) \Rightarrow W=\sqrt{1+1+4} \Rightarrow \sqrt{6}$

$$P_{V/W}=\frac{\vec{V}\cdot\vec{W}}{W} \Rightarrow \frac{-4+2+2t}{\sqrt{6}}$$

$$\Rightarrow \frac{-2+2t}{\sqrt{6}}=\sqrt{6}$$

$$-2+2t=\sqrt{6}\times\sqrt{6}$$

$$2t=6+2$$

$$t=\frac{8}{4}$$

$$t=4$$

a) VERDADERO

24.– $\vec{U}=(1,0)$ $\vec{V}=(0,1)$ > son perpendiculares

$$\vec{U}-\vec{V}=(1,-1)$$

$$|\vec{U}-\vec{V}|=\sqrt{1+1}$$

$$\Rightarrow \sqrt{2}$$

c) CORRECTO

25.– $\vec{V_1}=\hat{i}+4\hat{j}-2\hat{k}$

$\vec{V_2}=\hat{i}-\hat{j}+3\hat{k} \Rightarrow V_2=\sqrt{1+1+9}\Rightarrow\sqrt{11}$

$$P_{V_1/V_2}=\frac{\vec{V_1}\cdot\vec{V_2}}{V_2}\Rightarrow\frac{1-4-6}{\sqrt{11}}\Rightarrow\frac{9}{\sqrt{11}}\times\frac{\sqrt{11}}{\sqrt{11}}$$

$$\Rightarrow \frac{9\sqrt{11}}{11}$$

26.– a) $\vec{A}=(2,3,1)$

$\vec{B}=(-3,1,0)$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ -3 & 1 & 0\end{vmatrix}=(0-1)\hat{i}-(0+3)\hat{j}+(2+9)\hat{k}$$

$$\vec{A}\times\vec{B}=-\hat{i}-3\hat{j}+11\hat{k}\Rightarrow(-1,-3,11)$$

b) $\vec{A}=(2,1,-1)$

$\vec{B}=(-2,2,1)$

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ -2 & 2 & 1\end{vmatrix}=(1+2)\hat{i}-(2-2)\hat{j}+(4+2)\hat{k}$$

$$\vec{A}\times\vec{B}=(3,0,6)$$

c) $\vec{E}=(2,10,-2)$; $\vec{F}=(1,5,-1)$

$$\vec{E}\times\vec{F}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 10 & -2 \\ 1 & 5 & -1 \end{vmatrix}=(-10+10)\hat{i}-(-2+2)\hat{j}+(10-10)\hat{k}$$

$\vec{E}\times\vec{F}=(0,0,0)$ ⇒ son paralelos.

d) $\vec{P}=(0,0,1)$ $\vec{Q}=(1,0,1)$

$$\vec{P}\times\vec{Q}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix}=(0-0)\hat{i}-(0-1)\hat{j}+(0-0)\hat{k}$$

$\vec{P}\times\vec{Q}=(0,1,0)$

e) $\vec{R}=(1,0,0)$; $\vec{S}=(0,1,0)$

$$\vec{R}\times\vec{S}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}=(0-0)\hat{i}-(0-0)\hat{j}+(1-0)\hat{k}$$

$\vec{R}\times\vec{S}=(0,0,1)$

27.– Por definición

$$|\vec{A}\times\vec{B}|=AB\operatorname{sen}\theta$$

a) CORRECTO

28.– $\vec{V_1}\cdot(\vec{V_2}\times\vec{V_3})=0$

Forman un plano y no un cuerpo.

29.- $\vec{A} = \hat{i}+2\hat{j}+3\hat{k} \Rightarrow A=\sqrt{1+4+9} \Rightarrow \sqrt{14}$

$\vec{B} = 3\hat{i}+2\hat{j}+\hat{k} \Rightarrow B=\sqrt{9+4+1} \Rightarrow \sqrt{14}$

$\vec{C} = \hat{i}+4\hat{j}+\hat{k} \Rightarrow C=\sqrt{1+16+1} \Rightarrow \sqrt{18}$

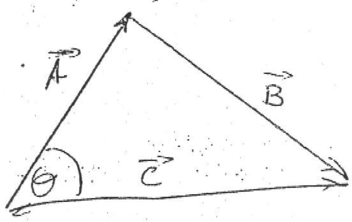
a) Si $\vec{A} \perp \vec{C}$ entonces $\vec{A}\cdot\vec{C}=0$

$\vec{A}\cdot\vec{C} = 1+8+3 \neq 0$

$12 \neq 0$; FALSO

b)

$\vec{A} =$



$$\theta = \cos^{-1}\frac{\vec{A}\cdot\vec{C}}{AC} \Rightarrow \cos^{-1}\frac{1+8+3}{\sqrt{14}\times\sqrt{18}} \Rightarrow \cos^{-1}\frac{12}{\sqrt{18\times14}}$$

$\theta = 40{,}89°$

el ángulo no es $60°$ o $120°$; por lo tanto NO FORMAN UN Δ EQUILATERO

c)

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = (2-6)\hat{i}-(1-9)\hat{j}+(2-6)\hat{k}$$

$\vec{A}\times\vec{B} = -4\hat{i}+8\hat{j}-4\hat{k}$

$$\vec{C}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 1 \\ 3 & 2 & 1 \end{vmatrix} = (4-2)\hat{i}-(1-3)\hat{j}+(2-12)\hat{k}$$

$\vec{C}\times\vec{B} = 2\hat{i}+2\hat{j}-10\hat{k}$

$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{B}) = -8+16+40$

$\neq 0$; FALSO

d) $(\hat{i}+2\hat{j}+3\hat{k})\cdot(-2\hat{i}-2\hat{j}+10\hat{k})$

$-2-4+30 \neq 1$; FALSO

e) $\theta = \cos^{-1} \dfrac{\vec{A}\cdot\vec{B}}{AB}$

$\theta = \cos^{-1} \dfrac{3+4+3}{\sqrt{14}\sqrt{14}} \Rightarrow \cos^{-1}\dfrac{10}{14}$

$\theta \neq 45^\circ$; FALSO

30.- $\vec{A} = -\hat{i}+2\hat{j}-\hat{k}$

$\vec{B} = 2\hat{i}+2\hat{j}-3\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ 2 & 2 & -3 \end{vmatrix} = (-6+2)\hat{i}-(3+2)\hat{j}+(-2-4)\hat{k}$$

$\vec{A}\times\vec{B} = -4\hat{i}-5\hat{j}-6\hat{k}$

$|\vec{A}\times\vec{B}| = \sqrt{16+25+36} \Rightarrow \sqrt{77}$

$\vec{U} = \dfrac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} \Rightarrow \dfrac{-4\hat{i}-5\hat{j}-6\hat{k}}{\sqrt{77}} \Rightarrow -\dfrac{\sqrt{77}}{77}(4,5,6)$

31.- FALTAN DATOS; NO SE PUEDE CALCULAR.

32.- a) $\vec{A} = (1,3,0)$

$\vec{B} = (-2,1,5)$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 0 \\ -2 & 1 & 5 \end{vmatrix} = (15-0)\hat{i}-(5-0)\hat{j}+(1+6)\hat{k}$$

$\Rightarrow 15\hat{i}-5\hat{j}+7\hat{k}$

área $= |\vec{A}\times\vec{B}| = \sqrt{15^2+5^2+7^2} = \sqrt{299}$

b) $\vec{C}=(1,-1,6)$

$\vec{D}=(-2,10,1)$

$$\vec{C}\times\vec{D}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -1 & 6\\ -2 & 10 & 1\end{vmatrix}=(-1-60)\hat{\imath}-(1+12)\hat{\jmath}+(10+12)\hat{k}$$

$$\vec{C}\times\vec{D}=(-61,-13,12)$$

$$\text{área}=|\vec{C}\times\vec{D}|=\sqrt{61^2+13^2+12^2}$$

$$\Rightarrow\sqrt{4034}$$

c) $\vec{E}=(1,-4,-2)$

$F=(0,7,0)$

$$\vec{E}\times\vec{F}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -4 & -2\\ 0 & 7 & 0\end{vmatrix}=(0+14)\hat{\imath}-(0-0)\hat{\jmath}+(7-0)\hat{k}$$

$$\vec{E}\times\vec{F}=(14,0,7)$$

$$\text{área}=|\vec{E}\times\vec{F}|=\sqrt{14^2+7^2}=\sqrt{245}$$

33.- a) $A=(1,1,1)$, $B(-1,2,3)$ $>$ $\vec{P}=(-2,1,2)$

$C(2,6,5)$ $>$ $\vec{Q}=(3,4,2)$

$$\vec{P}\times\vec{Q}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -2 & 1 & 2\\ 3 & 4 & 2\end{vmatrix}=(2-8)\hat{\imath}-(-4-6)\hat{\jmath}+(-8-3)\hat{k}$$

$$\vec{P}\times\vec{Q}=-6\hat{\imath}+10\hat{\jmath}-11\hat{k}$$

$$\text{area}\,\Delta=\frac{|\vec{P}\times\vec{Q}|}{2}=\frac{\sqrt{36+100+121}}{2}$$

$$\text{área}\,\Delta=\frac{\sqrt{257}}{2}$$

b) $A(1,4,3)$ $> \vec{P} = (-3,-2,-1)$
$B(-2,2,2)$ $> \vec{Q} = (1,4,3)$
$C(-1,6,5)$

$$\vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & -1 \\ 1 & 4 & 3 \end{vmatrix} = (-6+4)\hat{i} - (-9+1)\hat{j} + (-12+2)\hat{k}$$

$$\vec{P}\times\vec{Q} = (-2, 8, -10)$$

$$\text{área}\Delta = \frac{|\vec{P}\times\vec{Q}|}{2} \Rightarrow \frac{\sqrt{4+64+100}}{2} \Rightarrow \frac{\sqrt{4\cdot 42}}{2} \Rightarrow \frac{2\sqrt{42}}{\cancel{2}}$$

$$\Rightarrow \sqrt{42}$$

c) $A(0,0,1)$ $> \vec{P} = (-1,0,1)$
$B(-1,0,0)$ $> \vec{Q} = (-1,1,0)$
$C(0,1,0)$

$$\vec{P}\times\vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{vmatrix} = (0-1)\hat{i} - (0+1)\hat{j} + (-1+0)\hat{k}$$

$$\vec{P}\times\vec{Q} = (-1,-1,-1)$$

$$\text{área}\Delta = \frac{|\vec{P}\times\vec{Q}|}{2} \Rightarrow \frac{\sqrt{1+1+1}}{2} \Rightarrow \frac{\sqrt{3}}{2}$$

34.— a) $\vec{A} = (0,2,1)$
$\vec{B} = (3,-6,2)$
$\vec{C} = (1,0,3)$

$$\begin{vmatrix} 0 & 2 & 1 \\ 3 & -6 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & -6 & 2 \end{vmatrix} = 0+0+4-(-6+0+18)$$

$$\Rightarrow 4-12$$

$$\Rightarrow 8$$

b) $\vec{A}=(1,0,0)$
$\vec{B}=(0,-1,0)$
$\vec{C}=(1,7,2)$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 7 & 2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = -2+0+0-(0+0+0$$

$\Rightarrow 2$

c) $\vec{A}=(2,1,4)$
$\vec{B}=(-1,1,0)$
$\vec{C}=(3,2,4)$

$$\begin{vmatrix} 2 & 1 & 4 \\ -1 & 1 & 0 \\ 3 & 2 & 4 \\ 2 & 1 & 4 \\ -1 & 1 & 0 \end{vmatrix} = 8-8+0-(12+0-4)$$

$\Rightarrow 0-(8)$

$\Rightarrow 8$

35.- $\vec{V}_1=(1,1,0)$
$\vec{V}_2=(0,1,1)$
$\vec{V}_3=(1,1,1)$

$$\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 1+0+1-(0+1+0)$$

$\Rightarrow 2-1$

$\Rightarrow 1$

b) CORRECTO

36.- $P(a, 2a, 3a)$

$Q(3a, a, -2a)$ $\qquad \vec{PQ} = (2a, -a, -5a)$

$R(-a, a, 2a)$ $\qquad \vec{PR} = (-2a, -a, -a)$

$S(2a, 5a, a)$ $\qquad \vec{PS} = (a, 3a, -2a)$

$$\text{Volumen} \Rightarrow \begin{vmatrix} 2a & -a & -5a \\ -2a & -a & -a \\ a & 3a & -2a \\ 2a & -a & -5a \\ -2a & -a & -a \end{vmatrix} \Rightarrow 4a^3+30a^3+a^3-(5a^3-6a^3-4a^3)$$

$$\Rightarrow 35a^3-(-5a^3)$$

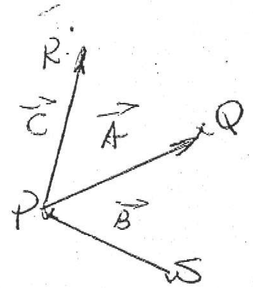
$$\Rightarrow 40a^3$$

37.- $P(2,1,3)$

$Q(4,-2,2)$

$R(1,1,3)$

$S(-4,0,2)$



$\vec{A} = (2, -3, -1)$

$\vec{B} = (-6, -1, -1)$

$\vec{C} = (-1, 0, 0)$

$$\text{Volumen} \begin{vmatrix} 2 & -3 & -1 \\ -6 & -1 & -1 \\ -1 & 0 & 0 \\ 2 & -3 & -1 \\ -6 & -1 & -1 \end{vmatrix} = 0+0-3-(-1+0+0)$$

$$\Rightarrow -3+1$$

$$\Rightarrow 2$$

$$\text{área base} = |\vec{A}\times\vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & -1 & -1 \end{vmatrix} = (3-1)\hat{i} - (-2-6)\hat{j} + (-2-18)\hat{k}$$

$$\vec{A}\times\vec{B} = 2\hat{i} + 8\hat{j} - 20\hat{k}$$

$$\text{área base} = |\vec{A}\times\vec{B}| = \sqrt{4+64+400} \Rightarrow \sqrt{468}$$

$$\Rightarrow \sqrt{4\times 117} = 2\sqrt{117}$$

$$\text{Volumen} = (\text{área base}) \times \text{altura}$$

$$\text{altura} = \frac{\text{Volumen}}{\text{área base}}$$

$$\text{altura} = \frac{\cancel{2}}{\cancel{2}\sqrt{117}} \times \frac{\sqrt{117}}{\sqrt{117}}$$

$$\text{altura} = \frac{\sqrt{117}}{117}$$

135

SOLUCION EJERCICIOS PROPUESTOS CAPITULO DIEZ MATEMATICAS GEOMETRIA ANALITICA

10.1 RECTAS EN EL PLANO

a) $(1,2)$; $(-2,3)$

$d = \sqrt{(-2-1)^2+(3-2)^2}$

$d = \sqrt{9+1}$

$d = \sqrt{10}$

$\bar{X} = \dfrac{-2+1}{2} \Rightarrow -1/2$

$\bar{Y} = \dfrac{3+2}{2} \Rightarrow 5/2$

$PM = (-1/2, 5/2)$

b) $(0,3)$; $(1,5)$

$d = \sqrt{(1-0)^2+(5-3)^2}$

$d = \sqrt{1+4}$

$d = \sqrt{5}$

$\bar{X} = \dfrac{1+0}{2} = 1/2$

$\bar{Y} = \dfrac{5+3}{2} = 4$

$PM = (1/2, 4)$

c) $(-2,-1)$; $(-3,4)$

$d=\sqrt{(-3+2)^2+(4+1)^2}$

$d=\sqrt{1^2+5^2}$

$d=\sqrt{26}$

$\bar{x}=\frac{-3-2}{2}=-5/2$

$\bar{y}=\frac{4-1}{2}=\frac{3}{2}$

$PM=(-5/2\ ,\ 3/2)$

d) $(2,4)$; $(3,4)$

$d=\sqrt{(3-2)^2+(4-4)^2}$

$d=\sqrt{1^2+0^2}$

$d\neq 1$

$\bar{x}=\frac{3+2}{2}=5/2$

$\bar{y}=\frac{4+4}{2}=4$

$PM=(5/2\ ,\ 4)$

e) $(-4,6)$; $(-7,6)$

$d=\sqrt{(-7+4)^2+(6-6)^2}$

$d=\sqrt{3^2+0^2}$

$d=3$

$\bar{x}=\frac{-7-4}{2}=-11/2$

$\bar{y}=\frac{6+6}{2}=6$

$PM=(-11/2\ ;\ 6)$

f.- $(a,1)$; $(2a,1)$

$d=\sqrt{(2a-a)^2+(1-1)^2}$

$\bar{X}=\frac{2a+a}{2}=\frac{3a}{2}$

$d=\sqrt{a^2+0^2}$

$\bar{Y}=\frac{1+1}{2}=1$

$d=a$

$PM=(3a/2;1)$

g.- $(5a,2a)$; $(a,3a)$

$d=\sqrt{(a-5a)^2+(3a-a)^2}$

$\bar{X}=\frac{a+5a}{2}=\frac{3a}{2}$

$d=\sqrt{(4a)^2+(2a)^2}$

$\bar{Y}=\frac{3a+2a}{2}=5/2\,a$

$d=\sqrt{16a^2+4a^2}$

$PM=(3a/2\,;\,5/2\,a)$

$d=\sqrt{20a^2}$

$d=\sqrt{4}\sqrt{a^2}\sqrt{5}$

$d=2a\sqrt{5}$

2.- a) $(-2,3)$; $(-3,1)$

$$m = \frac{1-3}{-3+2} \Rightarrow \frac{-2}{-1} \Rightarrow 2$$

Pto. Pendiente

$$y - y_1 = 2(x - x_1)$$

$$y - 3 = 2(x+2) \Rightarrow$$

$$y - 3 = 2x + 4$$

$$y = 2x + 7$$

Ec. Paramétrica

$$\frac{y-3}{2} = \frac{2x+4}{2}$$

$$\frac{y-3}{2} = \frac{x+2}{1} = t$$

Ec. General.

$$y = 2x + 7$$

$$2x - y + 7 = 0$$

b) $(2,0)$; $(4,5)$

$$m = \frac{5-0}{4-2} \Rightarrow \frac{5}{2}$$

$$y - y_1 = \frac{5}{2}(x - x_1)$$

$$y - 0 = \frac{5}{2}(x-2) \Rightarrow \Rightarrow \Rightarrow$$

$$y = \frac{5x}{2} - 5$$

Ec. Pto.-Pendiente

$$2y = 5(x-2)$$

$$2y = 5x - 10$$

$$5x - 2y - 10 = 0$$

Ec. General

$$\frac{2y}{10} = \frac{5x - 10}{10}$$

$$\frac{y}{5} = \frac{x-2}{2} = t$$

c) $(-1,1)$; $(1,1)$

$$m = \frac{1-1}{1+1} \Rightarrow 0$$

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = 0(x - x_1)$$

$y - 1 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$y = 1$

Ec. Pto/pendiente

d) $(-2,4)$; $(1,4)$

$$m = \frac{4-4}{1+2} \Rightarrow 0$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 0(x + 2)$$

$y - 4 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$y = 4$

Ec. Pto/Pendiente

e) $(-4,5)$; $(-4,2)$

$m = \dfrac{2-5}{-4+4} \Rightarrow$ NO HAY PENDIENTE

$x - x_1 = 0$

$x + 4 = 0$ Ec. General

Ec. Paramétrica

$\Downarrow$

$x = -4$

3.- $P(1,3)$

$m = 9$

$y - y_1 = m(x - x_1)$

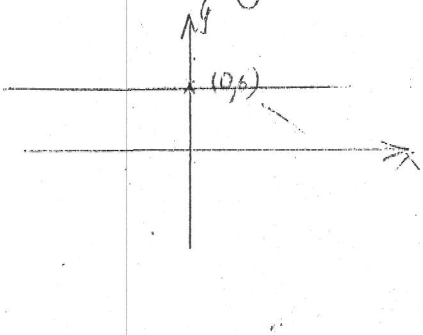
$y - 3 = 9(x - 1)$

$y - 3 = 9x - 9$

$y = 9x - 6$

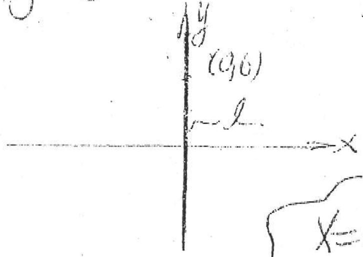
4.- $P(0,6)$

a) Paralela al eje x.



$y = 6$

b) Paralela al eje y



$x = 0$

c) Paralela a la recta $3x - 2y = 6$

$$-2y = -3x + 6$$

$$y = \frac{3}{2}x + 3$$

$$m = \frac{3}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{3}{2}(x - 0)$$

$$2(y - 6) = 3(x - 0)$$

$$2y - 12 = 3x$$

$$3x - 2y + 12 = 0$$

d) Perpendicular

$$-2x + y - 1 = 0$$

$$y = 2x + 1$$

$$m_1 = 2$$

$$m_2 = -\frac{1}{2} \qquad y - y_1 = -\frac{1}{2}(x - x_1)$$

$$y - 6 = -\frac{1}{2}(x - 0)$$

$$2(y - 6) = -x$$

$$2y - 6 = -x \Rightarrow x + 2y - 6 = 0$$

5.- a) $l: ax+by+c=0$ $\vec{v}=(a,b)$

$$ax=-by-c$$

$$\frac{ax}{ab}=\frac{-by-c}{ab}$$

$$\frac{x}{b}=-\frac{y+c/b}{a}=t$$

$$\frac{x}{b}=\frac{y+c/b}{-a}=t$$

un vector paralelo a l es: $(b,-a)$

(Falso)

b) $\vec{v}=(2a,2b)$

$$l: ax+by+c=0$$

$$ax=-by-c$$

$$\frac{ax}{ab}=-\frac{(by+c)}{ab}$$

$$\frac{x}{b}=-\frac{y+c/b}{a}=t$$

un vector paralelo a l: $(b,-a)$ $\vec{v}_1$

si $\vec{v}\perp\vec{v}_1$ entonces $\vec{v}\cdot\vec{v}_1=0$

$$(2a,2b)\cdot(b,-a)=0$$

$$2ab-2ab=0$$

$$0=0$$

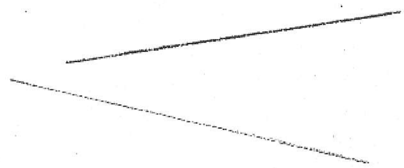
CORRECTO

c) $l_1 = A_1x + B_1y + C_1 = 0$

$l_2 = A_2x + B_2y + C_2 = 0$

$l_1 \cap l_2 = \phi$

no se intersectan.



no necesariamente son paralelas.

$\underbrace{A_1 = A_2K \quad ; \quad B_1 = B_2K \quad ; \quad C_1 = C_2K}$

Se cumple cuando son rectas paralelas.

(FALSO)

d) $y = kx + b$ → la pendiente es k

$y = mx + b$

VERDADERO

e) l_1: $x = x_0 + at$ $\qquad$ l_2: $y = y_0 + bt$

$l_1 \cap l_2 = (x_0, y_0)$

Al existir intersección entre $l_1 \wedge l_2$ se podrá trazar una recta paralela al vector (a, b), tomando en cuenta que por un punto pasan infinitos rectas.

6.- recta vertical.- $\boxed{x-k=0}$

$x+1/2=0$

7.- $P(5,-1)$

$\parallel l: 2x+y-1=0$

$y=-2x+1$

$m_1=-2$

$y-y_1=m(x-x_1)$

$y+1=-2(x-5)$

$y+1=-2x+10$

$2x+y-9=0$

8.- $\vec{V_1}=(2,-3)$

$\vec{V_2}=(4,1)$

$\vec{V_1}-\vec{V_2}=(-2,-4)$ $\quad$ $P(1,2)$

$$\frac{x-1}{\cancel{-2}}=\frac{y-2}{-4}=t$$

$$x-1=\frac{y-2}{2}$$

$2(x-1)=y-2$

$2x-\cancel{2}=y-\cancel{2}$

$2x-y=0$

9.- $P(3,-5)$

$l \parallel \vec{v}=(-4,2)$

$$\frac{x-3}{\cancel{-4}_{2}} = \frac{y+5}{\cancel{2}_{1}} = t$$

$$\frac{x-3}{-2} = y+5$$

$$x-3=-2(y+5)$$

$$x-3=-2y-10$$

$$x+2y+7=0$$

10.- $P_0(1,2)$

$l \perp \vec{v}=(-2,-4)$

$\vec{v}_1 \perp \vec{v} \Rightarrow l \parallel \vec{v}_1$

$(a,b)\cdot(-2,-4)=0$

$-2a-4b=0$

$-2(a+2b)=0$

$a=-2b$

$\vec{v}_1=(-2b,b) \parallel l$

$$\frac{x-1}{-2b} = \frac{y-2}{b} = t$$

$$\frac{x-1}{-2\cancel{b}} = \frac{y-2}{\cancel{b}}$$

$$\frac{x-1}{-2} = y-2$$

$x-1=-2(y-2)$

$x-1=-2y+4 \Rightarrow x+2y-5=0$ b) CORRECTO

11.- $P_1(-1,1)$

$P_2(3,9)$

$$m=\frac{9-1}{3+1} \Rightarrow \frac{8}{4} \Rightarrow 2$$

$y-y_1=2(x-x_1)$

$y-1=2(x+1)$

$y-1=2x+2$

$2x-y+3=0$ $\quad P(x,0) \Rightarrow 2x-0+3=0$

$2x=-3$

$x=-3/2$

a) CORRECTO

12.- $l_1: 2x-5y+6=0$ $\quad l_2: 4x-2y-5=0$

$5y=2x+6$ $\quad 2y=4x-5$

$P(x,0)$ $\quad y=2x-5/2$

$m=2$

$0=2x+6$

$2x=-6$ $\quad y-y_1=m(x-x_1)$

$x=\frac{-6}{2}$ $\quad y-0=2(x+3)$

$x=-3$ $\quad y=2x+6$

$P(-3,0)$

$2x-y+6=0$

d) CORRECTO.

13.- $l: 3x+2y-5=0$

$2y = -3x+5$

$y = -\frac{3}{2}x + \frac{5}{2}$ $\begin{cases} m = -3/2 \\ \text{Intersección eje y } (0, 5/2) \\ \text{Intersección eje x } (5/3, 0) \end{cases}$

a) Falso

b) Falso

c) VERDADERO; $2x-3y-15=0$

$3y = 2x-15$

$y = \frac{2}{3}x - 5$

$m_1 = \frac{2}{3}$ $\qquad m_1 \cdot m_2 = 1$

$\frac{2}{3}\left(-\frac{3}{2}\right) = 1$

14.- $f(x) = ax+b$

a) $a=0$

$\underbrace{f(x)=b}$ ⇒ función constante

⇓

función creciente; FALSO

b) $b=0$

$f(x) = ax$ ⇒ función lineal

⇓

NO ES FUNCION PAR; FALSO

c) $a<0$

$f(x) = -ax+b$ ⇒ función lineal

⇓

es función decreciente; VERDADERO

d) FALSO.

15.- $P_1(-3,0)$
$P_2(0,6)$

$$m = \frac{6-0}{0+3} \Rightarrow \frac{6}{3}$$

$m = 2$

$y - y_1 = 2(x - x_1)$

$y - 0 = 2(x+3)$

$y = 2x+6$ c) CORRECTO

16.-
$t = 6$
$P_1(0, 59000)$
$P_2(6, 95000)$

$$m = \frac{95000 - 59000}{6-0}$$

$m = 6000$

$y - y_1 = 6000(x - x_1)$

$y - 59000 = 6000(x-0)$

$y - 59000 = 6000x$

$y = 6000x + 59000$

a) CORRECTO.

17.- $P_0 = 80.000 \Rightarrow P_1(0, 80.000)$

$t = 10$ años

$P = 2000 \Rightarrow P_2(10, 2000)$

$$m = \frac{2000 - 80000}{10 - 0} \Rightarrow \frac{-7800\cancel{0}}{1\cancel{0}}$$

$$m = -7800$$

$$y - y_1 = m(x - x_1)$$

$$y - 2000 = -7800(x - 10)$$

$$y - 2000 = -7800x + 78000$$

$$y = -7800x + 80.000$$

18.-

$$y = -7800x + 80.000$$

$$x = 5$$

$$y = -7800(5) + 80.000$$

$$y = -39000 + 80.000$$

$y = 41000$ b) CORRECTO

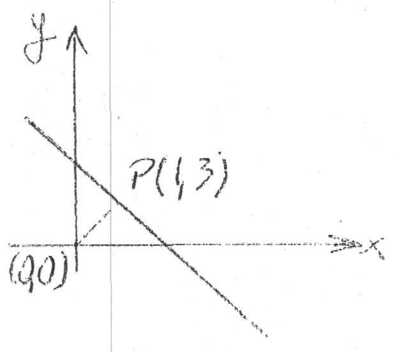
19.- $l: 3x - 4y + 12 = 0$

$P(4, -1)$

$$d = \frac{3(4) - 4(-1) + 12}{\sqrt{3^2 + 4^2}}$$

$$d = \frac{28}{5}$$

20.-

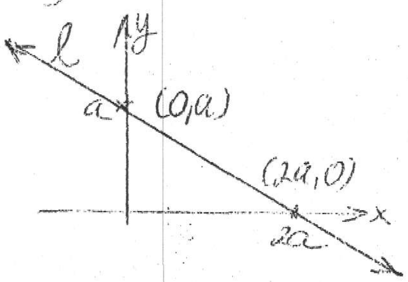


$$d = \sqrt{(1-0)^2 + (3-0)^2}$$

$$d = \sqrt{10}$$

a) FALSO; faltan datos
b) FALSO; faltan datos
c) FALSO; faltan datos
d) FALSO; faltan datos
e) CORRECTO

21.- $P\left(\frac{3}{2}, \frac{1}{2}\right) \in l.$



$$m = \frac{0-a}{2a-0} \Rightarrow -\frac{\cancel{a}}{2\cancel{a}}$$

$$m \Rightarrow -\frac{1}{2}$$

$$y - y_1 = m \cdot (x - x_1)$$

$$y - \frac{1}{2} = -\frac{1}{2}\left(x - \frac{3}{2}\right)$$

$$\frac{2y-1}{\cancel{2}} = -\frac{1}{\cancel{2}}\left(x - \frac{3}{2}\right)$$

$$2y - 1 = -\left(\frac{2x-3}{2}\right)$$

$$\Rightarrow 2(2y-1) = -(2x-3)$$

$$4y - 2 = -2x + 6$$

$$2x + 4y - 8 = 0 \Rightarrow x + 2y - 4 = 0$$

22.- $l_1 = 3x - 4y = 1 \rightarrow P(\frac{1}{3}, 0)$

$l_2 = 3x - 4y = 10$

distancia punto-recta P, l_2

$$d = \frac{|(\frac{1}{3})(3) + 0(-4) - 10|}{\sqrt{3^2 + 4^2}}$$

$$d = \frac{9}{5}$$

23.- $m_1 = -2$

$l: ax + by + c = 0$

$d = 4$

$P(-2, 3)$

$$m = +\frac{a}{b} = +2$$

$$\boxed{a = 2b}$$

$$d = \frac{-2a + 3b + c}{\sqrt{a^2 + b^2}} = \pm 4$$

$$\Rightarrow \frac{-2(2b) + 3b + c}{\sqrt{(2b)^2 + b^2}} = \pm 4$$

$$\Rightarrow \frac{-b + c}{\sqrt{5}\, b} = \pm 4$$

$$-b + c = \pm 4\sqrt{5}\, b$$

$$\boxed{c = \pm 4\sqrt{5}\, b + b}$$

$ax + by + c = 0$

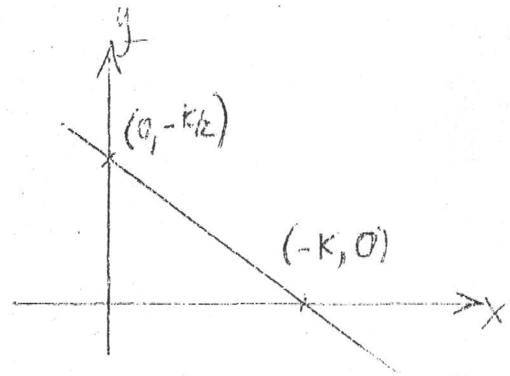
$\rightarrow 2bx + by \pm 4\sqrt{5}\, b + b = 0$

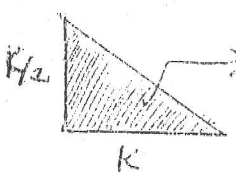
$b(2x + y \pm 4\sqrt{5} + 1) = 0$

$2x + y \pm 4\sqrt{5} + 1 = 0$

24.- $l: x+2y+k=0$

$\text{área} = 16\,u^2$





$\text{área} = \frac{1}{2} ab.$

$\text{área} = \frac{1}{2} k\left(\frac{k}{2}\right) = 16$

$\Rightarrow \frac{k^2}{4} = 16$

$\Rightarrow k^2 = 16 * 4$

$\Rightarrow k = \sqrt{16 * 4}$

$\Rightarrow k = 8$

25.- $l: 3x + ky - 2 = 0 \qquad P(1,1)$

a) $d = 2$

$d = \frac{3(1) + k(1) - 2}{\sqrt{3^2 + 1^2}} = 2$

$\frac{1+k}{\sqrt{10}} = 2$

$k = 2\sqrt{10} - 1$

b) $d < 5$

$d = \frac{3(1) + k(1) - 2}{\sqrt{3^2 + 1^2}} < 5$

$\frac{k+1}{\sqrt{10}} < 5$

$k + 1 < 5\sqrt{10}$

$k < 5\sqrt{10} - 1$

c) $d > 1$

$$d = \frac{3(1) + K(1) - 2}{\sqrt{3^2+1^2}} > 1$$

$$\frac{1+K}{\sqrt{10}} > 1$$

$$K+1 > \sqrt{10}$$

$$K > \sqrt{10} - 1$$

26.– $x^2+y^2-4x+6y-3=0$

$(x^2-4x+\underline{4})+(y^2+6y+\underline{9})-3=0+13$

$(x-2)^2+(y+3)^2=13+3$

$(x-2)^2+(y+3)^2=16$

$C(2,-3)$

$R=4$ d) CORRECTO

27.– $P_1(5,-2)$

$P_2(-3,-2)$

$\bar{x} = \frac{5-3}{2} = 1$

$\bar{y} = \frac{-2-2}{2} = -2$

$C(1,-2)$

$R = d = \sqrt{(1-5)^2+(-2+2)^2}$

$R = \sqrt{4^2+0}$

$R = 2$

$(x-1)^2+(y+2)^2=4$

$x^2-2x+1+y^2+4y+4=4$

$x^2+y^2-2x+4y+1=0$

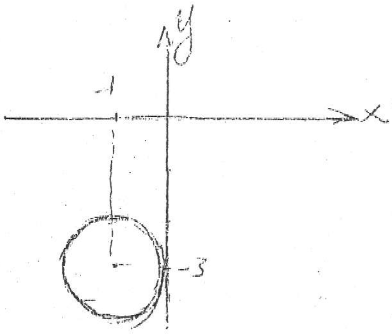
28.– $C_1: x^2+y^2-2x+6y+9=0$

$C_2: x^2+y^2-4x-4y+4=0$

$\underline{\underline{C_1}}$

$(x^2-2x+1)+(y^2+6y+9)\cancel{+9}=1+\cancel{9}$

$(x-1)^2+(y+3)^2=1$

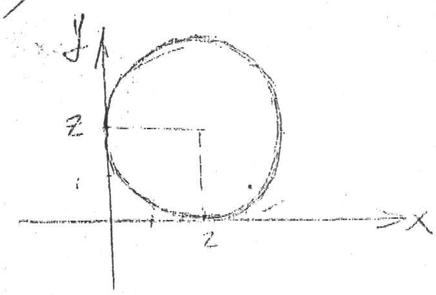


$\begin{cases} C:(1,-3) \\ R=1 \end{cases}$

$\underline{\underline{C_2}}$

$(x^2-4x+4)+(y^2-4y+4)\cancel{+4}=\cancel{4}+4$

$(x-2)^2+(y-2)^2=4$



$\begin{cases} C:(2,2) \\ R=2 \end{cases}$

a) FALSO

b) FALSO

c) CORRECTO

d) FALSO

e) FALSO

29.– $(x+1)^2+(y-9)^2=3$

$C(-1,9)$

$l: 3x-6y+1=0$

$6y=3x+1$

$y=\frac{3}{6}x+\frac{1}{6}$

$y=\frac{1}{2}x+\frac{1}{6}$

$m_1=\frac{1}{2}$

$l \perp l_1$

$m_1 \times m=-1$

$\left(\frac{1}{2}\right)m=-1$

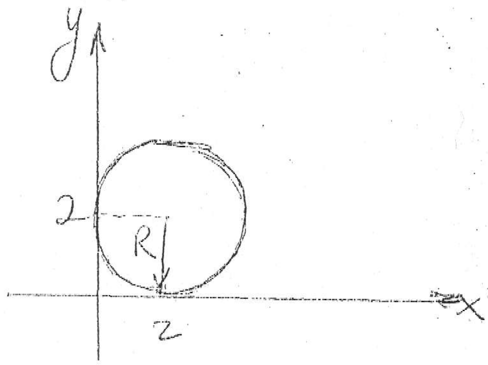
$\underline{m=-2}$

$(y-9)=-2(x+1)$

$y-9=-2x-2$

$2x+y-7=0$

30.-



$C(2,2)$

$R=2$

$(x-2)^2+(y-2)^2=2^2$

$x^2-4x+4+y^2-4y+4=4$

$x^2+y^2-4x-4y+4=0$

31.- $l: 2x-y+3=0$

$y=2x+3$

$(0,3)$ $(-1/5, 13/5)$

$(0,1)$ $(-1/5, 7/5)$

$(0,3)$ ✓

$y=0+3$

$y=3$

$(0,1)$ ✗

$(-1/5, \frac{13}{5})$

$y=2\left(-\frac{1}{5}\right)+3$

$y=-\frac{2}{5}+3$

$y=\frac{13}{5}$

$c: x^2+y^2-3x-4y+3=0$

$x^2+(2x+3)^2-3x-4(2x+3)+3=0$

$x^2+4x^2+12x+9-3x-8x-12+3=0$

$5x^2+x=0$

$x(5x+1)=0$

$x=0$ $5x+1=0$

$5x=-1$

$x=-1/5$

$x^2+y^2-3x-4y+3=0$

$0+y^2-4y+3=0$

$(y-3)(y-1)=0$

$y=3$ $y=1$

$\left(-\frac{1}{5}\right)^2+y^2-3\left(-\frac{1}{5}\right)-4y+3=0$

$\frac{1}{25}+y^2+\frac{3}{5}-4y+3=0$

$y^2-4y+\frac{91}{25}=0$

$y=\frac{4\pm\sqrt{16-4(1)(91/25)}}{2(1)}$

$y=\frac{4\pm\frac{6}{5}}{2}$

$y=\frac{\frac{20\pm6}{5}}{\frac{2}{1}}$

$y=\frac{20\pm6}{10}=\frac{10\pm3}{5}$

2 puntos de la recta coinciden con la circunferencia

a) CORRECTO

32.- $C(4,1)$

tangente a $l: 2x-3y=15$.

$$d=\frac{|2(4)-3(1)-15|}{\sqrt{2^2+3^2}}$$

$$d=R=\frac{10}{\sqrt{13}}$$

$$(x-4)^2+(y-1)^2=\left(\frac{10}{\sqrt{13}}\right)^2$$

33.- $P(0,0)$

$$C_1 = x^2+y^2-6x+2y-5=0 \Rightarrow C$$

$$(x^2-6x+9)+(y^2+2y+1)=5+9+1$$

$$\underbrace{(x-3)^2+(y+1)^2=15}_{C:(3,-1)}$$

$$(x-3)^2+(y+1)^2=R^2$$

$$(0-3)^2+(0+1)^2=R^2$$

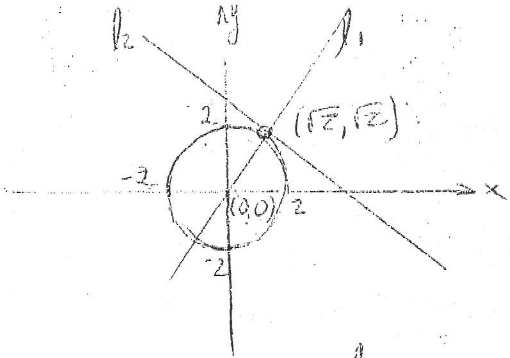
$$R^2=10$$

$$(x-3)^2+(y+1)^2=10$$

$$x^2-6x+\cancel{9}+y^2+2y+\cancel{1}=\cancel{10}$$

$$x^2+y^2-6x+2y=0$$

34.-



l_1

$m = \frac{\sqrt{2}-0}{\sqrt{2}-0} = 1$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 0)$

$y = x$

l_2

$m_2 = -\frac{1}{m_1}$

$m_2 = -1$

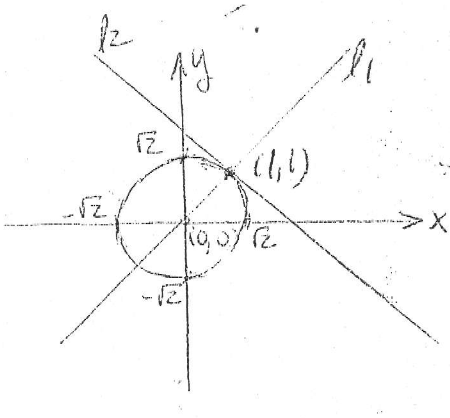
$y - y_1 = m_2(x - x_1)$

$y - \sqrt{2} = -(x - \sqrt{2})$

$y - \sqrt{2} = -x + \sqrt{2}$

$x + y - 2\sqrt{2} = 0$

35.-



l_1

$P_1(1,1)$

$P_0(0,0)$

$m = \frac{1-0}{1-0}$

$m = 1$

$l_1 \perp l_2$

$l_2 =$

$m_2 = -\frac{1}{m_1}$

$m_2 = -1$

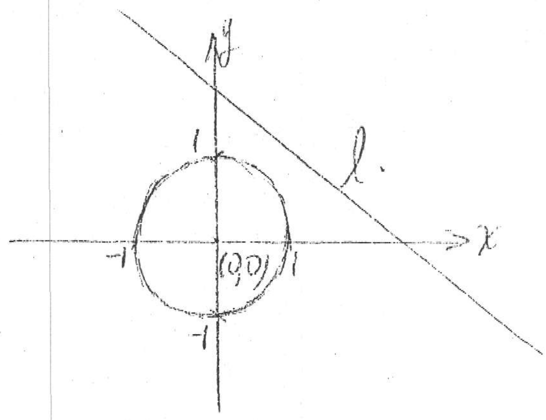
$y - y_1 = m(x - x_1)$

$y - 1 = -1(x - 1)$

$y - 1 = -x + 1$

$x + y - 2 = 0$

36.—



distancia pto-recta

$l: 4x+3y-15=0$

$P:(0,0)$

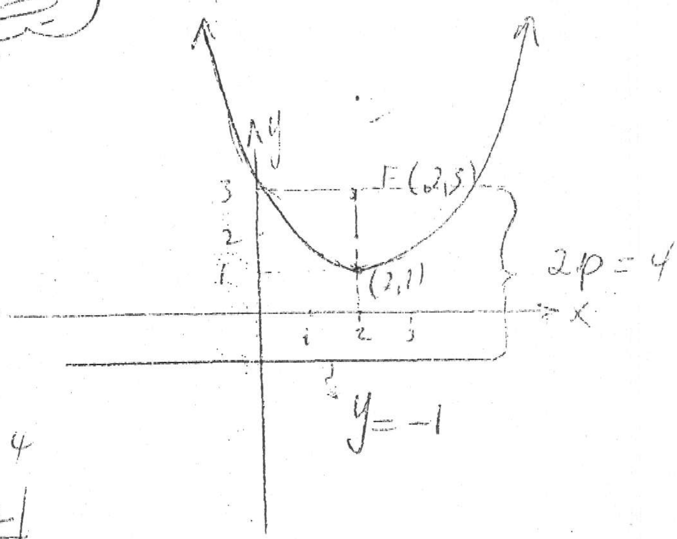
$$d=\frac{|4(0)+3(0)-15|}{\sqrt{4^2+3^2}}=\frac{\cancel{15}}{\cancel{5}}=3$$

distancia $=3-R$

$\Rightarrow 2$

37.- $F(2,3)$

$y=-1$



$2p=4$

$\boxed{p=2}$

$$(x-h)^2=4p(y-k)$$

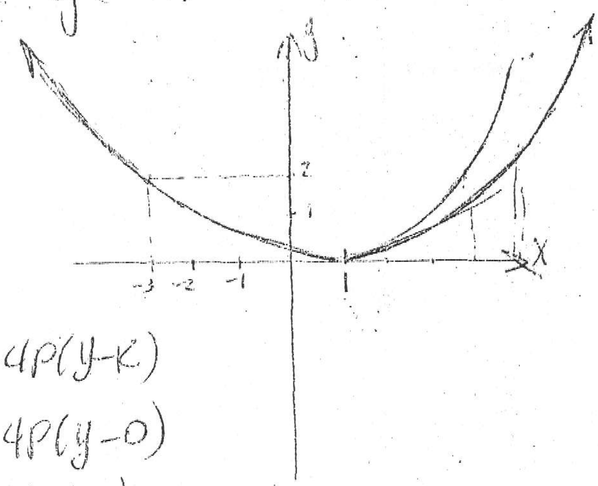
$$(x-2)^2=4(2)(y-1)$$

$$(x-2)^2=8(y-1)$$

$$x^2-4x+4=8y-8$$

$$x^2-4x-8y+12=0$$

38.- Parábola eje vertical



$V(1,0)$

$P:(-3,2)$

$(x-h)^2 = 4p(y-k)$

$(x-1)^2 = 4p(y-0)$

$(-3-1)^2 = 4p(2)$

$16 = 8p$

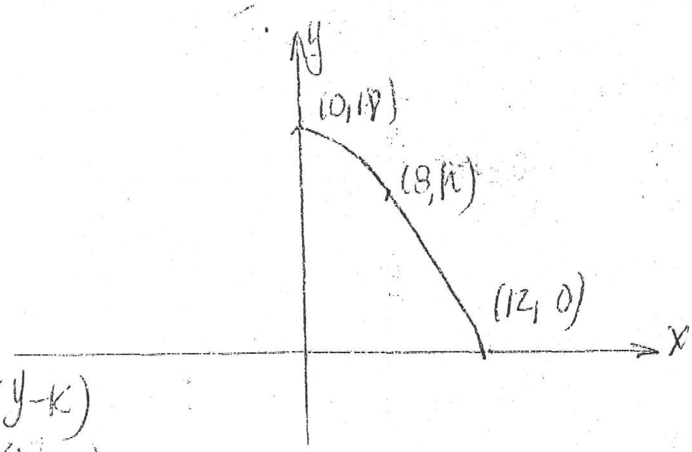
$\boxed{p=2}$

$(x-1)^2 = 4(2)y$

$x^2 - 2x + 1 = 8y$

$x^2 - 2x - 8y + 1 = 0$

39.-



$V:(0,18)$

$P:(12,0)$

$(x-h)^2 = 4p(y-k)$

$(x-0)^2 = 4p(y-18)$

$(12)^2 = 4p(0-18)$

$\cancel{144} = \cancel{4}p(-\cancel{18})$

$\cancel{36}$

2

$2 = -p$

$\boxed{p=-2}$

$x^2 = 4(-2)(y-18)$

$x^2 = -8y + 144$

$x^2 + 8y - 144 = 0$

$P(8,h)$

$8^2 + 8h - 144 = 0$

$64 + 8h - 144 = 0$

$8h = 80 \Rightarrow h = 10$

40.- F=¿?

$$2x^2-4x+y+4=0 \Rightarrow (x-h)^2=4p(y-k)$$

$$2x^2-4x=-y-4$$

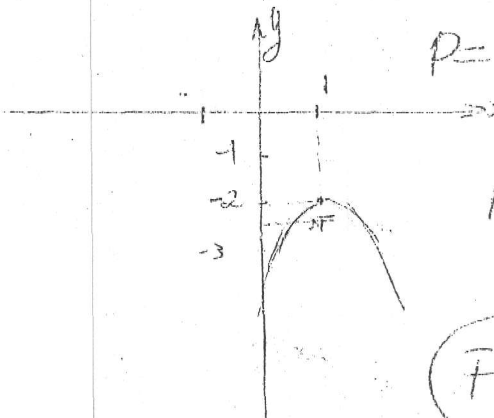
$$2(x^2-2x+1)=-y\underbrace{-4+2}_{-2}$$

$$2(x-1)^2=-(y+2)$$

$$(x-1)^2=-\frac{1}{2}(y+2)$$

$V(1,-2)$ $\qquad$ $4p=-\frac{1}{2}$

$p=-\frac{1}{8}$



$F \Rightarrow -2-p$

$\Rightarrow -2-\frac{1}{8} \Rightarrow \frac{-16-1}{8}$

$F(1, -17/8)$

41.- $C: x^2+y^2+2x-8y+9=0$

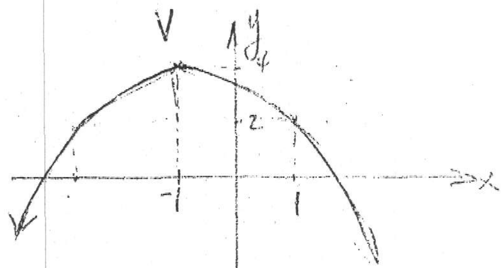
$$(x^2+2x+1)+(y^2-8y+16)=-9+1+16$$

$$(x+1)^2+(y-4)^2=8$$

$C(-1,4)$

$V(-1,4)$

$A(1,2)$



$(x-h)^2=4p(y-k)$

$(x+1)^2=4p(y-4)$

$(1+1)^2=4p(2-4)$

$\cancel{4}=\cancel{4}p(-2)$

$\boxed{p=-\frac{1}{2}}$

$F(-1, 4-\frac{1}{2})$

$F(-1, 7/2)$

c) CORRECTO

42- $y=-x-1$

a) $(x+2)^2+(y+1)^2=4$

$(x+2)^2+(-x\cancel{-1}\cancel{+1})^2=4$

$x^2+4x+\cancel{4}+x^2=\cancel{4}$

$2x^2+4x=0$

$2x(x+2)=0$

$2x=0 \qquad x+2=0$

$\boxed{x=0} \qquad \boxed{x=-2}$

$y=-x-1$

$P_1(0,-1)$; $P_2(-2,-3)$

$(x+2)^2+(y+1)^2=4$

$P_1(0,-1)$ ✓ válido

$P_2(-2,-3)$ ✓ válido

b) $(x-2)^2=y-1$

$(x-2)^2=-x-1-1$

$x^2-4x+4=-x-2$

$x^2-3x+6=0$

$x=\dfrac{3\pm\sqrt{3^2-4(1)(6)}}{2(1)}$

NO HAY INTERSECCIÓNES

c) $-(y+1)^2 = x$; $y = -x-1$

$-(-x\cancel{-1+1})^2 = x$

$-(-x)^2 - x = 0$

$-x^2 - x = 0$

$-x(x+1) = 0$

$\boxed{x=0}$; $x+1=0$

$\boxed{x=-1}$

$y = -x-1$

$P_1(0,-1)$; $P_2(-1,0)$

$-(y+1)^2 = x$

$P_1(0,-1)$ válido

$P_2(-1,0)$ válido

43.- $C:(-3,1)$

$R: 3$

$(x+3)^2+(y-1)^2=3^2$

$x^2+6x+\cancel{9}+y^2-2y+1=\cancel{9}$

$x^2+y^2+6x-2y+1=0$

$V(-3,1)$

$P(1,3)$

$(y-1)^2 = 4p(x+3)$

$(3-1)^2 = 4p(1+3)$

$4 = 4p(\cancel{4})$

$p = \frac{1}{4}$

$(y-1)^2 = \cancel{4}\left(\frac{1}{\cancel{4}}\right)(x+3)$

$y^2-2y+1 = x+3$

$y^2-2y-x-2=0$

44.- $V_1(0,2)$
$V_2(4,2)$ $\Big\}$ $2a=4$
$\boxed{a=2}$

Centro $(2,2)$

$2b=2$
$\boxed{b=1}$

$a^2=b^2+c^2$
$c^2=a^2-b^2$
$c^2=4-1$
$\boxed{c=\sqrt{3}}$

$$\frac{(x-2)^2}{4}+\frac{(y-2)^2}{1}=1$$

$$\frac{(x-2)^2+4(y-2)^2}{4}=1$$

$$x^2-4x+4+4(y^2-4x+4)=4$$

$$x^2-4x+4y^2-16x+16=0$$

$$x^2+4y^2-4x-16x+16=0$$

45.- Centro: $(2,3)$
$2b=4$
$\boxed{b=2}$
$a=\sqrt{13}$

$$\frac{(x-2)^2}{13}+\frac{(y-3)^2}{4}=1$$

a) $P(4,5)$ $\quad \frac{(4-2)^2}{13}+\frac{(5-3)^2}{4}\neq 1$

b) $P(3,0)$ $\quad \frac{(3-2)^2}{13}+\frac{(0-3)^2}{4}\neq 1$

c) $P(2,1)$ $\quad \frac{(2-2)^2}{13}+\frac{(1-3)^2}{4}=1$ CORRECTO

d) $(5,3)$ $\quad \frac{(5-2)^2}{13}+\frac{(3-3)^2}{4}\neq 1$

e) $(4,\sqrt{10})$ $\quad \frac{(4-2)^2}{13}+\frac{(\sqrt{10}-3)^2}{4}\neq 1$.

46.— $3x^2-4y^2+16y-18=0$

$3x^2+(-4y^2+16y-\quad)=18$

$3x^2-4(y^2-4y+4)=18-16$

$3x^2-4(y-2)^2=2$

$\frac{3x^2}{2}-\frac{\cancel{4}^{2}(y-2)^2}{\cancel{2}_{1}}=\frac{\cancel{2}^{1}}{\cancel{2}}$

$\frac{(x-0)^2}{2/3}-\frac{(y-2)^2}{1/2}=1$ HIPERBOLA

$C(0,2)$

b) CORRECTO

47.— $6y^2-4x^2+12y+16x-34=0$

$(6y^2+12y)-(4x^2-16x\quad)=34$

$6(y^2+2y+1)-4(x^2-4x+4)=34+6-16$

$6(y+1)^2-4(x-2)^2=24$

$\frac{\cancel{6}(y+1)^2}{\cancel{24}\ \cancel{6}\,3}-\frac{\cancel{4}^{1}(x-2)^2}{\cancel{24}\ 6}=\frac{\cancel{24}^{1}}{\cancel{24}}$

$\frac{(y+1)^2}{3}-\frac{(x-2)^2}{6}=1$

$a^2=3 \Rightarrow a=\sqrt{3}$

$b^2=6 \Rightarrow b=\sqrt{6}$

$c = \sqrt{a^2+b^2}$

$c = \sqrt{3+6}$

$\boxed{c = 3}$

centro $(2,-1)$

$e = \frac{c}{a} \Rightarrow \frac{3}{\sqrt{3}}$

$e = \sqrt{3}$

$V_1 = (2, -1+\sqrt{3})$

$V_2 = (2, -1-\sqrt{3})$

$F_1 = (2, 2)$

$F_2 = (2, -4)$

asintotas

$y - k = \pm \frac{a}{b}(x - h)$

$y + 1 = \pm \frac{3}{6}(x - 2)$

$y + 1 = + \frac{1}{2}(x - 2)$

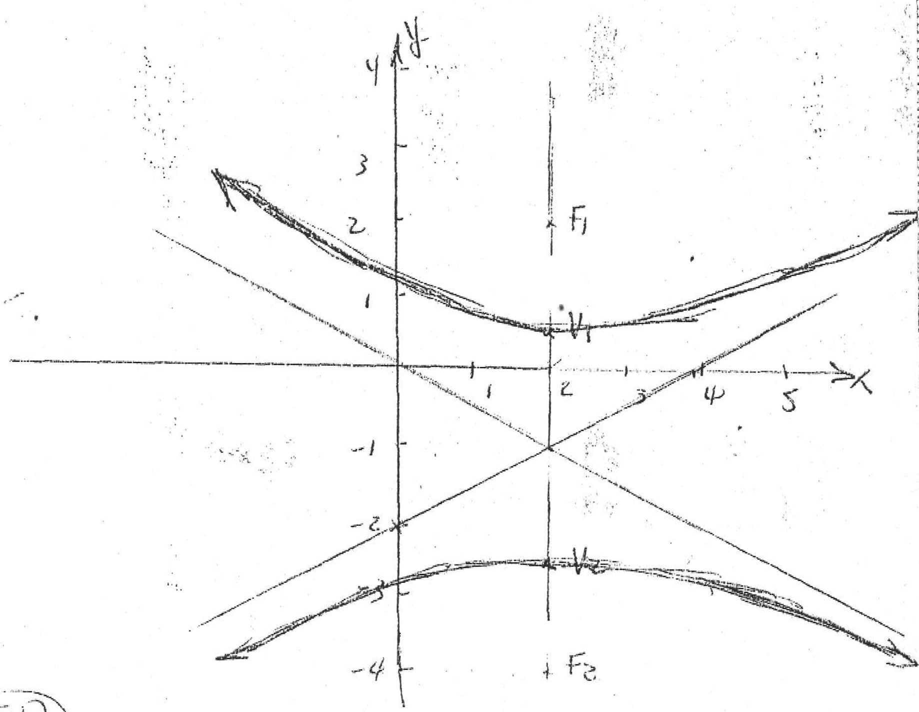
$y = \frac{x}{2} - 1 - 1$

$\boxed{y = \frac{x}{2} - 2}$ $x=0$, $y=-2$ $y=0$, $x=4$

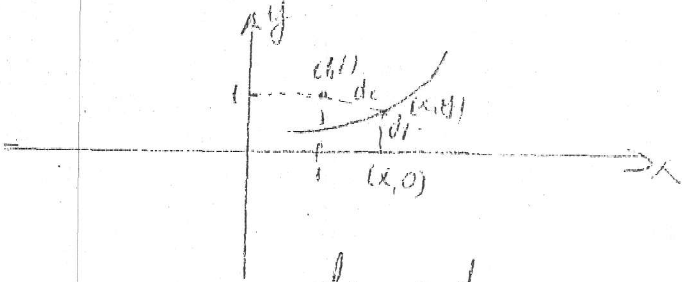
$y = -\frac{1}{2}(x-2) - 1$

$y = -\frac{x}{2} + \cancel{1} \cancel{-1}$

$\boxed{y = -\frac{x}{2}}$ $x=0$, $y=0$ $x=2$, $y=-1$



4 8.—



$$d_2 = 2\,d_1$$

$$\sqrt{(x-1)^2+(y-1)^2} = 2\sqrt{(x-x)^2+(y-0)^2}$$

$$\sqrt{(x-1)^2+(y-1)^2} = 2\sqrt{y^2}$$

$$\left(\sqrt{(x-1)^2+(y-1)^2}\right)^2 = (2y)^2$$

$$(x-1)^2+(y-1)^2 = 4y^2$$

$$x^2-2x+1+y^2-2y+1-4y^2=0$$

$$x^2-3y^2-2x-2y+2=0$$

$$3y^2+2y-x^2+2x=2$$

$$3\left(y^2+\frac{2}{3}y+\frac{1}{9}\right)-\left(x^2-2x+1\right)=2+\frac{1}{3}-1$$

$$3\left(y+\frac{1}{3}\right)^2-(x-1)^2=\frac{4}{3}$$

$$\frac{3\left(y+\frac{1}{3}\right)^2}{4/3}-\frac{(x-1)^2}{4/3}=\frac{\cancel{4/3}}{\cancel{4/3}}$$

$$\frac{\left(y+\frac{1}{3}\right)^2}{4/9}-\frac{(x-1)^2}{4/3}=1$$

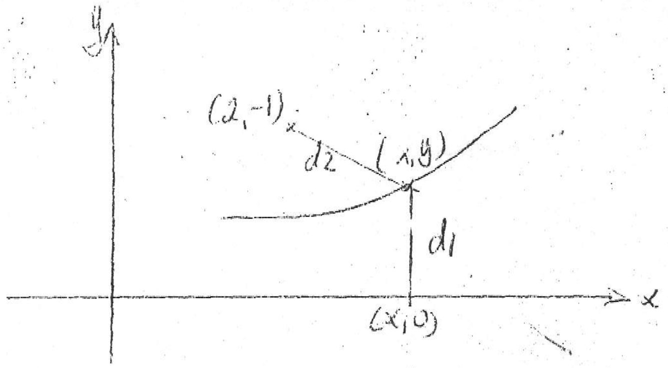
$\Downarrow$

hipérbola

centro$(1, -1/3)$

b) CORRECTO.

49.—



$$d_1 = 3d_2$$

$$\sqrt{(x-x)^2+(y-0)^2} = 3\sqrt{(x-2)^2+(y+1)^2}$$

$$\sqrt{y^2} = 3\sqrt{(x-2)^2+(y+1)^2}$$

$$(y)^2 = \left(3\sqrt{(x-2)^2+(y+1)^2}\right)^2$$

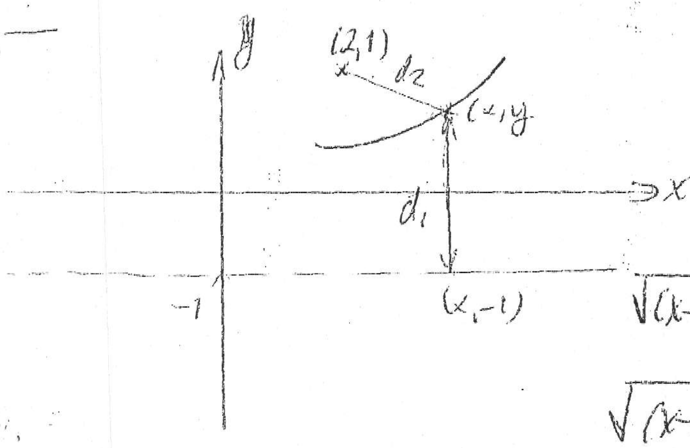
$$y^2 = 9\left[(x-2)^2+(y+1)^2\right]$$

$$y^2 = 9\left(x^2-4x+4+y^2+2y+1\right)$$

$$y^2 = 9x^2+9y^2-36x+18y+45$$

$$9x^2+8y^2-36x+18y+45=0$$

50.—



$$d_2 = 2d_1$$

$$\sqrt{(x-2)^2+(y-1)^2} = 2\sqrt{(y+1)^2+(x-x)^2}$$

$$\sqrt{(x-2)^2+(y-1)^2} = 2\sqrt{(y+1)^2}$$

$$\left(\sqrt{(x-2)^2+(y-1)^2}\right)^2 = \left(2(y+1)\right)^2$$

$$(x-2)^2+(y-1)^2 = 4(y+1)^2$$

$$x^2-4x+4+y^2-2y+1=4(y^2+2y+1)$$

$$x^2-4x+y^2-2y+5=4y^2+8y+4$$

$$x^2-3y^2-4x-10y+1=0$$